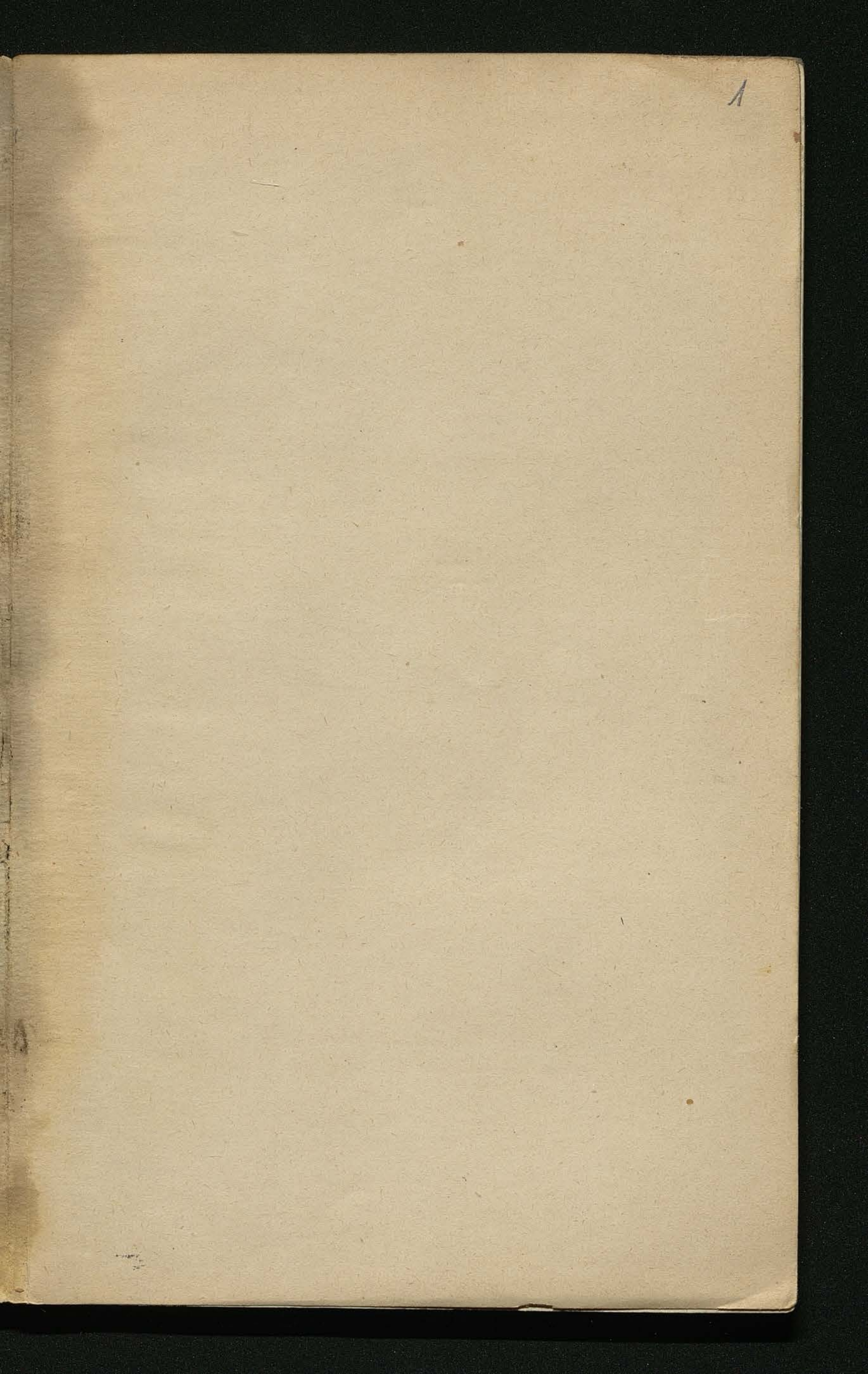
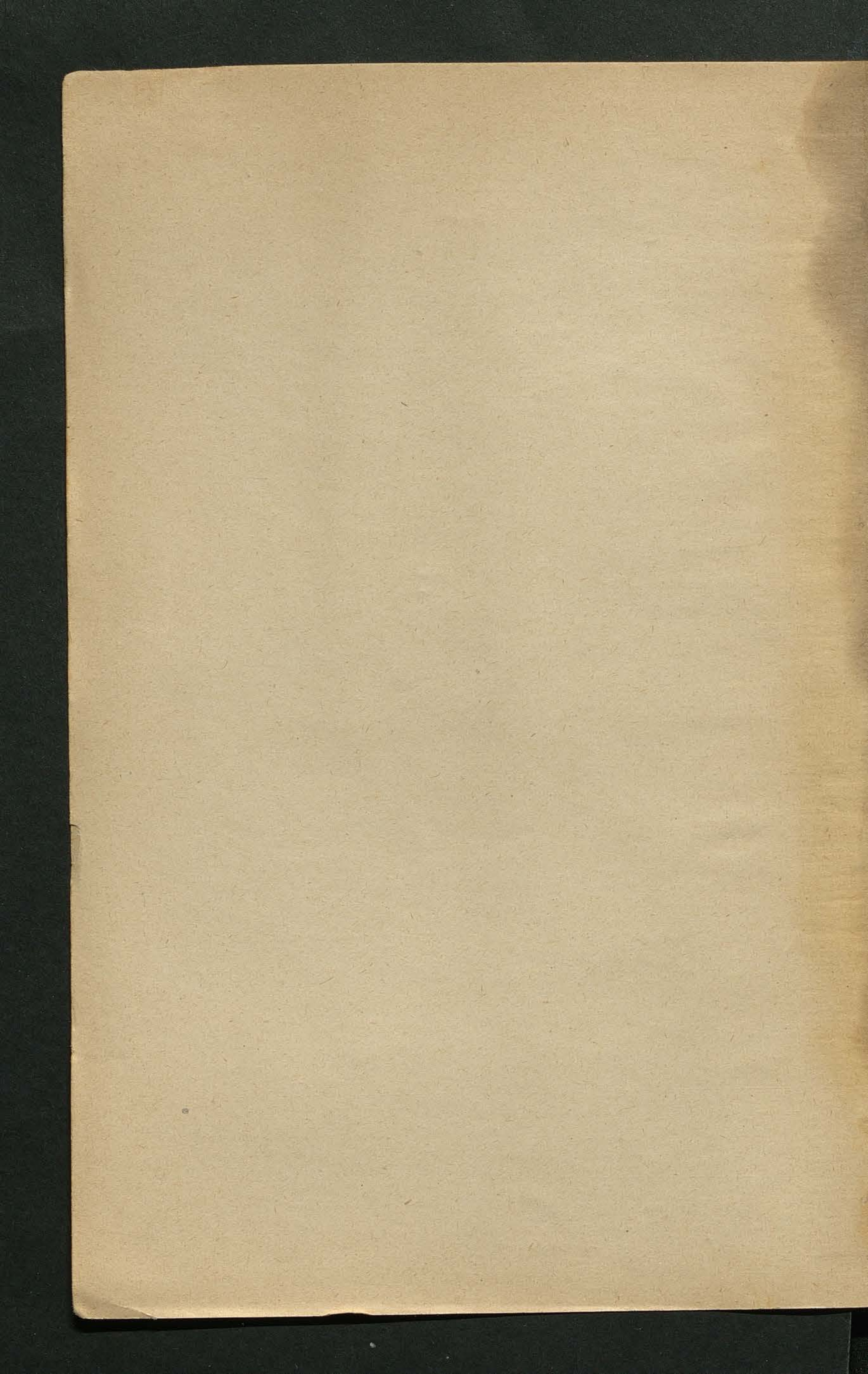
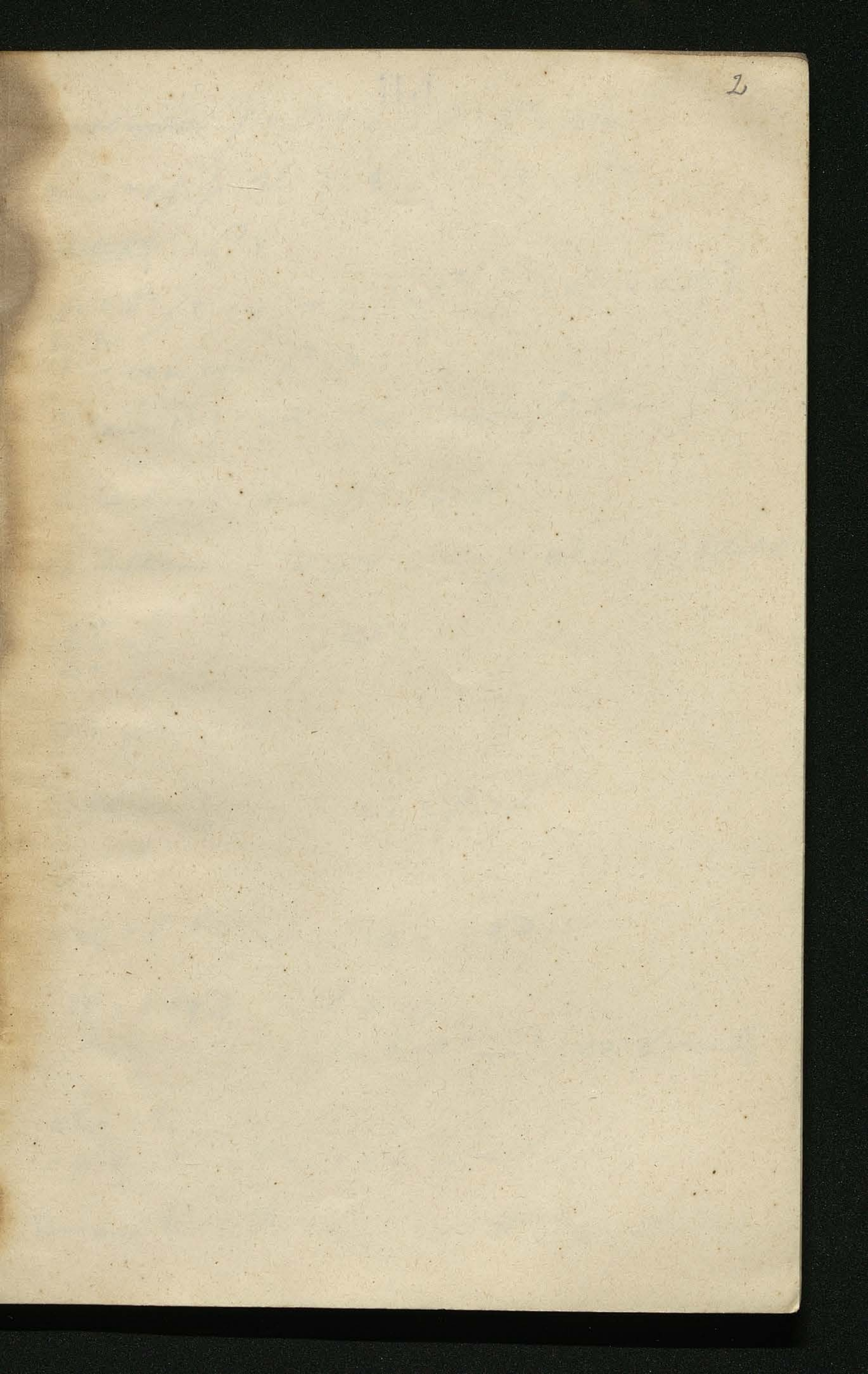
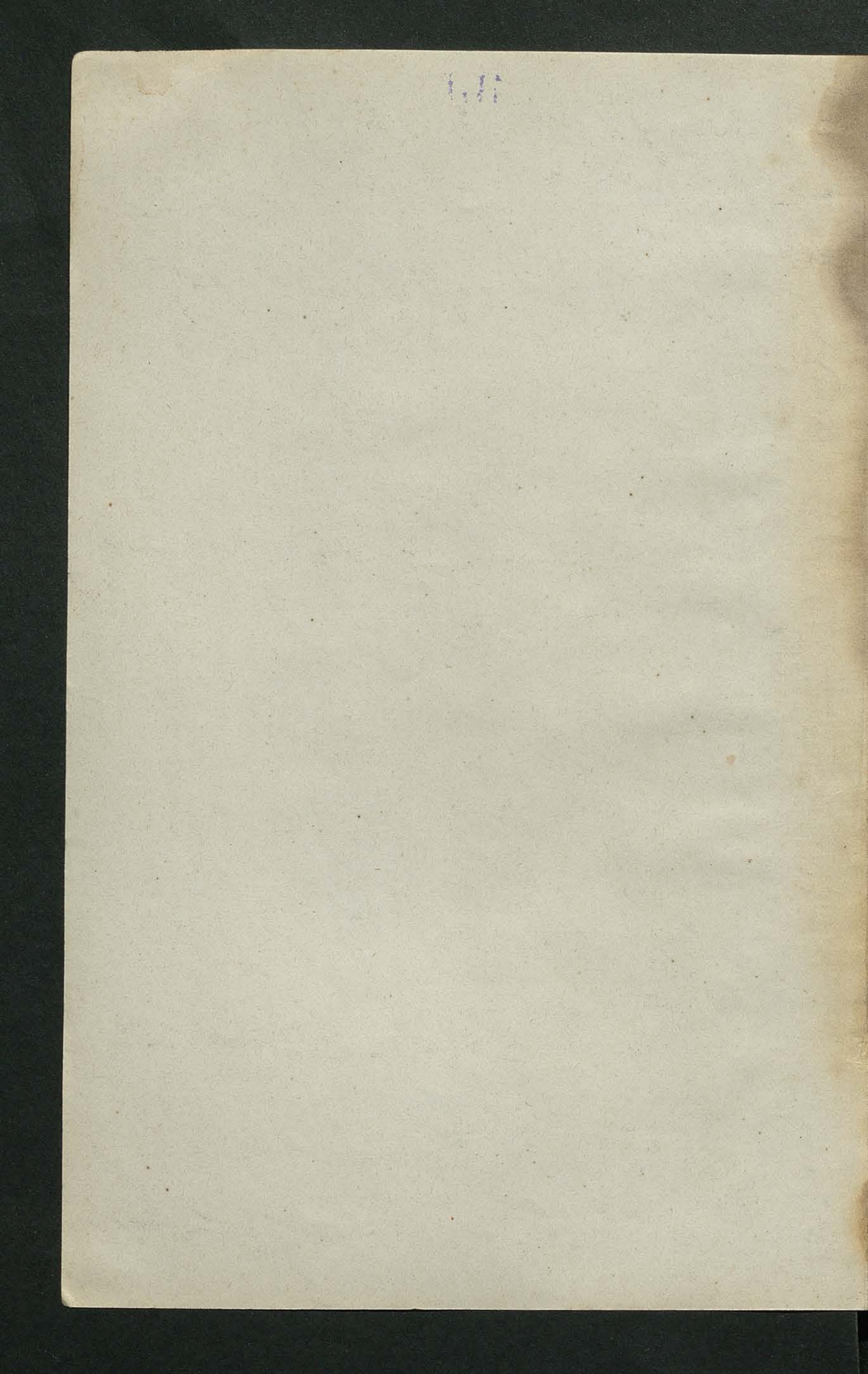










Lemonnier [illegible] Atm.

[illegible]

~~[illegible]~~ [illegible]

[illegible]

[illegible] norm [illegible]

F. Exner [illegible] norm [illegible] Atm [illegible]

1). [illegible]

2). Influenz [illegible] Potential

$$\frac{\Delta V}{\Delta s} = 1$$ [illegible] Pot. [illegible]

[illegible] Pot. [illegible]

$V + \Delta V$ [illegible]

V

[illegible] V [illegible] ; 1 D

$1V = 0{\cdot}89\,D$ = 10^8 [illegible]

[illegible] $V = 0$ [illegible]

[illegible] Pot. [illegible]

[illegible] V [illegible]

Ermann [illegible] Infl. [illegible]

[illegible] Peltier [illegible]

[illegible]

Dellmann, Palmieri, Secchi

V [illegible]

$$V_1 + \frac{e_1}{R} = \text{[illegible]} = 0.$$

[illegible]

$$V_2 + \frac{e_2}{R} = 0$$

[illegible] isolirt [illegible]

[illegible]

$e_2 - e_1$ [illegible] Elektrometer [illegible]

$$e_2 - e_1 = R.(V_1 - V_2)$$

[illegible]: [illegible]; [illegible],
[illegible], [illegible] Elektr., Lunten

Exner'sche Anordnung:

Exner's Elektrometer

Aluminium [illegible]

Aichung [illegible] gelbe Batterie

[illegible] 200V ; [illegible] Pot. [illegible]

[illegible] 4

Im Luftballon [illegible] Elektr. [illegible] ; [illegible] Elek.

[illegible]

[illegible]

[2 [illegible] Elektr.]

[illegible] 20m ([illegible] Ballon)

[illegible] Pot. [illegible]

[illegible] Pot. [illegible]

[illegible] N[illegible] [illegible]

~~Saussure~~ [illegible] Pot. [illegible]

Hof:

Pot. Fl.

Exner:

	[illegible]		[illegible]	
2m	20m	48		68 V.
	15m	17		32
	10m	7		11
	5m	2		5
	0	0		0

[illegible] Felswand:

40 m	[illegible]	0	35 m [illegible]	80 V.
30 m		0		80
25		0		70

[illegible] 100 m:

230 V.

200

150 [illegible] Niveau [illegible].

[illegible] Niveaufläche 0 V.

[illegible] Riedschon[?] [illegible].

/IV

[illegible]:

[illegible] Pol. [illegible] [[illegible]

[illegible]

[illegible]

Elster [illegible] Sertel [illegible] Wolfenbüttel

[illegible] 16 [illegible] 2 m

1'03 [illegible] Ried [illegible] 5 mark[?] [illegible]

[illegible] 50 –

[shorthand]:

Jänner	391 Volt pro 1m
Feber	339
März	294
April	138
Mai	110
Juni	102
Juli	123
August	121
September	121
October	188
November	260
December	470

absolute Min 81 [im Juni
Max 638 [- Dec

[shorthand] 2 Max u. 2 Min.
[shorthand]
[shorthand] Baro.
ob registr. [shorthand]
in Perpignan

	1	2	3	4	5	6	7	8	9	10	11	12
M	43	40 (44)	39	40	43	50	59	63	61	55	53	55
A.	54	53	54	58	62	69	72	71	66	60	53	48

Eintritt der t. M. [shorthand]:

	Morgen-Maximum	Abend Max	Früh Min	Nachm. Min.
[shorthand]	8 3/4h	7h	2 1/2	2h
[shorthand]	7 1/2	7 1/2h	3h	2h
[shorthand]	7 1/2	8h	3h	3h
[shorthand]	8h	6 3/4h	3h	12h

[shorthand] 13ten Max [shorthand]

[illegible] Vol. [illegible] ?

Wolfenbüttel [illegible] 220 V. pro 1m Elster Geitel

Wien
St. Gilgen
Lido bei Venedig } 140

Ceylon, Bombay 55 [illegible]

Sonnblick 1100 [illegible] Geitel [[illegible] Thomas]

Ägypten Pyramide 1100 Exner

Eiffelth. 10000 Vol. [illegible]

[illegible] Siemens [illegible] Cheops Pyr. [illegible]

Palmieri [illegible] 2 Vesuv [illegible]

[illegible] Thomson [illegible]

VI [illegible]

[illegible] Vol. [illegible]

[illegible]

[illegible]

W. Kohn [illegible] −1000 V.

[illegible] −50 V.

[illegible] Cl.

[illegible]

[illegible]

3°1 gr pro 1 cm³ — 5.31

5°2 — 92

7°8 — 48

Théorie

Volta . ——— Ermann et Peltier

[illegible]

[illegible]

Pouillet [illegible]

[illegible] condensat [illegible]

[illegible]

Ermann et Peltier [illegible] = Influence

[illegible]

[illegible]

7/2

$$V = \frac{Q}{D} \qquad D = R + h$$

$$V = \frac{Q}{R+h} \qquad \frac{\partial V}{\partial h} = -\frac{Q}{(R+h)^2} = -\frac{QR^2}{R^2(R+h)^2}$$

$$V_0 = \frac{Q}{R} \qquad\qquad = -\frac{V_0}{R}\left(\frac{R}{R+h}\right)^2$$

$\frac{\partial V}{\partial h} = -\frac{V_0}{R}$ [illegible] const [illegible]

[illegible]

R. 200 Volt per m = $-V_0$ (Elster's Geitel)

$$V_0 = 200 \,.\, 6400\,000$$

$$= -130,000\,000 \text{ Volt}$$

also [illegible] Peltier'sche [illegible]

Modification [illegible] Thomson

[illegible]

[illegible]

$$V = -\frac{Q}{R+h} + \frac{Q}{R_2} \text{ [illegible]}$$

$\frac{\partial V}{\partial h}$ [illegible]

[illegible]

Neuere Theorien

1. Edlund [illegible]

[illegible]

[illegible] [[illegible]]

[illegible]

2 [illegible] Pol [illegible]

Th[illegible] unipolare Induction

[illegible] Edlund [illegible]

[illegible] 0'023 Volt

Exner, Tatermach [illegible]

Schr[illegible]

---

4. Simplex sigillum veri F. Exner

[illegible] Peltier [illegible] 2

[illegible]

[illegible] Exner [illegible]

[illegible] 2.

[illegible] Hypothese

[illegible] electr. ; [illegible]

[illegible]

[illegible] Mascart:

[illegible] Elektr. [illegible]

Mascart [illegible]

indirekten [illegible]

Mascart [illegible]

[illegible]

Exner [illegible]

[illegible]

[illegible]

Mascart [illegible]

[illegible]

[illegible] Exner [illegible] Alkohol [illegible] Äther

Lecher [illegible]

Exner [illegible]:

[illegible]	[illegible]	
2·3	325	V pro 1 m
3·8	2~~1~~7	
4·4	197	
5·5	166	
6·8	116	
8·4	106	
9·5	87	
10·4	84	
11·4	74	
12·5	68	

$\frac{\partial V}{\partial h} = -4\pi\sigma$ [illegible] Conductors

[illegible] $\frac{\partial V}{\partial h} = -4\pi\sigma_0$

[illegible] $\sigma_0 - \sigma'$

[illegible]

[illegible]

[illegible] $\sigma_0 - \sigma'$ [illegible]

[illegible] σ' prop. [illegible] $\sigma' = l\,\frac{\partial V}{\partial h}$

[illegible]

$$\frac{\partial V}{\partial h} = -4\pi\sigma_0 + 4\pi\,k\,l\,\frac{\partial V}{\partial h}$$

$$\frac{\partial V}{\partial h}\left[\underbrace{1 - 4\pi k l}_{\text{const}}\right] = -4\pi\sigma_0$$

$$\frac{\partial V}{\partial h} = -\frac{4\pi\sigma_0}{1 - 4\pi k l}$$

[illegible]

$4\pi\sigma_0$ [illegible] $\left(\frac{\partial V}{\partial h}\right)_0$ [illegible] k [illegible]

$$\frac{\partial V}{\partial h} = \frac{\left(\frac{\partial V}{\partial h}\right)_0}{1 + 4\pi k l}$$

[illegible]

$l\,\sigma' = -$ [illegible]

[illegible]

28/5

$k = 1{\cdot}31$ $\qquad$ $\frac{\partial V}{\partial r} = \frac{A}{1+4/p_0}$

$A = 1330$

$\left(\frac{\partial V}{\partial r}\right)_0 = 1330$ $\qquad$ $\frac{\partial V}{\partial r} = -4\pi 6 = \frac{E}{r^2} = -\frac{V_0}{r}$

$V_0 = -8 \cdot 10^9$ Volt [illegible] E. [illegible]

[illegible] Elster [illegible] Geitel [illegible]

[illegible] 1·6 mm [illegible] $\qquad$ 4π6 V pro m $\qquad$ 420 [illegible] Dauer

10·4 " — $\qquad$ 99 $\qquad$ 89

[illegible]:

$A = 1550$ V. $\qquad$ [illegible]

$k = 1{\cdot}31$

[illegible] A [illegible]

[illegible] A [illegible] Monaten:

[illegible] November $\qquad A = 1170$ Dez 1930

Jänner 2640 $\qquad$ Febr–Apr 1180

[illegible]

Exner [illegible] 2.17^{6} [illegible] $\frac{s}{c}$

[illegible] $A = 1410$

$k = 1.5$

[illegible]	Volt [illegible]	berechnet
1.7 m	432	441
3.4	323	276
6.3	191	168
9.3	112	120
12.4	82	93
15.4	76	76
17.5	66	69
19.5	60	62
21.4	56	57
25.5	51	52

[illegible]

V_0 [illegible] $= -9 \cdot 10^{9}$ V.

Theorie von Sohncke

[illegible]

[illegible] – [Faraday]

[illegible]

[illegible] Isotherme [illegible] 0°

[illegible] Sohncke [illegible]

Isoth. [illegible]

[illegible]

[illegible]

[illegible] Isoth. [illegible]

[illegible]

[illegible] Theorie [illegible]

Theorie von Arrhenius

[illegible] ultravi. [illegible]

[illegible]

[illegible] ultravi. [illegible]

[illegible] [[illegible] ultravi. [illegible]]

[illegible shorthand]

[illegible shorthand]

[illegible shorthand] statisch [illegible]

1m²

1m

$\frac{\partial V}{\partial n}$

1m²

$\left(\frac{\partial V}{\partial n}\right)_0$

$$\frac{\partial V}{\partial n} - \left(\frac{\partial V}{\partial n}\right)_0 = -4\pi m$$

[illegible shorthand]

[illegible shorthand] Niveau [illegible]

[illegible shorthand]

$$\frac{\partial V}{\partial h} = \left(\frac{\partial V}{\partial h}\right)_0 - 4\pi m$$

[illegible shorthand] $m = \rho h$ [illegible]

$$\left(\frac{\partial V}{\partial h}\right)_0 = -4\pi\sigma$$

↑ [illegible shorthand] ♁

$$\frac{\partial V}{\partial h} = -4\pi\sigma - 4\pi\rho h$$

← [illegible shorthand] $\int \rho\, dh$

[illegible shorthand]

[illegible shorthand] E

[illegible shorthand]

[illegible shorthand]

4/6

$$U = \int \frac{\sigma\, d0}{u}$$

[illegible] U_a [illegible] U_0

[illegible] U_i

$V = U + W$ W = Pot. [illegible] Atmosph. [illegible] kosm.

[illegible] Elstatik

$$\frac{U_a}{a} + 2\left(\frac{\partial U_i}{\partial r}\right)_{r=a} = 4\pi\sigma$$

[illegible] V const [illegible]

$$U_i + W_i = \text{const} = k$$

$$U_a + W_a = k$$

$$U_0 + W_0 = k$$

$$U_a = W_0 + U_0 - W_a$$

$$\frac{\partial U_i}{\partial r} = -\frac{\partial W_i}{\partial r}$$

$$\frac{W_0 - W_a + U_0}{a} - 2\left(\frac{\partial W_i}{\partial r}\right)_{r=a} = 4\pi\sigma$$

$$U_0 = \frac{1}{a}\int \sigma\, d0 = \frac{E}{a}$$

$$4\pi\sigma = \frac{E}{a^2} + \frac{W_0 - W_a}{a} - 2\,\frac{\partial W_e}{\partial r}$$

$$\frac{\partial V}{\partial h} = -\frac{E}{a^2} + \frac{W_a - W_0}{a} + 2\cdot\frac{\partial W_0}{\partial r} - 4\pi\int\rho\,dh$$

[shorthand]

[shorthand]

[shorthand]

$$\frac{\partial V}{\partial h} = C \ldots - 4\pi\rho h$$ [shorthand]

Exner [shorthand] 60 m [shorthand]

[shorthand] $4\pi\rho h$ [shorthand]

[shorthand] Baltimore [shorthand]

Lecher:

[shorthand]	93 V pro 1 m
440 m	193
550 m	193
660 m	193

[shorthand]

Tuma

410 m	370 V
500	406
750	434
820	480
1000	490
1120	508
1300	554
1900	647

[shorthand] 636 [shorthand]

[shorthand] $4\pi\rho h$ + [shorthand]

[illegible] 277 [illegible] 1000 m

[illegible] 0'18 V. pro 1m. [illegible] 60m [illegible] 11 Volt

[illegible] 400 [illegible] [5%]

Pasch [illegible]

[illegible]

Börnstein [illegible]

Andrée [illegible]

[illegible]

Elster u Geitel

Peter Lechner am Sonnblick

6h ~~8h~~ 10h 12h 2h 4h 6h 8h 10h

[illegible] E. u. G. [illegible] Wolfenbüttel [illegible]

[illegible]

Jährl. Gang

	Okt.	N.	D.	J.	F.	M.	Ap.	Mai	Juni
Wolf.	0'85	1'17	<u>2'12</u>	1'77	1'53	1'33	0'62	0'50	<u>0'46</u>
Sonnbl.	0'87	0'84	1'05	0'98	1'13	1'00	1'06	1'07	1'01

[illegible] constant

[illegible]

$$\frac{\partial V}{\partial h} = \left(\frac{\partial V}{\partial h}\right)_0 - 4\pi\rho h$$

constant ↑

[illegible] $\left(\frac{\partial V}{\partial h}\right)_0$ [illegible]

[illegible] ρ − [illegible]

[illegible] $\left(\frac{\partial V}{\partial h}\right)_0$ [illegible] ρ + [illegible]

$h=0$

$$\left(\frac{\partial V}{\partial h}\right)_{h=0} = -\frac{E}{a^2} + \frac{W_a - W_0}{a} + 2\frac{\partial W_a}{\partial r}$$

[illegible] E [illegible]

[illegible]

[illegible] constant;

[illegible]

[illegible]

[illegible]

/6

$$\left(\frac{\partial V}{\partial \lambda}\right)_{\rho_0} = -\frac{E}{A^2} + \frac{W_e - W_0}{A} + 2\left(\frac{\partial W}{\partial \lambda}\right)_{r=A}$$

[illegible] E [illegible] I [illegible]

II [illegible] NSS [illegible]

[illegible]

[illegible] c [illegible]

Newcastle

C

a b

φ

θ

$W_0 - CA$

W_0

$\{\varphi, \theta\}$

$W_0 - CA + cp$ ← Proj. [illegible] $W_0 - CA$

$p = \overline{bc} - \text{Proj. } \overline{ab} \text{ ; } \overline{bc}$

$ab = e \sin\varphi$

[illegible] $= e \sin\varphi \cos\theta$

[illegible] bc [illegible] $x \cos\delta$

$= e \sin\varphi \cos\theta \cos\delta$

$\overline{bc} = a - e \sin\varphi \sin\delta$

$p = a - e \sin\varphi \sin\delta - e \sin\varphi \cos\delta \cos\theta$

$$W = W_0 - c\,a\left[\sin\varphi \sin\delta + \cos\varphi \cos\delta \cos t\right]$$

[illegible] $\odot$ $\quad$ $\underbrace{\cos z = \sin h}$

Zenithdist. $\quad$ $\odot$ [illegible]

[illegible]

$$W_a = W_0 - c a\,[\;.\;]$$

$$\frac{W_a - W_0}{a} = -c\left[\sin\varphi \text{———}\right]$$

$$\frac{\partial W}{\partial r}\,dr = -c\left[\sin\varphi \text{———}\right]$$

$$\frac{W_a - W_0}{a} + 2\left(\frac{\partial W}{\partial r}\right)_a = -3c\left[\sin\varphi\sin\delta + \cos\varphi\cos\delta\cos t\right]$$

$$\left(\frac{\partial V}{\partial h}\right)_\rho = -\frac{E}{A^2} - 3c\left[\text{———}\right]$$

[illegible]

[illegible] $\qquad \theta = 0$

$$\left(\frac{\partial V}{\partial h}\right)_0 = -\frac{E}{A^2} - 3c\,\cos(\varphi - \delta)$$

[illegible] $\varphi - \varepsilon$ [illegible] $\qquad \varphi + \varepsilon$

[illegible]

[illegible]

[illegible] $c > 0$ [illegible] $\odot$ [illegible] $\left(\frac{\partial V}{\partial h}\right)$ [illegible]

[illegible]

[illegible] ?

[illegible] $-3c[\,\ldots\cos\delta\cos\theta]$

[illegible] $\varphi = 0$ [illegible] $\delta = 0$

[illegible] $-3c[\cos\theta]$

[illegible] $-3c$

MN $+3c$

[illegible] Ampl. $6c$

[illegible]

[illegible] ♂ ♂ ☿

[illegible] ♀ ♀ [illegible]

[illegible]

[illegible]

$$\frac{W_a - W_0}{2a} + 2\,\frac{dW}{\ldots} = 0$$

[illegible] ?

[illegible]

A

[illegible]

[illegible]

[illegible]

[illegible shorthand] A

$$W = \int \frac{e \cos\varphi \, d\theta \, \sigma}{u} \qquad u = \sqrt{n^2+m^2+a^2\cos^2\varphi - 2am\cos\varphi\cos\theta}$$

$$\int \frac{\sigma a \cos\varphi \; d\theta}{\sqrt{n^2+m^2+a^2\cos^2\varphi}\,\sqrt{1+p\cos\theta}} \qquad p = \frac{-2am\cos\varphi}{n^2+m^2+a^2\cos^2\varphi} \quad (p<1)$$

$$W = \frac{\sigma a \cos\varphi}{\sqrt{n^2+m^2+a^2\cos^2\varphi}} \int_0^{2\pi} \frac{d\theta}{\sqrt{1+p\cos\theta}}$$

$$W = \frac{\sigma a \cos\varphi}{\sqrt{}}\, 2\pi \left\{1 + k_1 p^2 + k_2 p^4 + \cdots\right\}$$

[illegible shorthand] σ [illegible shorthand]

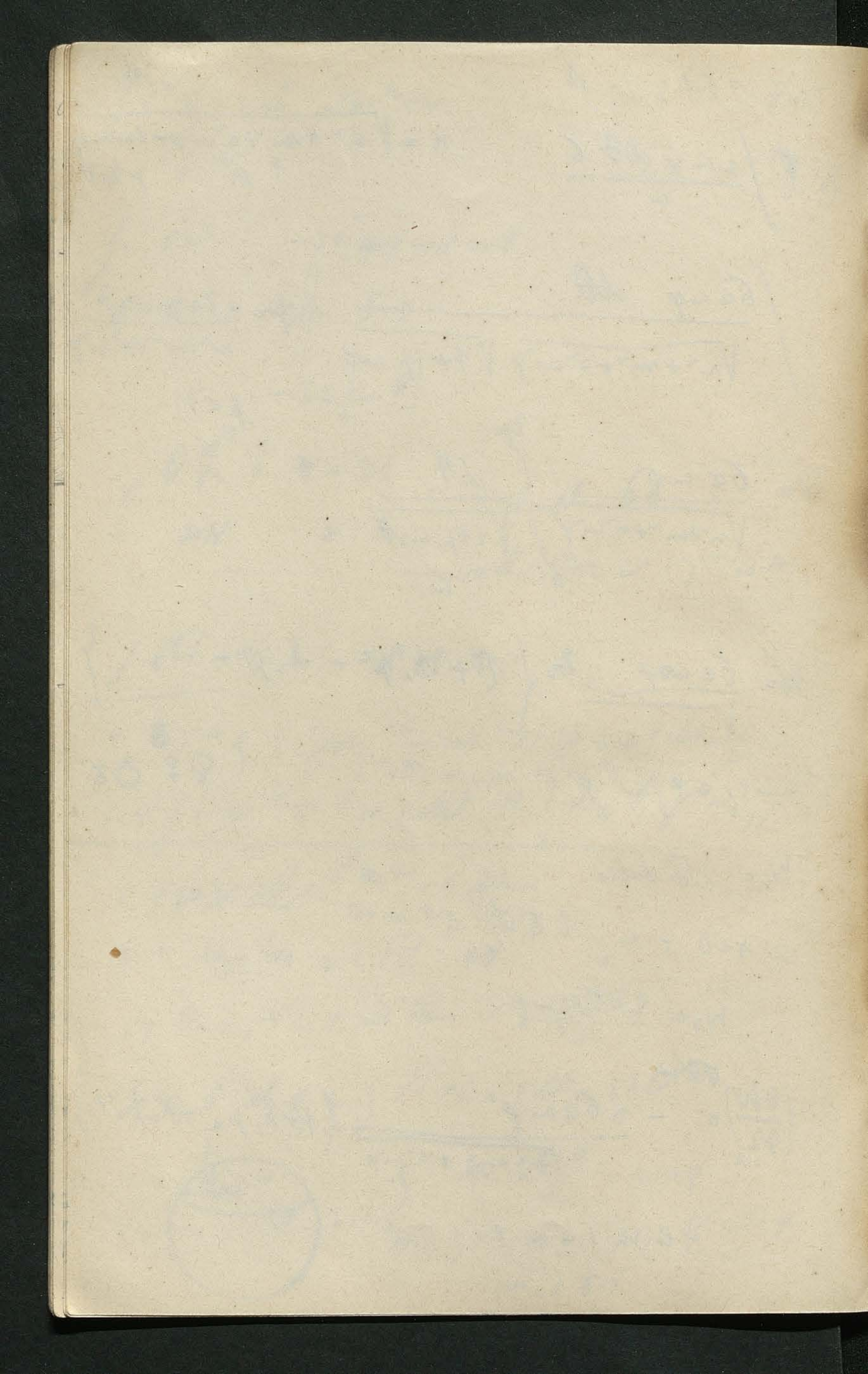
$W_a =$ – Indices

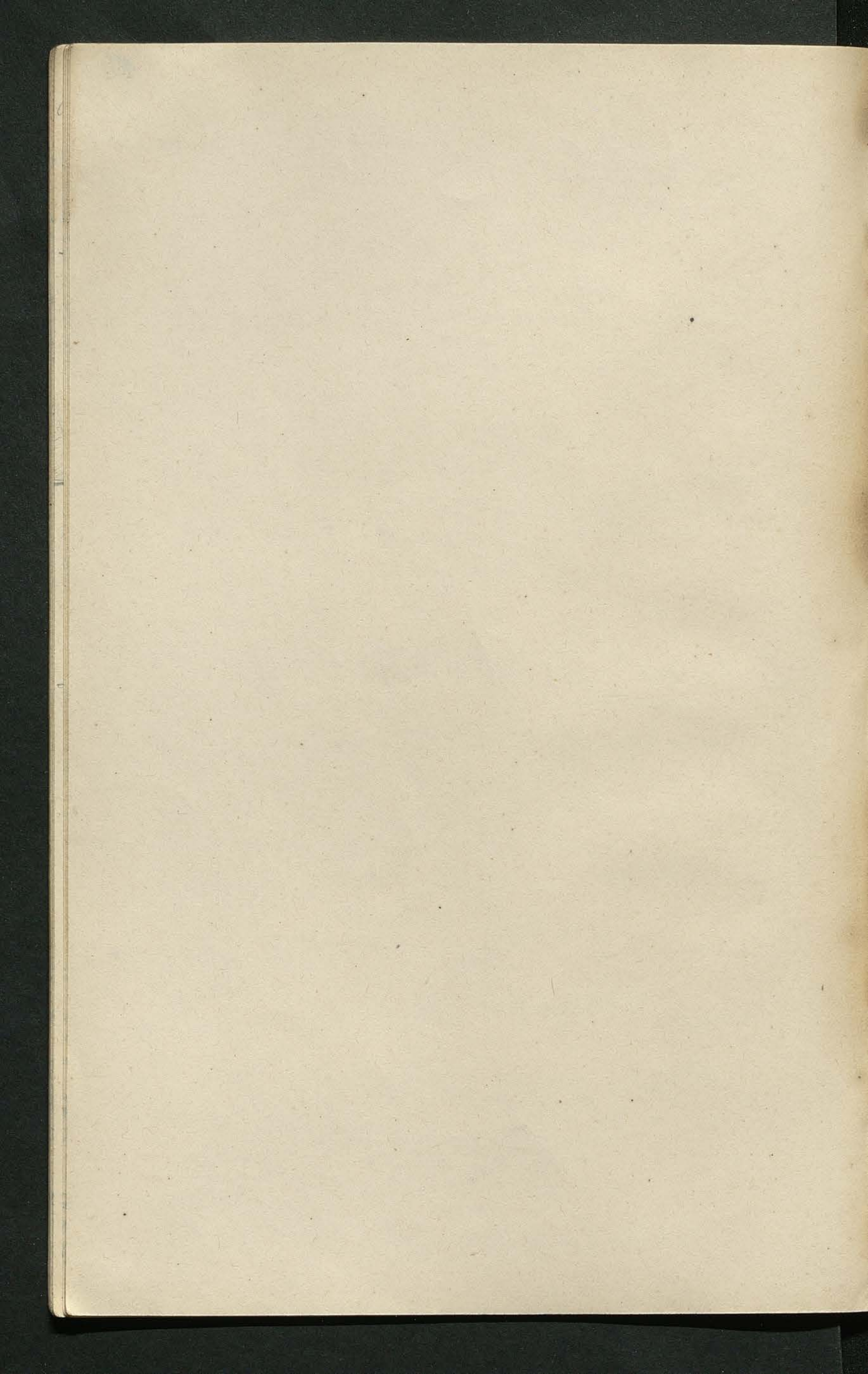
[illegible shorthand] $r=0$ [illegible shorthand] σ $m=0$ [illegible shorthand]

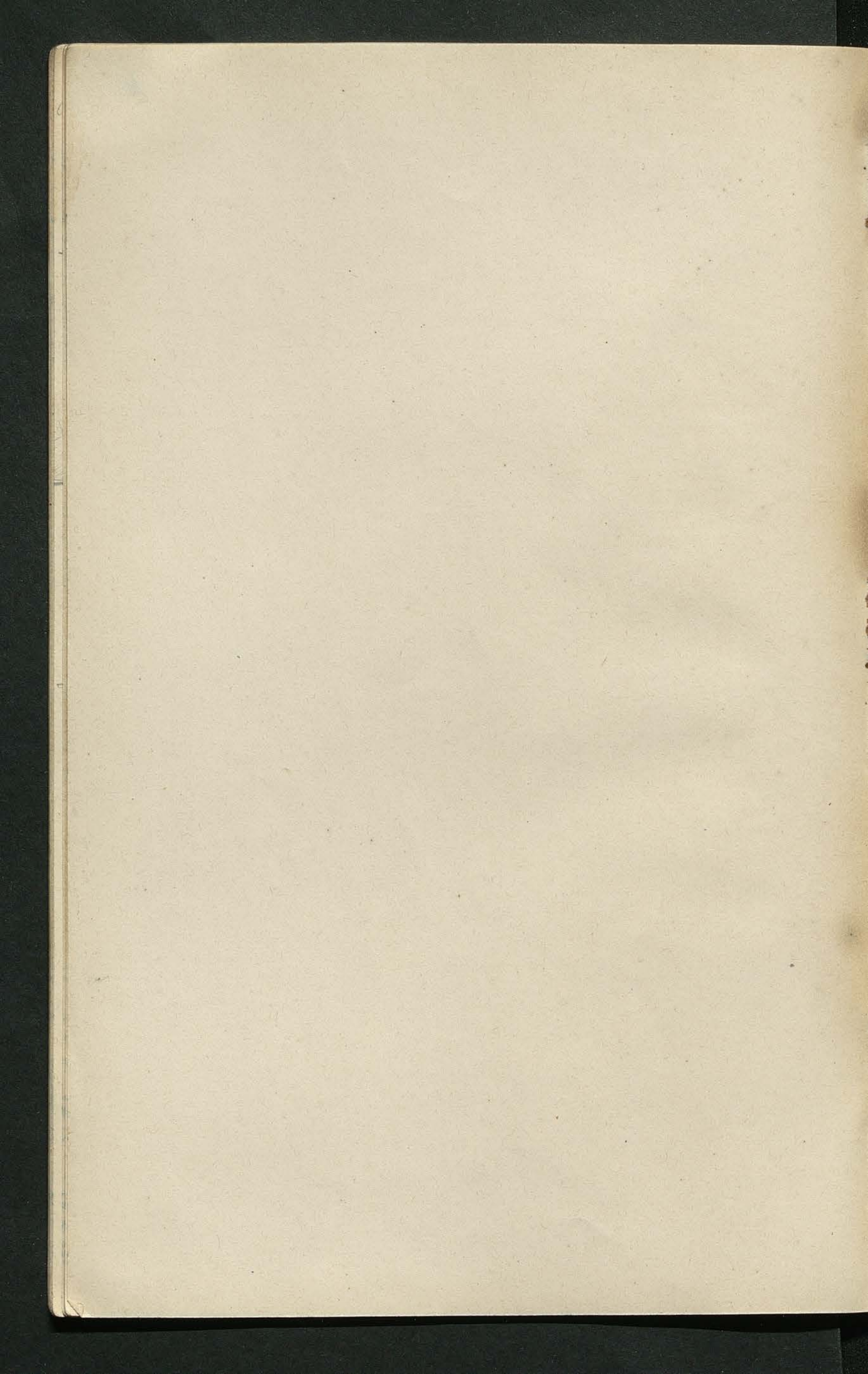
$$W_0 = \frac{2\pi\sigma a \cos\varphi}{a}$$

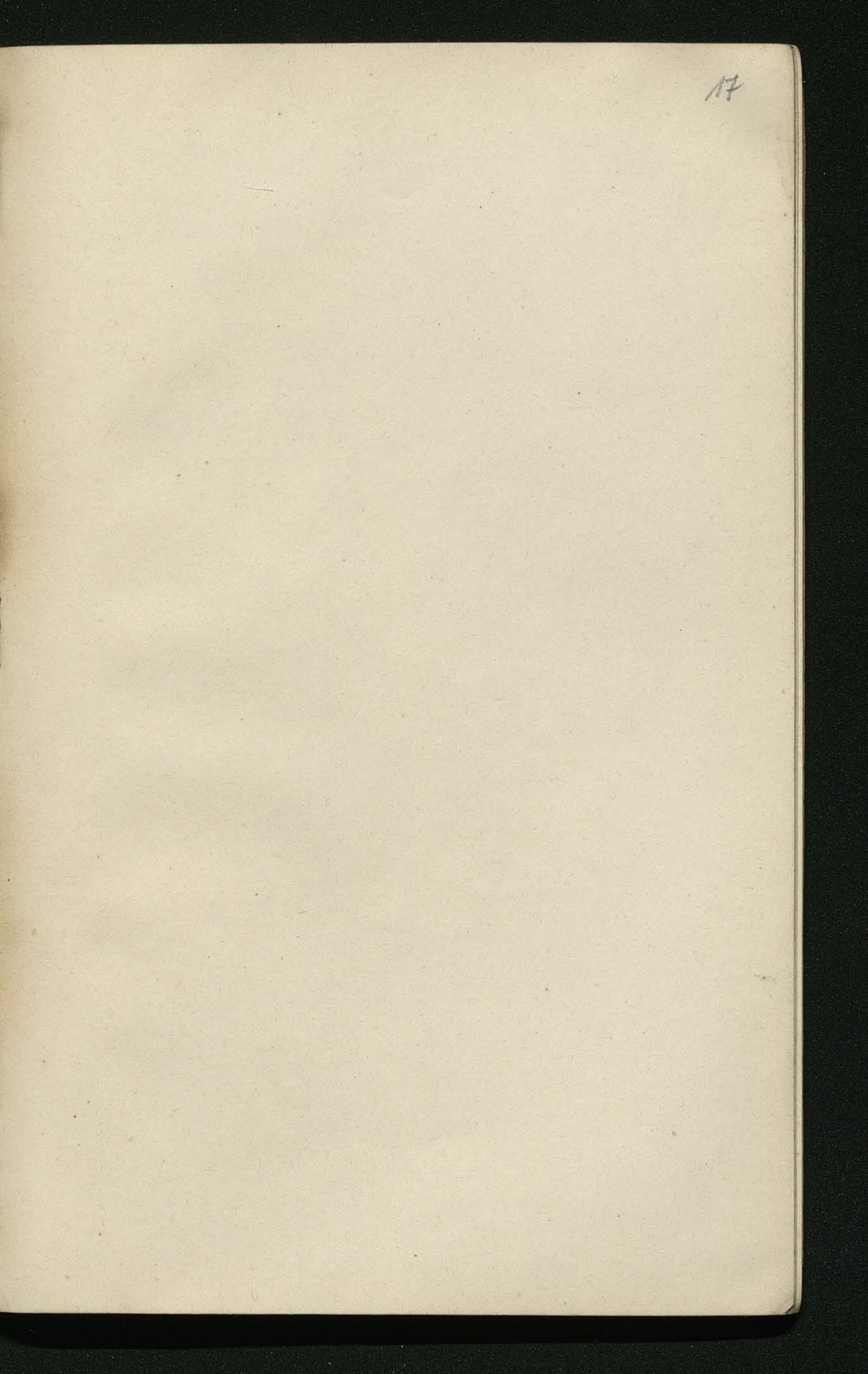
$$\left(\frac{\partial W}{\partial r}\right)_{r=a} = -\frac{\pi\sigma a\cos\varphi}{a\sqrt{n_a^2+m_a^2+a^2\cos^2\varphi}}\left\{1+k_1 p_a^2+k_2 p_a^4+\cdots\right\}$$

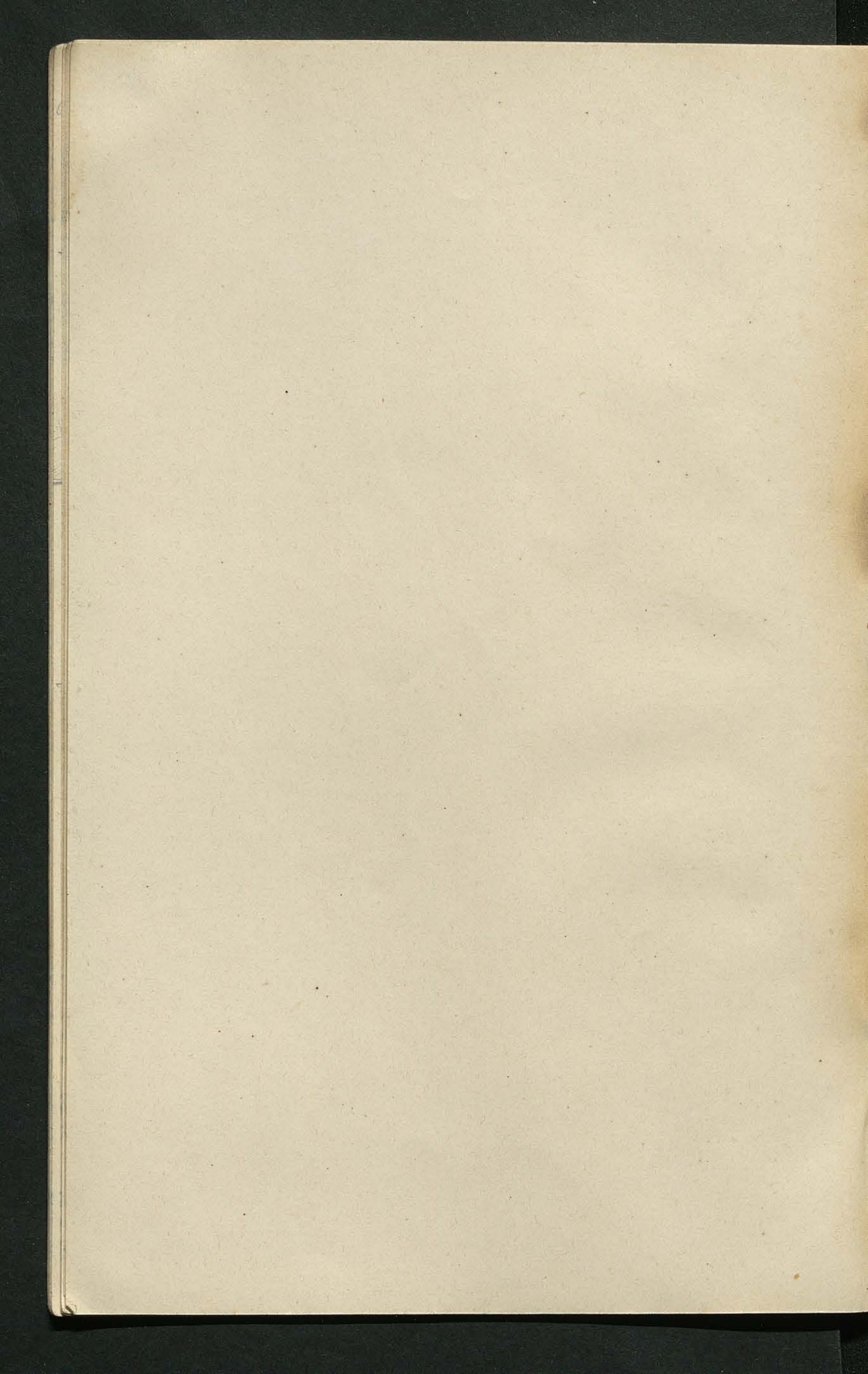
[illegible shorthand]

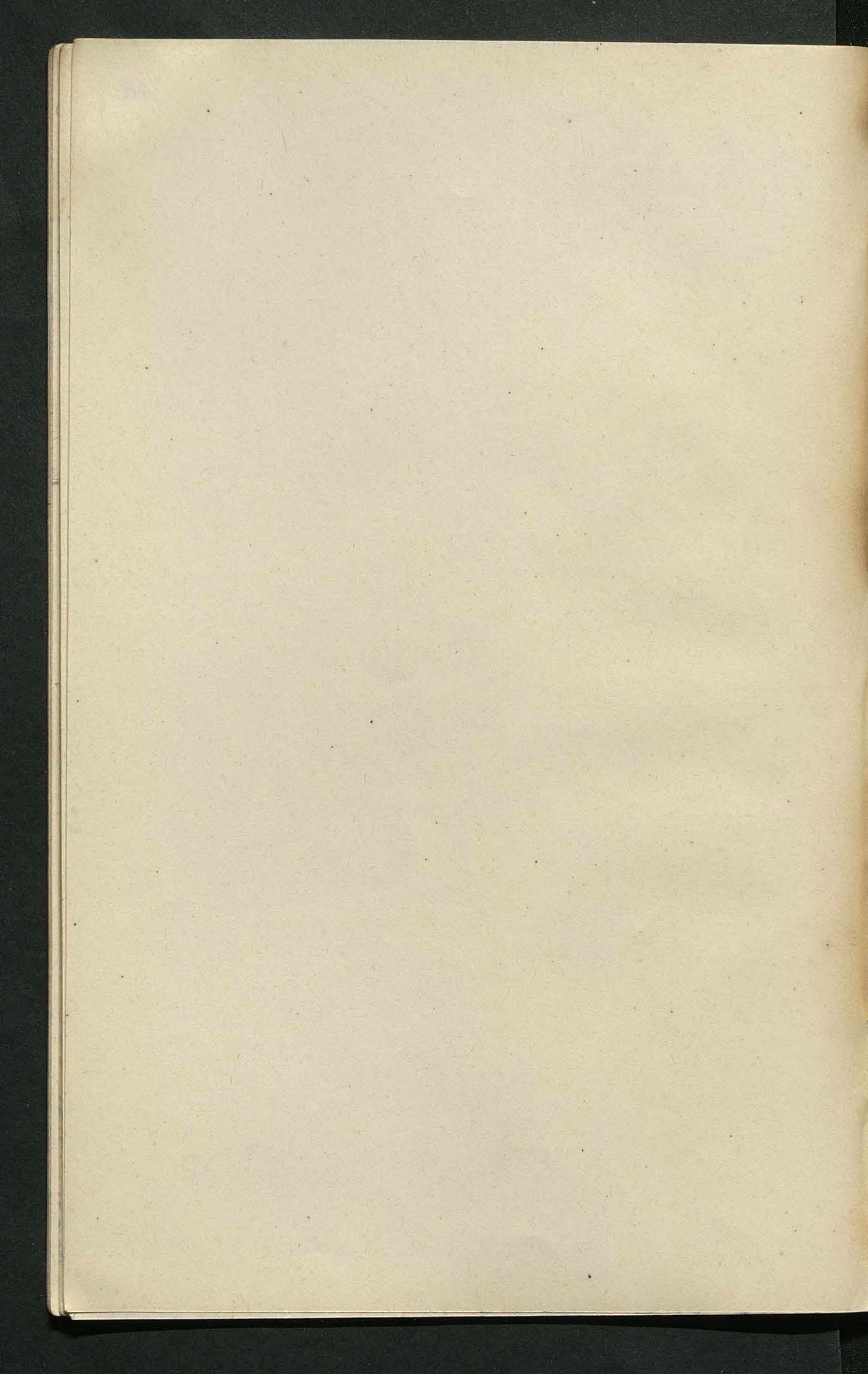


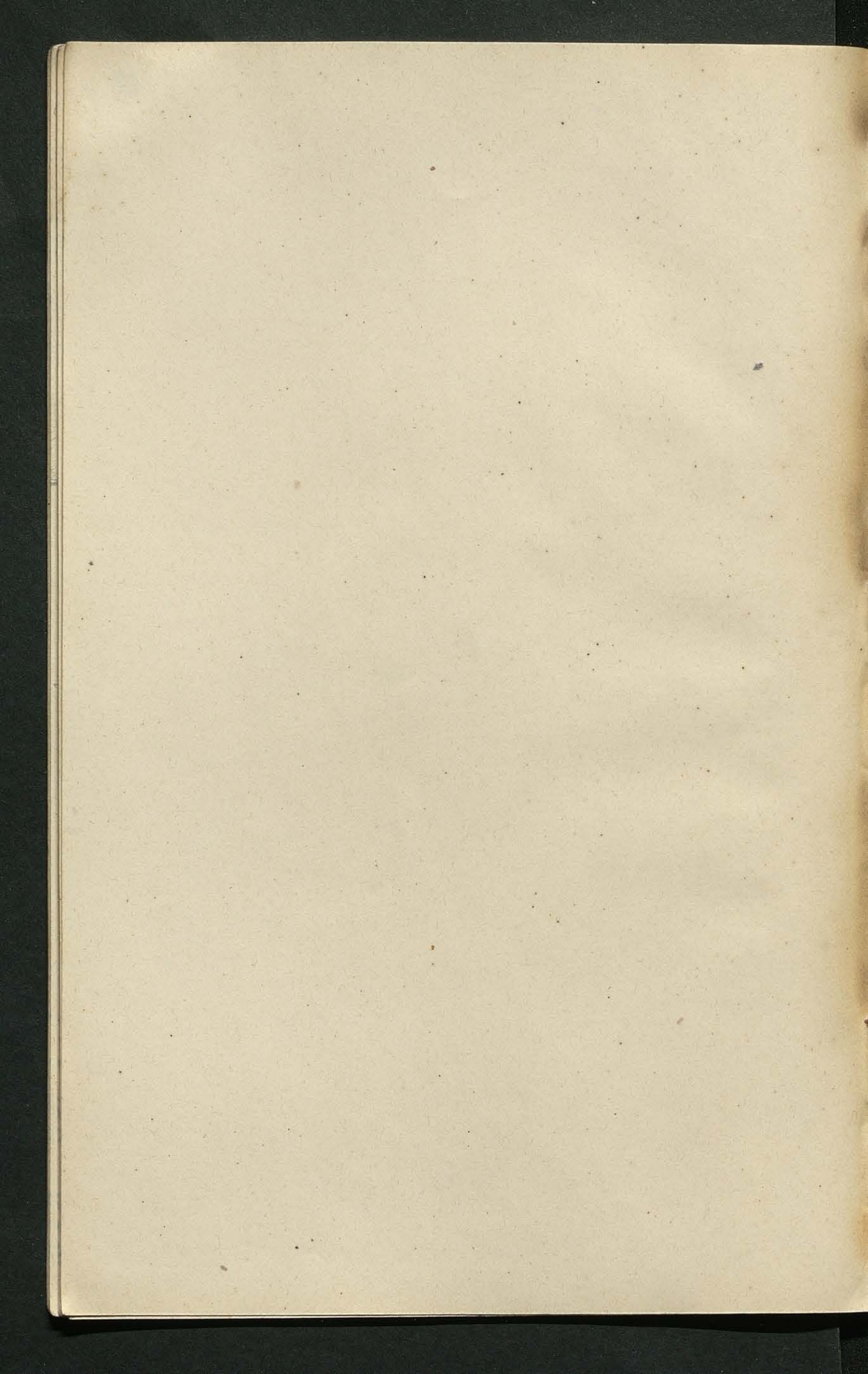


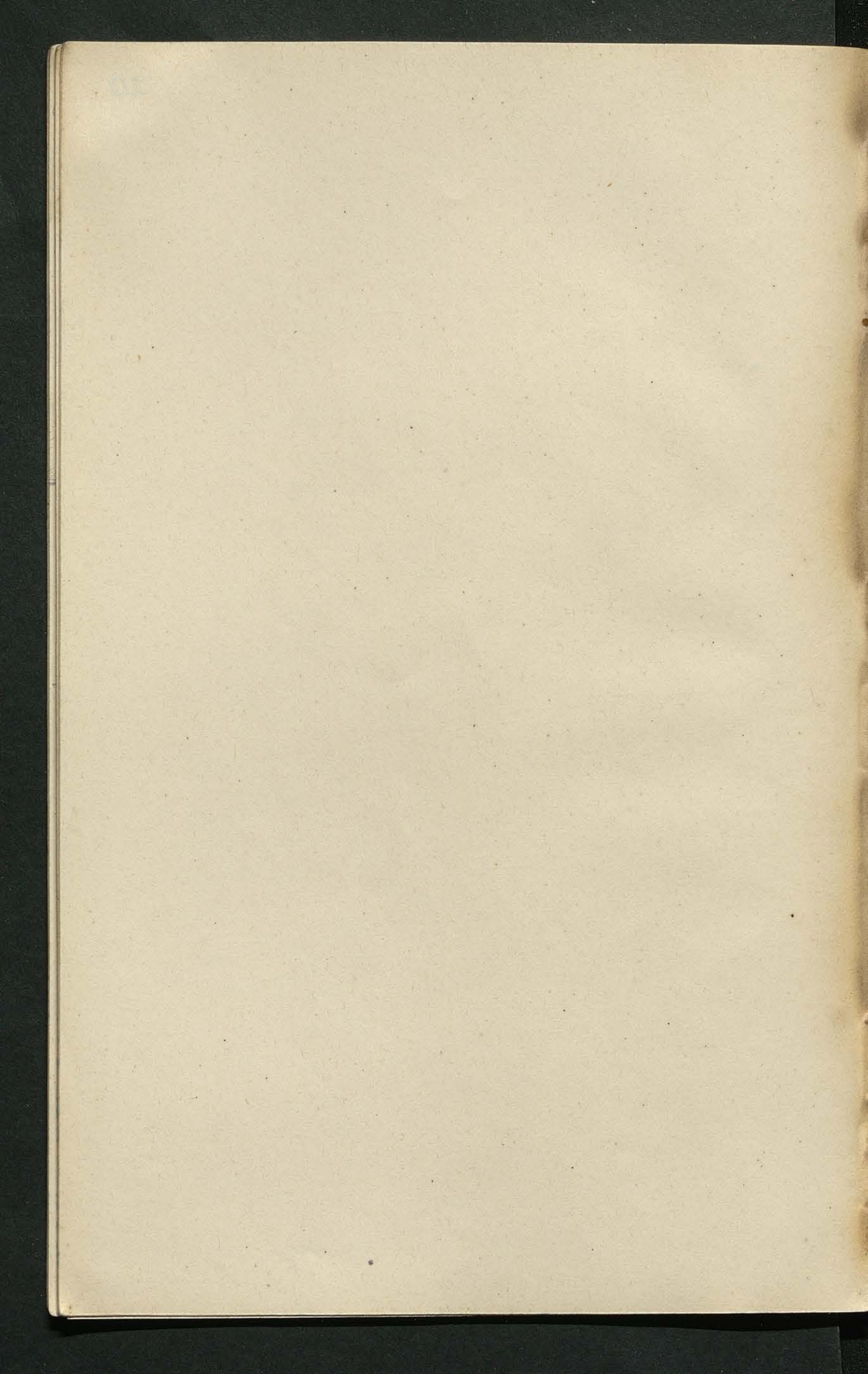


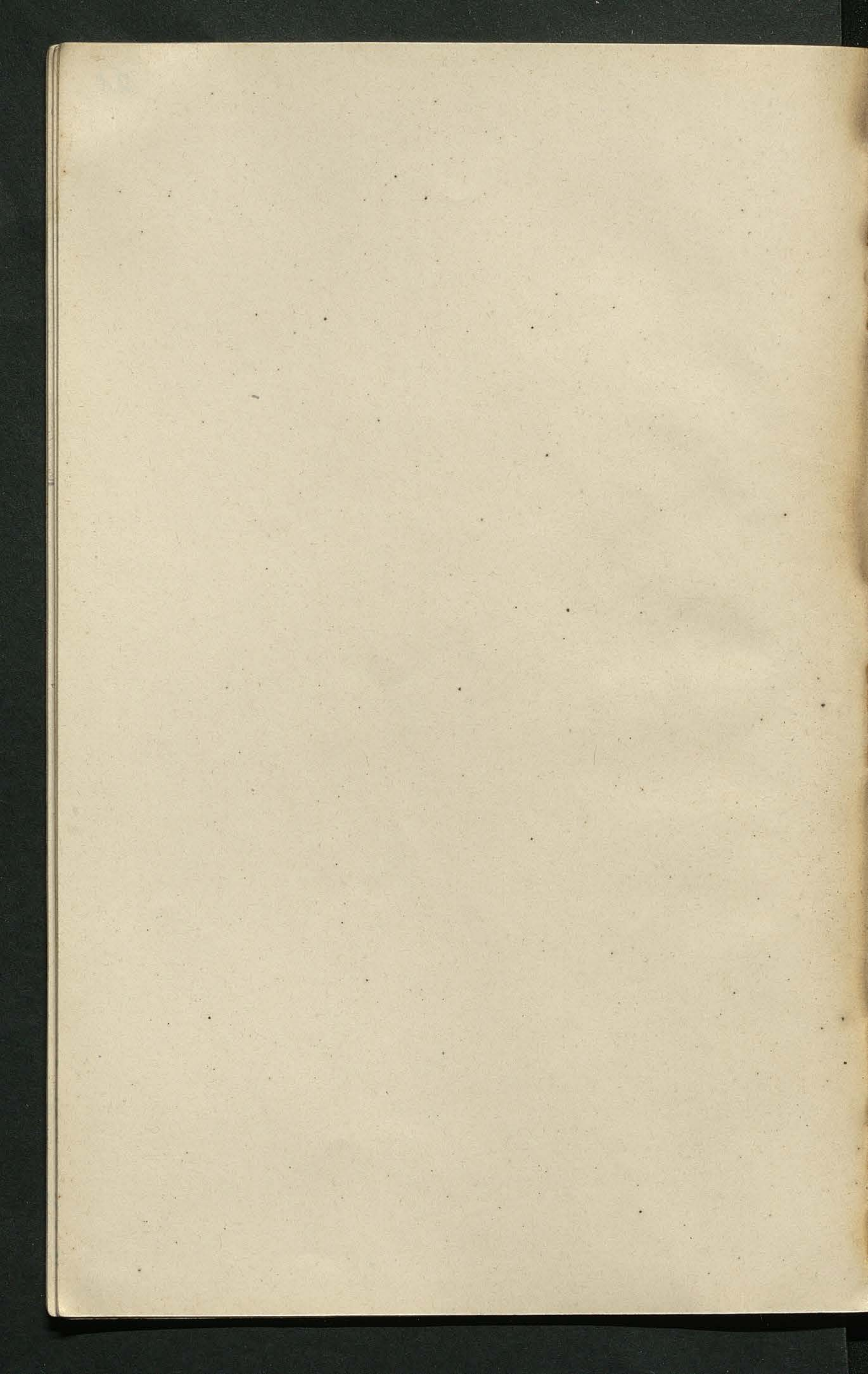


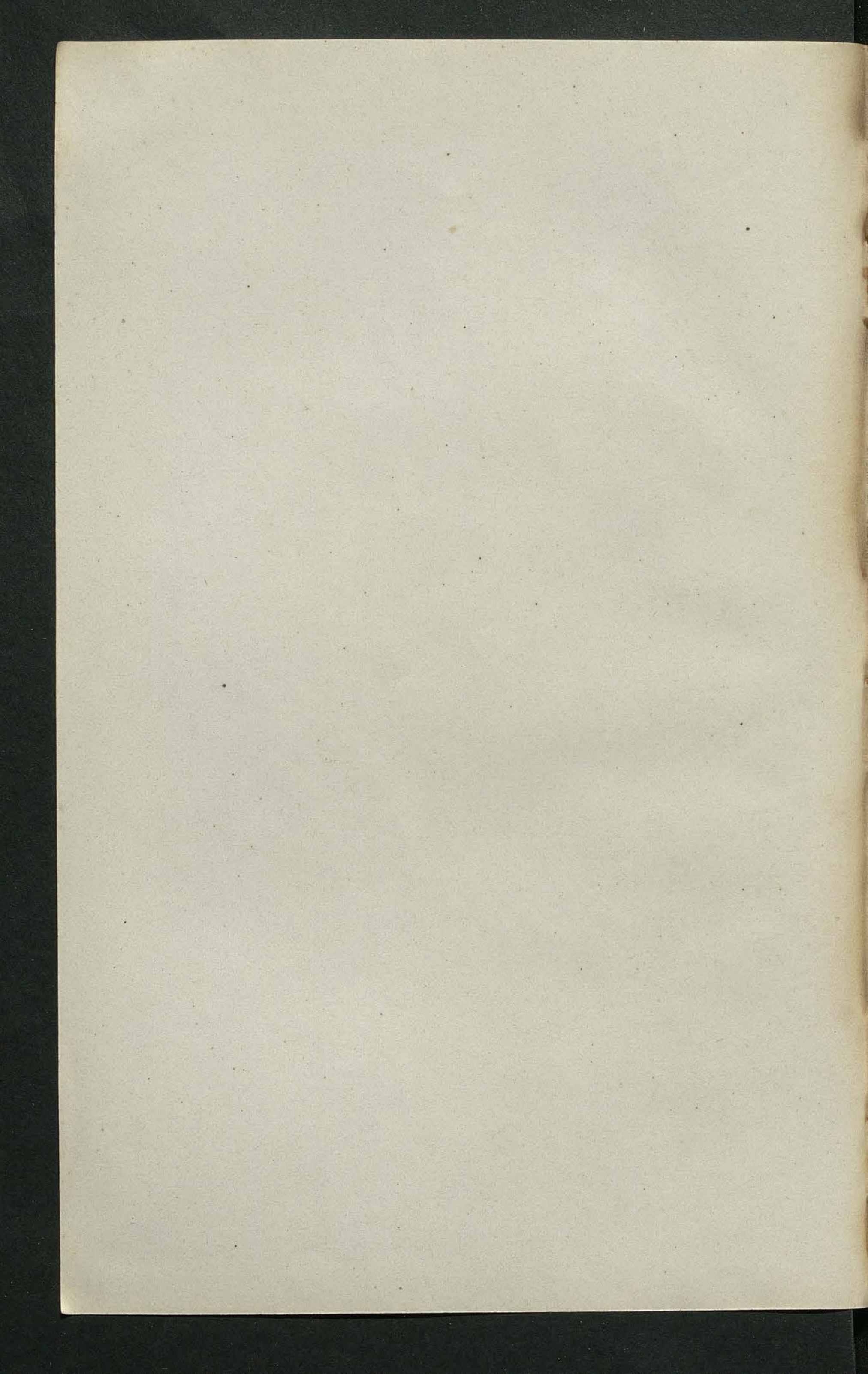


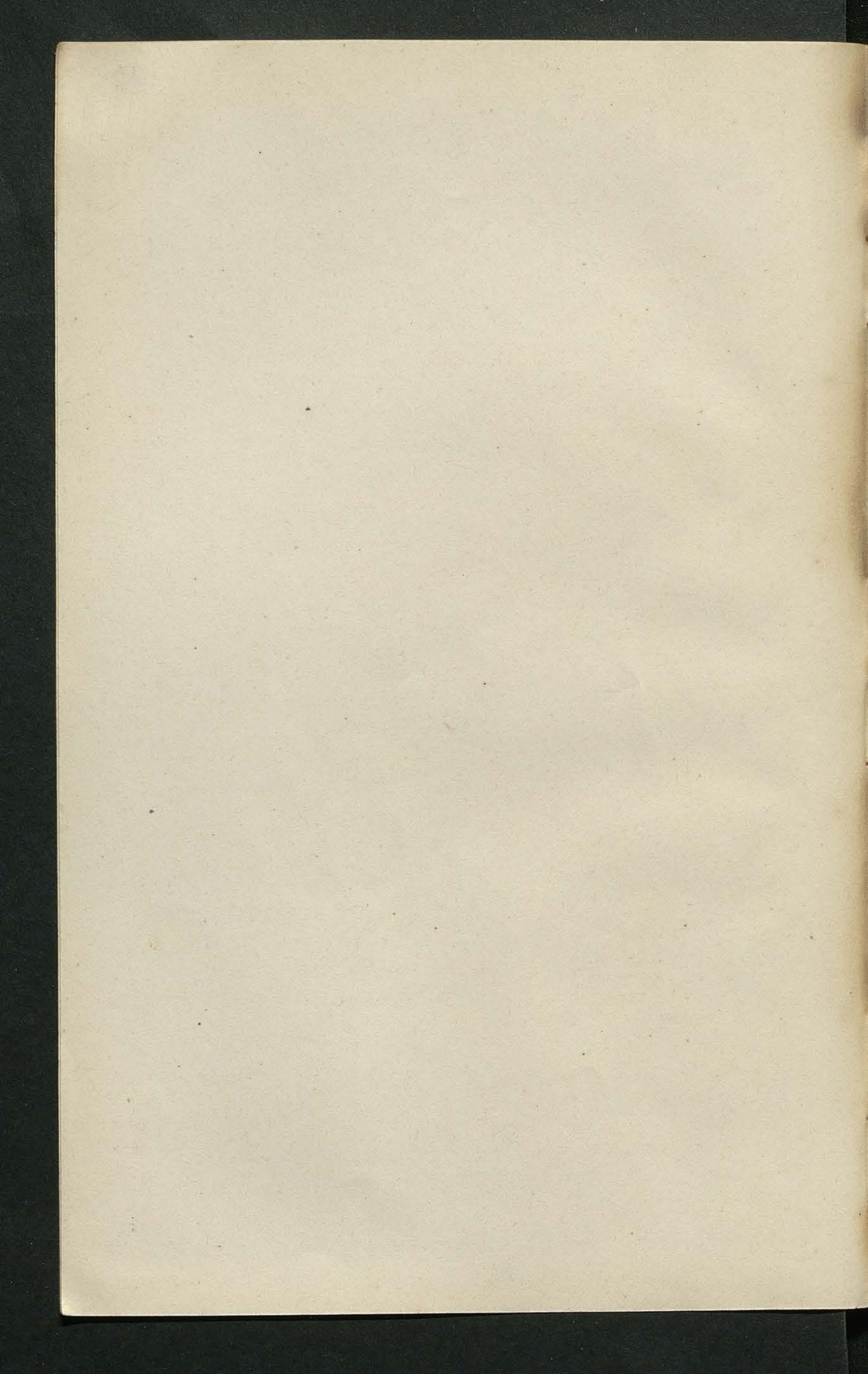


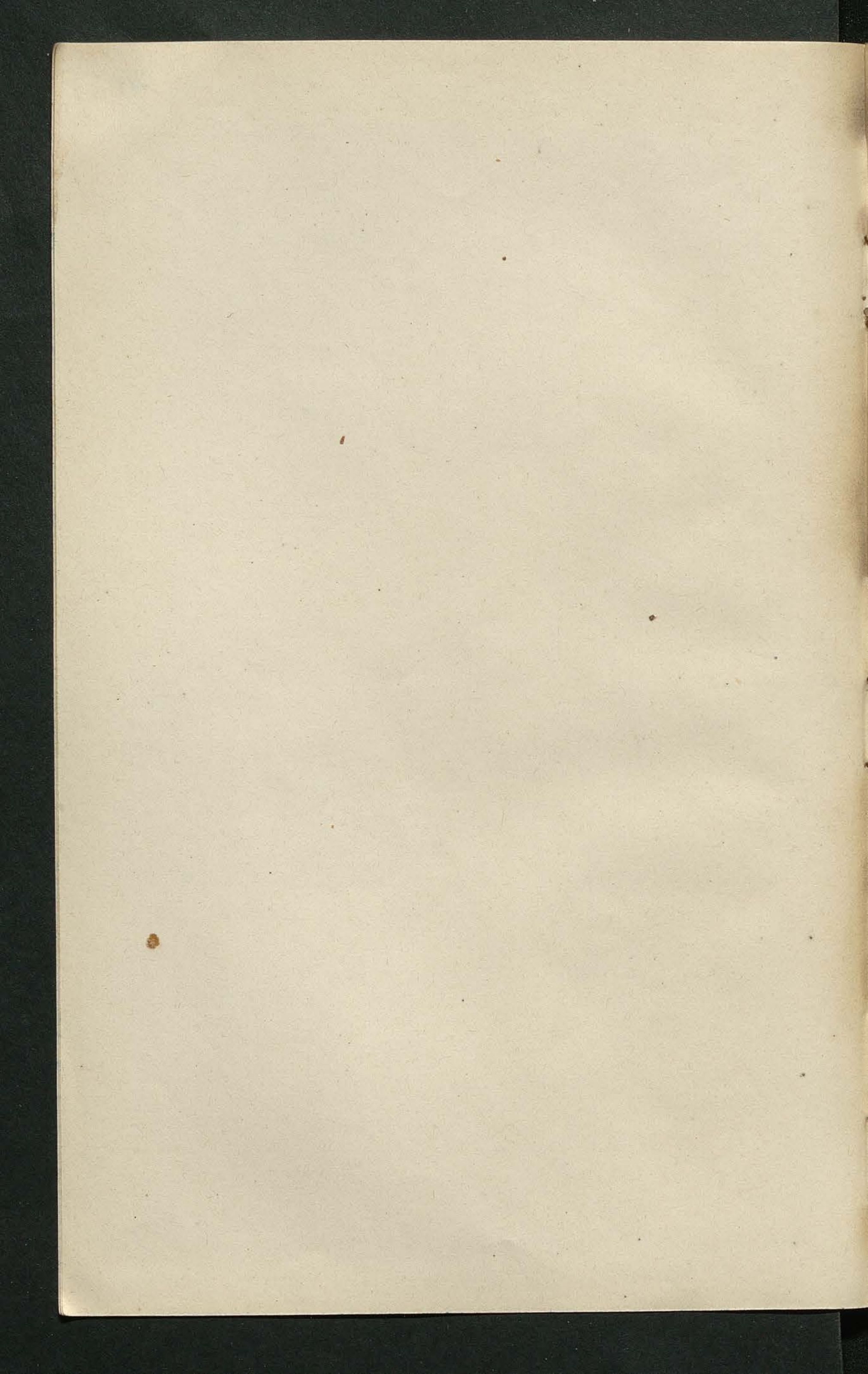


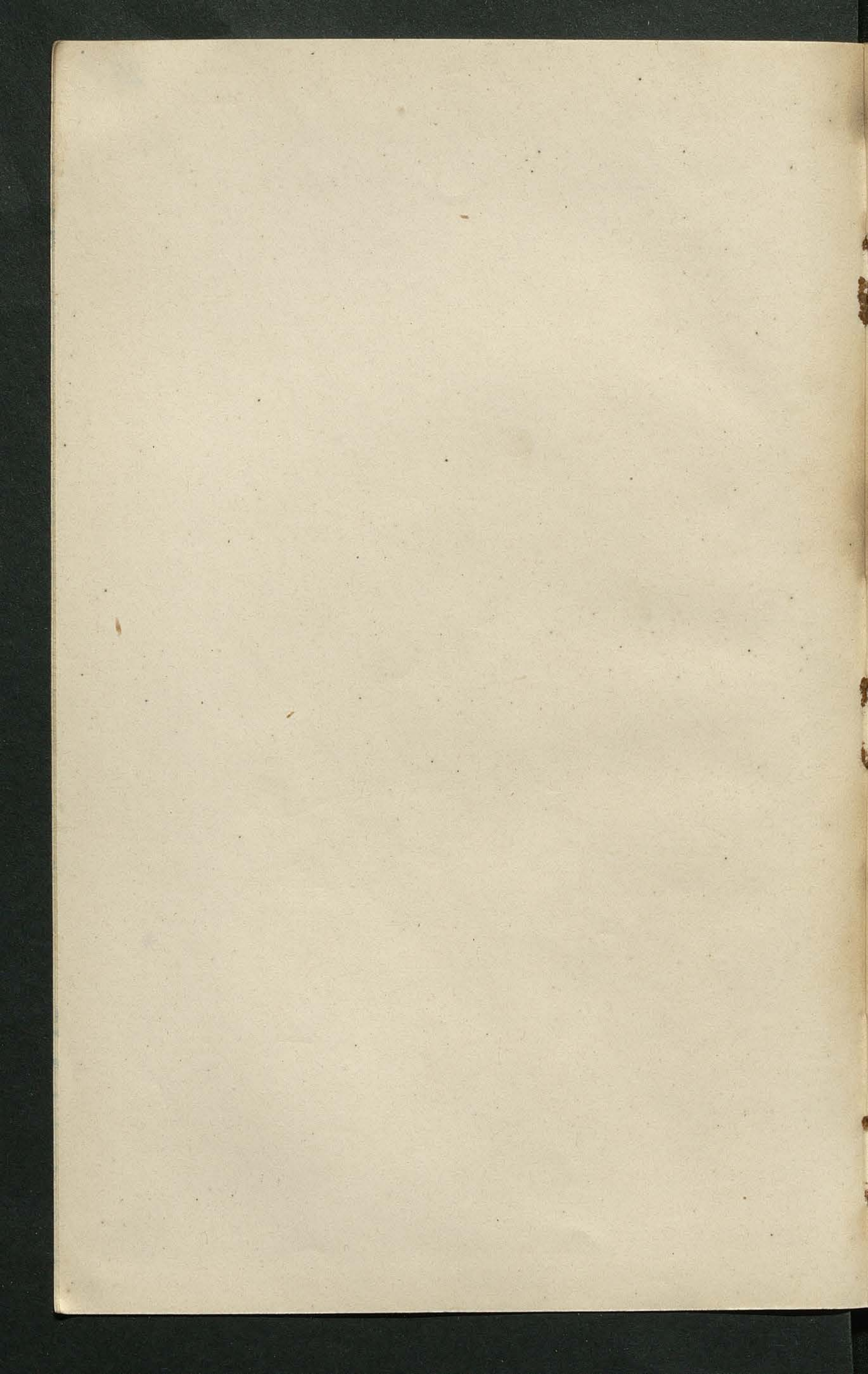












$$\varphi(u+2\omega) = \varphi(u)$$

$$\varphi(u+2\omega') = \rho^{-1} e^{-\frac{i\pi}{\omega}u} \varphi(u)$$

[illegible]

u — $\omega'-\omega$

$$\varphi(\omega'+\omega) = \varphi(\omega'-\omega)$$

$$\varphi(\omega+\omega') = \rho^{-1} e^{-\frac{i\pi}{\omega}(\omega-\omega')} \varphi(\omega-\omega')$$

$$= -\rho^{-1} \varphi(\omega-\omega') = -\varphi(\omega'-\omega)$$

$$\therefore \varphi(\omega+\omega') = \varphi(\omega-\omega') = 0$$

[illegible]

[illegible]

[illegible]

$\varphi(u)$ [illegible] III ✓ [illegible]

$$= \varphi(u) = e^{Au^2+Bu+C}\,\sigma(u-a)$$

$$\varphi(u) = \sum_{0}^{k-1} \left[a_r e^{\frac{i\pi}{\omega}(u-\beta) + \frac{i\pi}{\omega} r\beta} \; \theta_{00}\left((u-\beta)\frac{k i\pi}{2\omega} \;\; k\tau \right) \right]$$

$z = (u-\beta)\frac{ki\pi}{2\omega}$

$\tau = \log q = \frac{\omega'}{\omega}$

$\varphi(u)$ [illegible] θ –

[illegible] $k = 1$, $r = 0$,

$\tau + \log \mu = 0$

$\mu = e^{-\tau} = \frac{1}{\rho}$ $\quad \beta = 0$

$z = u \frac{i\pi}{2\omega}$

$$\varphi\left(\frac{2\omega z}{i\pi}\right) = a_0 \, e^{2z + \frac{i\pi}{\omega}} \; \theta_{00}(z, \tau)$$

[illegible] θ [illegible]

f^2

$$a_0 \sum q^{\rho^2} e^{\rho 2z}$$

$$= a_0 \sum_{\rho=-\infty}^{+\infty} e^{\rho^2 \tau + 2\rho z}$$

$$\varphi(u) = a_0 \sum_{\rho=-\infty}^{+\infty} e^{\rho^2\tau + \frac{i\pi u}{\omega}\rho}$$

$$= 1 + \sum_{\rho=+1}^{\infty} e^{\rho^2\tau} \underbrace{\left[e^{\frac{i\pi u}{\omega}\rho} + e^{-\frac{i\pi u}{\omega}\rho} \right]}_{2\cos\frac{\pi u \rho}{\omega}}$$

[illegible] $\varphi(-u) = \varphi(u)$

$$\mu^\rho a_{r-\rho\alpha} = \rho^{2\rho r + \rho\overline{\rho-1}\, k}$$

$$\varphi(u) = \sum_{r=0}^{k-1}\left[a_r \sum_{-\infty}^{+\infty} \mu^\rho\, \rho^{2\rho r + \rho\overline{\rho-1}\, k}\, e^{(k\rho + r)\frac{i\pi u}{\omega}}\right)$$

$$\psi(r, u) = \sum_{-\infty}^{+\infty} \frac{1}{\mu^\rho}\, e^{2\rho r \tau + \rho^2 k - \rho k\tau + (k\rho + r)\frac{i\pi u}{\omega} - \rho \log\mu}$$

$u = \alpha v + \beta$

[illegible]

$$\psi(r, \alpha v + \beta) = \sum_{-\infty}^{+\infty} e^{2\rho r\tau + \rho^2 k\tau - \rho k\tau + (\alpha k\rho + \alpha r)\frac{i\pi v}{\omega} + \frac{i\pi}{\omega}(k\rho\beta + r\beta) - \rho\log\mu}$$

~~$2\rho r\tau - \rho k\tau +$~~

$$2r\tau + ~~[illegible]~~ - k\tau + \frac{i\pi v\alpha k}{\omega} - \frac{i\pi k\beta}{\omega} - \log\mu = 0$$

$$\psi(r, \alpha v + \beta) = \sum_{-\infty}^{+\infty} e^{\rho^2 k\tau + \alpha k\rho\frac{i\pi v}{\omega}}\, e^{\alpha\rho\frac{i\pi v}{\omega} + \frac{i\pi}{\omega} r\beta}$$

$k\tau = \tau'$ $\quad \alpha\frac{k i\pi v}{\omega} = 2z$

$$= e^{\frac{\alpha r i\pi v}{\omega} + \frac{i\pi}{\omega} r\beta} \underbrace{\sum_{-\infty}^{+\infty} e^{\rho^2\tau' + 2\rho z}}_{\vartheta_{00}(z, \tau')}$$

$= ~~\psi(r~~$

w. $\alpha = 1$

$$\mu^{\rho} a_{\rho k} = a_0 \, q^{k(\rho-1)\rho}$$

$$\mu a_0 = a_{-k} \, q^{2k}$$

$m = -k$

$$\mu a_{-k} = a_{-2k} \, q^{-4k}$$

$$\vdots$$

$$\mu a_{-(\rho-1)k} = a_{-\rho k} \, q^{-2\rho k}$$

$$\mu^{\rho} a_0 = a_{-\rho k} \, q^{k\rho(\rho+1)}$$

$$\sum a_m e^{\frac{m\,i\pi u}{\omega}} = \mu e^{\frac{k i \pi u}{\omega}} \sum a_m e^{\frac{m i\pi u}{\omega}}$$

$$= \mu \sum a_m e^{\frac{(m-k)\, i\pi u}{\omega}}$$

$$= \mu \sum a_m e^{m 2\frac{i\pi\omega'}{\omega}} \cdot e^{\frac{m i\pi u}{\omega}}$$

$$e^{+\frac{i\pi}{\omega}\omega'} = \rho$$

$$= \sum a_m \rho^{2m} e^{+\frac{m i\pi u}{\omega}}$$

$$= \sum \mu a_{m+k} e^{+\frac{m i \pi u}{\omega}}$$

$$\mu a_{m+k} = a_m \rho^{2m}$$

$a_1 \; a_2 \; \ldots \; a_{k-1}$ [illegible]

false

ρ coefficient a_0 ? $m = 0$

$$\mu a_k = a_0 \qquad a_k = \frac{a_0}{\mu}$$

$$\mu a_{2k} = a_k \rho^{2k}$$

$$\mu a_{3k} = a_{2k} \rho^{4k}$$

$$\mu a_{pk} = a_{(p-1)k} \rho^{2(p-1)k}$$

$$\mu^p a_{pk} = a_0 \rho^{2k[1+2 \cdots + p-1]}$$

$$4e\omega = -2\eta\kappa$$

$$e = -\frac{\eta\kappa}{2\omega}$$

$1°$: $$a = \frac{\lambda - \eta'\kappa}{2\omega'} = -\frac{\eta\kappa}{2\omega}$$

$$-\lambda\omega = \kappa(\omega'\eta - \eta'\omega) = \frac{\kappa}{\omega}\frac{i\pi}{2}$$

$$\lambda = -\frac{\kappa}{\omega}\frac{i\pi}{2}$$

$$\varphi(u+2\omega) = \varphi(u)$$

$$\varphi(u+2\omega') = \mu e^{2\lambda\omega'}\varphi(u)$$

$$\lambda = -\frac{\kappa}{\omega}\frac{i\pi}{2}$$

$$\varphi(u) = \sum_{-\infty}^{+\infty} a_m e^{\frac{m i\pi u}{\omega}} = \sum a_m \rho^{mu}$$

$$\varphi(u+2\omega') = \mu \sum a_m e^{\frac{m i\pi u}{\omega} + \frac{m i\pi 2\omega'}{\omega}}$$

$$e^{\frac{i\pi}{\omega}} = \rho$$

$$= \sum a_m \rho^{mu + 2m\omega'}$$

$$= \mu \rho^{\frac{-\kappa u}{2}} \sum a_m \rho^{mu}$$

$$= \mu \sum a_m \rho^{mu - \kappa u}$$

$\varphi(u+2\omega') = \sigma(u \ldots \quad \sigma(u-\alpha_k)(-1)^k e^{2\eta'\kappa(u+\omega')}$

$-2\eta'\Sigma\alpha_\lambda + g(u+2\omega')$

$= \varphi(u)\, e^{2\lambda u + \ell}$

$\sigma(u \ldots \quad e^{g(u)}$

$2\eta'\kappa(u+\omega') + 2\eta'\Sigma\alpha_\lambda + g(u+2\omega') - g(u) + (2\nu'+1)i\pi -$

$-2\lambda\omega - \ell = 2m'i\pi$

$g''(u+2\omega') = g''(u)$

$g''(u) \ldots = \text{const}$

$g(u) = au^2 + 2bu + c$

a, b, c

$g(u+2\omega') - g(u) = \ldots + 4au\omega' + 4\omega'^2 + 4b\omega'$

$= (2\rho+1)i\pi - 2\eta'\kappa(u+\omega') + 2\eta'\Sigma\alpha_\lambda$

$+ 2\lambda u + \ell$

$4a\omega' = -2\eta'\kappa + 2\lambda$

$a = \dfrac{-\eta'\kappa + \lambda}{2\omega'}$ … b … c

$= 4a\omega u + 4\omega^2 + 4b\omega = (2\rho'+1)i\pi - 2\eta\kappa(u+\omega) - 2\eta\Sigma\alpha_\kappa$

[illegible] φ

[illegible] φ [illegible] [illegible]

$\varphi(u)$ [illegible]

[illegible] k [illegible] : $\alpha_1 \quad \alpha_2 \quad \dots \quad \alpha_k$

[illegible]

$$\varphi(u) = \sigma(u-\alpha_1)\,\sigma(u-\alpha_2)\dots\sigma(u-\alpha_k)\,e^{g(u)}$$

[illegible] φ : [illegible]

[illegible] $=$ [illegible]

$= e^{g(u)}$

$$\varphi(u+2\omega) = \sigma(u-\alpha_1)\dots\sigma(u-\alpha_k)\,(-1)^k\,e^{2\eta[k(u+\omega)\,-2\eta\sum\alpha_k\,+g(u+2\omega)}$$

$$= \varphi(u) = \sigma(u-\alpha_1)\dots e^{g(u)}$$

$$(-1)^k = e^{2n_1 i\pi}$$

$$2\eta\, k(u+\omega) - 2\eta\sum\alpha_k + g(u+2\omega) - g(u) + (2\mu+1)i\pi = 2ni\pi$$

$$g''(u+2\omega) = g''(u)$$

$$\varphi(u+2\omega') = \sigma u -$$

$$F(u+2\omega) = e^{\alpha u^2 + a + \beta u + b - \alpha u^2 - 4\alpha\omega u - 4\alpha\omega^2 - 2\beta u - 4\beta\omega} F(u)$$

[illegible] $F(u+2\omega) = F(u)$

[illegible] $au + b - 4\alpha\omega u - 4\alpha\omega^2 - 4\beta\omega = 2n\pi i$

[illegible] $u = 0$

$$a - 4\alpha\omega = 0$$

$$\alpha = \frac{a}{4\omega}$$

$$b - 4\alpha\omega^2 - 4\beta\omega = 2n\pi i$$

[illegible]

[illegible]

$F(u+2\omega) = F(u)$

$$F(u+2\omega) = e^{A\omega + Bu + C} F(u)$$

[illegible] $A = 0$

[illegible]

$$\sigma(u+2\omega) = e^{2\eta(u+\omega)} \sigma(u) \qquad \eta = \frac{\sigma'(\omega)}{\sigma(\omega)}$$

périodicité $2i\pi$

$$\varphi(z+2i\pi) = \varphi(z)$$

$$\varphi(z+2\tau) = e^{2\lambda z+\tau}\varphi(z)$$

fonction ... $\varphi(z+2\omega) = \varphi(z)$

$$\varphi(z+2\omega') = \mu\, e^{2\lambda z}\varphi(z)$$

... ϑ ... σ ...

---

pér. ... $f(u+2\omega) = \mu f(u)$

$f(u+2\omega') = \mu' f(u)$

$$\varphi(u+2\omega) = \varphi(u)$$ pér. ...

$$\varphi(u+2\omega') = \mu e^{2\lambda u}\varphi(u)$$

...: $\varphi(u+2\omega) = A e^{2bu}\varphi(u)$

$\varphi(u+2\omega') = A' e^{2b'u}\varphi(u)$

fonction ... $f(u+2\omega) = f(u)$

$$f(u+2\omega') = e^{Au+2Bu}f(u)$$

...

$$f(u+2\omega) = e^{au+b}f(u)$$

$$f(u+2\omega') = e^{a'u+b'}f(u)$$

$$f(u) = e^{\alpha u^2+2\beta u+\gamma}F(u)$$

$$e^{au+b}f(u) = e^{\alpha(u+2\omega)^2+2\beta(u+2\omega)+\gamma}F(u+2\omega)$$

$$f(u+2\omega) = e^{\alpha u^2+(2\beta+a)u+\gamma+b}F(u)$$

$$\vartheta_{01}(z+\tfrac{1}{2}i\pi) = \sum (-1)^n e^{n^2\tau + 2nz + ni\pi}$$

$$= \underline{\vartheta_{00}(z)}$$

$$\vartheta_{10}(z+\tfrac{1}{2}\tau) = \sum e^{\left(\frac{2n+1}{2}\right)^2\tau + \frac{2n+1}{2}(2z+\tau)}$$

$$= \sum e^{\left(\frac{2n+1}{2}\right)^2\tau + \frac{2n+1}{2}\tau + \frac{2n+1}{2}2z}$$

$$= e^{-\frac{\tau}{4}-z} \sum e^{\left[\left(\frac{2n+1}{2}\right)+\frac{1}{2}\right]^2\tau + \frac{2n+1}{2}2z + \left[\frac{2n+1}{2}+1\right]z}$$

$$= e^{-\frac{\tau}{4}-z} \sum e^{[n+1]^2\tau + 2(n+1)z}$$

$$= \underline{e^{-\frac{\tau}{4}-z}\, \vartheta_{00}(z)}$$

$$\vartheta_{10}(z+\tfrac{1}{2}i\pi) = \sum e^{\left(\frac{2n+1}{2}\right)^2\tau + \frac{2n+1}{2}(2z+i\pi)} =$$

$$= \sum e^{\left(\frac{2n+1}{2}\right)^2\tau + \frac{2n+1}{2}2z} + e^{\frac{2n+1}{4}i\pi}$$

$$= e^{\left(\frac{2n+1}{2}\right)^2\tau +} \qquad = \underline{\vartheta_{11}(z)}$$

$$\vartheta_{11}(z+\tfrac{1}{2}\tau) = i\sum(-1)^n e^{\left(\frac{2n+1}{2}\right)^2\tau + \frac{2n+1}{2}2z + \frac{2n+1}{2}\tau}$$

$$= i e^{-\frac{\tau}{4}-z} \sum (-1)^n e^{(n+1)^2\tau + (n+1)2z}$$

$$= \underline{i e^{-\frac{\tau}{4}-z}\, \vartheta_{01}(z)}$$

$$\vartheta_{11}(z+\tfrac{1}{2}i\pi) = i\sum(-1)^n e^{\left(\frac{2n+1}{2}\right)^2\tau + \frac{2n+1}{2}2z + 2\frac{2n+1}{2}i\pi}$$

$$= \underline{-\vartheta_{10}(z)}$$

$= e \text{---} \vartheta_{00}(z + \frac{1}{4}\rho\, i\pi + \frac{1}{2}\mu\tau)$

ϑ_{00}

Charakteristik

ϑ_{11}

τ = Modul

$\frac{\rho}{2}$

e^{2z}

$\frac{4}{0}$ 2 — $z + \frac{\tau}{2}$

$$\vartheta_{00}(z + \tfrac{1}{2}\tau) = \sum e^{n^2\tau + 2n\tau + 2nz} = \sum e^{-\tau} e^{(n+1)^2\tau + 2nz}$$

$$= e^{-\tau} e^{-2z} \sum e^{(n+1)^2\tau + 2(n+1)z}$$

$$= e^{-\tau - 2z} \vartheta_{00}(z)$$

$$\vartheta_{00}(z + \tfrac{1}{2} i\pi) = \sum e^{n^2\tau + 2nz + n i\pi}$$

$$= \sum (-1)^n e^{n^2\tau + 2nz} = \vartheta_{01}(z)$$

$$\vartheta_{01}(z + \tfrac{1}{2}\tau) = \sum (-1)^n e^{n^2\tau + 2nz + 2n\tau}$$

$$= e^{-n\tau - 2z} \sum (-1)^n e^{(n+1)^2\tau + 2(n+1)z}$$

$$= -e^{-\tau - 2z} \vartheta_{01}(z)$$

$$= A\, e^{z+\frac{1}{2}\pi i+\frac{1}{4}\tau}\left(1-e^{-2z}\right)\prod_{1}^{\infty}\left(1-2q^{2n}\cos 2iz+q^{4n}\right)$$

$$= A\, q^{\frac{1}{4}}\, 2.\sin(iz)\prod^{\infty}\left(1-q^{2n}\cos 2iz+q^{4n}\right)$$

allgemein:

$$\vartheta_{\mu\varrho}(z)=\sum_{-\infty}^{+\infty} e^{\left(\frac{2n+\mu}{2}\right)^2\tau+\left(\frac{2n+\mu}{2}\right)(2z+\varrho i\pi)}$$

[illegible]

e [illegible]

$$\mu = 2\mu+\mu_1 \qquad \varrho = 2\nu+\varrho_1$$

$n+$ [illegible]

$$\vartheta_{2\mu+\mu_1,\,2\nu+\varrho_1}(z)=\sum_{-\infty}^{+\infty} e^{\left(\frac{2n+\mu_1}{2}\right)^2\tau+\frac{2n+\mu_1}{2}\left(2z+\varrho_1 i\pi+2\nu i\pi\right)}$$

$$e^{\frac{2n+\mu_1}{2}2\nu i\pi}=e^{\mu_1\nu i\pi}=(-1)^{\mu_1\nu}$$

$$= (-1)^{\mu_1\nu}\,\vartheta_{\mu_1\varrho_1}(z)$$

[illegible]

$$\vartheta_{\mu\varrho}(z)=\sum_{-\infty}^{\infty} e^{n^2\tau+n\mu\tau+\mu^2\frac{\tau}{4}+2nz+\mu z+n\varrho i\pi+\frac{\mu\varrho i\pi}{2}}$$

$$\vartheta_{00}(z)= e^{\mu^2\frac{\tau}{4}+\mu z+\frac{1}{2}\mu\varrho i\pi}\sum_{-\infty}^{\infty} e^{n^2\tau+n\mu\tau+2nz+n\varrho i\pi}$$

$$\sum e^{n^2\tau+2n\left[z+\frac{1}{2}\varrho i\pi+\frac{1}{2}\mu\tau\right]}$$

$z \sim z + \frac{\tau}{2}$ — x

ϑ_{10}

$= e^{z + \frac{\tau}{4}} \vartheta_{00}(z + \frac{1}{2}\tau)$

$e^{z+\frac{1}{4}\tau} \sum e^{n^2\tau + 2nz +}$

$+ n\tau$

$= e^{z+\frac{1}{4}\tau} \sum e^{\tau (n+\frac{1}{2})^2 + (2n+1)z}$

$= \sum e^{\tau (\frac{2n+1}{2})^2 + \frac{2n+1}{2} 2z}$

$\vartheta_{10}(z) = A e^{z + \frac{1}{4}\tau} \prod_0^\infty \left(1 + \rho^{2n+1} e^{2z+\tau}\right)\left(1 + \rho^{2n+1} e^{-2z-\tau}\right) =$

$= A e^{z+\frac{1}{4}\tau} \prod_0^\infty \left(1 + \rho^{2n+2} e^{2z}\right)\left(1 + \rho^{2n} e^{-2z}\right) =$

fuori c. e 2 prod.

$= A e^{z+\frac{1}{4}\tau} (1 + e^{-2z}) \prod_1^\infty \left(1 + 2\rho^{2n} \cos 2iz + \rho^{4n}\right)$

$= A \rho^{\frac{1}{4}} 2i \cos iz \prod \left(1 + 2\rho^{2n} \cos 2iz + \rho^{4n}\right)$

$\vartheta_{11}(z)$ e ϑ_{10} ove z — $z + \frac{i\pi}{2}$

$\vartheta_{11} = \vartheta_{10}\left(z + \frac{i\pi}{2}\right) = \sum_{-\infty}^{\infty} e^{\left(\frac{2n+1}{2}\right)^2 \tau + \frac{2n+1}{2} \cdot 2\left(z + \frac{1}{2}\pi i\right)}$

$\frac{2n+1}{2} (2z + i\pi)$

$=$

$A e^{z + \frac{1}{2} i\pi + \frac{1}{4}\tau} \prod_0^\infty \left[1 + \rho^{2n+2} e^{2z}\right] \left[1 + \rho^{2n} e^{-2z}\right]$

$v(z) = \prod^{\infty} (1-q^{n}z)$

$f(z) = v(z)\, w(\frac{q}{z}) = A_0 \sum_{-\infty}^{+\infty} (-1)^n \rho^{\frac{1}{2}n(n-1)} z^n$

$\rho \quad \rho^2$

$w(z)\, (\frac{\rho^2}{z}) = A_0 \sum_{-\infty}^{+\infty} (-1)^n \rho^{n^2} (\frac{z}{\rho})^n$

$\frac{z}{\rho} = -e^{2z}$

$\vartheta_{00}(z) = \sum_{-\infty}^{\infty} \rho^{n^2} e^{2nz} =$

$= \sum_{-\infty}^{\infty} e^{n^2\tau + 2nz}$

$\vartheta_{01}(z) = \sum_{-\infty}^{\infty} (-1)^n \rho^{n^2} e^{2nz} = \quad \sum +1\, e^{n^2\tau + \dots}$

$\vartheta_{00} = A \prod \left[(1+\rho^{2n+1} e^{2z})(1+\rho^{2n+1} e^{-2z})\right.$

$w(z) = \prod_1^{\infty} (1+\rho^{2n-1} e^{2z})$

$w\frac{\rho^2}{z} = \prod_0^{\infty} (1+\rho \;)$

$= A \prod \left[1 + \rho^{2n+1} \cos 2iz + \rho^{4n+1}\right]$

$\vartheta_{01} = 1 - 2(n+1) e$

$= A \prod \left[1 - \rho^{(2n+1)} \cos 2iz + \rho^{(4n+2)}\right]$

Jacobi:

$$\vartheta(z) = \sum_{-\infty}^{+\infty} e^{n^2\tau + 2nz} = A\, w(-\rho e^{2z})\, w(-e^{-2z}\rho)$$

$$\vartheta_{0,1}(z) = \sum_{-\infty}^{+\infty} (-1)^n e^{n^2\tau + 2nz} = A\, w(\rho e^{2z})\, w(e^{-2z}\rho)$$

$$= \prod_0^{\infty} \left(1+\rho^{2n+1} e^{2z}\right) \times \prod_0^{\infty} \left(1+\rho^{2n+1} e^{-2z}\right)$$

$$1 + \rho^{2n+1}\underbrace{\left(e^{2z} + e^{-2z}\right)}_{2\cos 2iz} + \rho^{4n+2}$$

$$= \prod\left(1 + 2\rho^{2n+1} \cos 2iz + \rho^{4n+2}\right)$$

$$= \prod\left(1 - 2\rho^{2n+1} \cos 2iz + \rho^{4n+2}\right)$$

$\vartheta_{1,0}$ ω_1 $\vartheta_{(0)}$ — z — $z + \frac{\pi}{2}$; $\times e^{2z+\frac{\tau}{4}}$

$\omega_2 f\, 6^{\rho}$ z — $z + \frac{i\pi}{4}$ 6 ϑ_{11}

f $f\rho$; $\vartheta_{\nu,\rho}(z)$.

$b_n z^{-n} + b_0 + b_1 z + \ldots = -\{ b_{-n} \rho^{\frac{-n-n+1}{2}} + \ldots + b_0 \rho z + \ldots$

$- b_1 z^2 - \ldots - b_n z^{n+1}$

~~$b_1 = b_0$~~ $b_0 = - b_1 \rho^{-1}$ — $b_{-n} = - b_{-n+1} \rho^{-n+1}$

$b_1 = - b_0$ $\qquad b_{-n-1} = - b_{-n-2} \rho^{-n-2}$

$b_2 = - b_1 \rho$

$b_{-1} = b_{-2} \rho^{-2}$

$b_n = - b_{n-1} \rho^{n-1}$

$b_n = (-1)^n \rho \rho^2 \rho^{n-1} b_0$

$b_n = (-1)^n \rho^{\frac{n(n+1)}{2}} b_0$ $\qquad$ ~~$b_0 = (-1)^n b$~~

$b_{-n} = (-1)^n \rho^{n\left(\frac{n-1}{2}\right)} b_0$

$$f(z) = b_0 \sum_{n=-\infty}^{+\infty} (-1)^n \rho^{\frac{n(n-1)}{2}} z^n = \vartheta(z)\ \vartheta\left(\frac{\rho}{z}\right)$$

$$f(z\rho^2) = b_0 \sum_{n=\infty} (-1)^n \rho^{n(n-1)} z^n = \vartheta(z)\ \vartheta\left(\frac{\rho^2}{z}\right)$$

$$= b_0 \sum (-1)^n \rho^{n^2} \left(\frac{z}{\rho}\right)^n$$

NB:

$-\frac{z}{\rho} = e^{2z}$

$\frac{z}{\rho} = e^{2z}$

$\log \rho = \tau$

$\rho = e^{\tau}$

$$f(\rho z) = w(\rho z)\, w\left(\tfrac{1}{z}\right) = \frac{w(z)}{1-z}\, w\left(\tfrac{1}{z}\right) = \frac{w(z)}{1-z}\, \frac{z-1}{z}\, w\left(\tfrac{\rho}{z}\right)$$

$$= -\frac{f(z)}{z}$$

$$f(z) = -z f(\rho z)$$

$$f(z) = A \quad \cdots \quad A_n z^{n} + \cdots$$

$$f(z+2\omega) = \mu\, e^{2az} f(z)$$

$$f(z+2\omega') = \mu'\, e^{2a'z} f(z)$$

$$f(z+\omega) = \cdots$$

$$f(z+\omega') = e^{az+bz^2} \cdots$$

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$$f(z) = w(z)\, w\left(\frac{\rho}{z}\right) = \left[a_0 + a_1 z + \cdots\right]\left[a_0 + a_1 \frac{\rho}{z} + \cdots\right]$$

$$f(\rho z) = \frac{w(z)}{1-z}\, \frac{z-1}{z}\, w\left(\frac{\rho}{z}\right) = -\frac{f(z)}{z} \qquad f(z) = -z f(\rho z)$$

$$f(z) = \cdots + \frac{b_{-1}}{z} + b_0 + b_1 z + b_2 z^2 + \cdots$$

$$f(\rho z) = \cdots - \frac{b_{-n}}{z^n \rho^{-n}} + \cdots + b_0 + b_1 \rho z + b_2 \rho^2 z^2 + \cdots$$

$v(\rho z) - f(z)$

~~$v(\rho z) = (1-\rho z)(1-z) v(z)$~~ $\quad v(z) = (1-z)\, v(\rho z)$

$$= A_0 + A_1 z + \cdots$$

$$A_0 + A_1 z + A_2 z^2 + \cdots = (1-z)\left[A_0 + A_1 \rho z + A_2 \rho^2 z^2 + \cdots\right]$$

$$= A_0 + A_1 \rho z + A_2 \rho^2 z^2 + A_3 \rho^3 z^3 + \cdots$$
$$- A_0 z - A_1 \rho z^2 - A_2 \rho^2 z^3 \cdots$$

$A_0 = A_0$

$A_1 = A_1 \rho - A_0 \qquad A_1 = \dfrac{A_0}{\rho - 1}$

$A_2 = A_2 \rho^2 - A_1 \rho \qquad A_2 = \dfrac{A_1 \rho}{\rho^2 - 1} = \dfrac{A_0 \rho}{(\rho^2-1)(\rho-1)}$

$A_3 = A_3 \rho^3 - A_2 \rho^2 \qquad A_3 = \dfrac{A_2 \rho^2}{\rho^3 - 1} = \dfrac{A_0 \rho^3}{(\rho^3-1)(\rho^2-1)(\rho-1)}$

$A_n = A_n \rho^n - A_{n-1} \rho^{n-1}$

$A_0 = 1$

$$A_n = \frac{A_0 \rho \rho^2 \cdots \rho^{n-1}}{(\rho-1)(\rho^2-1) \cdots (\rho^n-1)}$$

$$A_n = \frac{\rho^{n(n-1)/2}}{(\rho^n-1) \cdots (\rho-1)}$$

$v \rho < 1$ for $v(z)\; v\left(\frac{\rho}{z}\right)$

$X_0 a$ = [illegible]

[illegible] $\varrho = 1$

[illegible]

θ Funktionen

[illegible]

[illegible] $2m\omega + 2n'\omega'$

[illegible] $\pm 2 \; \pm 1 \quad 0 \quad +1 \; 2$ ——

[illegible] $a, \; a\varphi, \; a\varphi^2 \quad = 0$

$$f(u) = \left(1 - \frac{u}{a}\right)\left(1 - \frac{u}{a\varphi}\right)\left(1 - \frac{u}{a\varphi^2}\right) \text{——}$$

[illegible]

$$\frac{1}{|a|} + \frac{1}{|a\varphi|} + \frac{1}{|a\varphi|^2} + \text{——} \qquad \text{[illegible]}$$

$$= \frac{1}{|a|}\left[1 + \frac{1}{|\varphi|} + \frac{1}{|\varphi|^2} + \text{—}\right] = \frac{1}{|a|} \qquad |\varphi| > 1$$

$\frac{u}{a} = z \qquad \frac{1}{\varphi} = \rho \qquad$ [illegible]

$$w(z) = (1-z)(1-\rho z)(1-\rho^2 z) \text{——}$$

$$\frac{\sigma u}{\sigma_3 u} = 1 - e_3 \sigma$$

[illegible] $\sin^2 \mathrm{am} \sqrt{e_1 - e_3}$

$$\sigma_1(u)^2 - \sigma_3(u)^2 + [e_1 - e_3]\,\sigma(u)^2 = 0$$

$$\left(\frac{\sigma_1(u)}{\sigma_3 u}\right)^2 - 1 + \frac{[e_1 - e_3]}{e_1 - e_3} \sin^2 \mathrm{am} \sqrt{\;}$$

$$\left(\frac{\sigma_1 u}{\sigma_3 u}\right)^2 = 1 - (\ldots) \sin^2 \mathrm{am} \sqrt{\;}$$

$$= \sin^2 \mathrm{am} \left(\sqrt{e_1 - e_3}\, u\, k\right)$$

$$\sigma_1(u)^2 - \sigma_3(u)^2 + (e_2 - e_3)\,\sigma u^2 = 0$$

$$\left(\frac{\sigma_2(u)}{\sigma_3 u}\right)^2 - 1 + \underbrace{\frac{(e_2 - e_3)}{e_1 - e_3}}_{k^2} \sin^2 \mathrm{am} \left(\sqrt{e_1 - e_3}\, u, k\right) = 0$$

$$\left[\frac{\sigma_2(u)}{\sigma_3(u)}\right]^2 = 1 - k^2 \sin^2 \mathrm{am} \left(\sqrt{e_1 - e_3}\, u, k\right)$$

$$\frac{\sigma_2(u)}{\sigma_3(u)} = \Delta\, \mathrm{am}$$

Periode

$$K = \int_0^1 \frac{dt}{\sqrt{1-t^2}\,\sqrt{1-k^2t^2}} \quad \sim \sqrt{\;}\ldots \quad -\varepsilon_2\ldots$$

$$\left[\frac{\sigma_\alpha(u)}{\sigma(u)}\right]^2 = \wp(u) - e_\alpha$$

$$\left[\frac{\sigma_3(u)}{\sigma u}\right]^2 = \wp(u) - e_3 \qquad \text{für } w_1 = \frac{w}{2} \quad u$$

$$\frac{\sigma_3(w_1)}{\sigma(w_1)} = \sqrt{e_1 - e_3}$$

$$\left(\frac{dz}{du}\right)^2 = \left[1-(e_\beta - e_\alpha)\, z^2\right]\left[1-(e_\gamma - e_\alpha)\, z^2\right]$$

$$X_{0\alpha}$$

$$\alpha = 3$$

$$\beta = 2$$

$$\gamma = 1$$

$$\left(\frac{dz}{du}\right)^2 = \left[1-(e_2-e_1)\, z^2\right]\left[1-(e_3-e_1)\, z^2\right]$$

$$\sqrt{e_1-e_3}\; z = \xi$$

$$\frac{1}{e_1-e_3}\left(\frac{d\xi}{du}\right)^2 = \Big[1 - \underbrace{\frac{e_2-e_3}{e_1-e_3}}_{k^2}\,\xi^2\Big]\left[1-\xi^2\right]$$

$$\frac{e_2-e_3}{e_1-e_3} = k^2$$

$$\frac{d\xi}{d\underbrace{\sqrt{e_1-e_3}\,u}_{v}} = \left[1-k^2\xi^2\right]\left[1-\xi^2\right]$$

$$\left(\frac{d\xi}{dv}\right)^2 = \left[1-k^2\xi^2\right]\left[1-\xi^2\right]$$

[illegible]

$$\xi = \sin am\, v$$

$$= \sqrt{e_1-e_3}\; X_{0\alpha}$$

$$\frac{\sigma(u)}{\sigma_3(u)} = \frac{1}{\sqrt{e_1-e_3}}\,\sin am\, v = \frac{1}{\sqrt{e_1-e_3}}\,\sin am\left(\sqrt{e_1-e_3}\;u, k\right)$$

$$\int^{z} \frac{dz}{\sqrt{(z-e)(z-e)(z-e)}} = \wp^{-1} u \qquad z = \wp(u)$$

$$= \int^{y} \frac{dy}{\sqrt{(1-y)(1-k^2y)}} = \int^{\sqrt{y}} \frac{dy}{\sqrt{(1-y^2)(1-k^2y^2)}}$$

$$\sqrt{y} = \operatorname{sn} u$$

?

$\sigma_\nu(-w) = \sigma_\nu(w)$ ∓ ...

$\sigma_1(w)\,\sigma(w)$ – ...

$\sigma_1(w)$ ... $\frac{w}{2}$ $\sigma_2 \| \frac{w}{2} + \frac{w'}{2}$ $\sigma_3(w) \| \frac{w}{2} + \frac{w'}{2}$

$\sigma_1^2(w) = \quad \wp(w) - \wp(\frac{w}{2})$

$e_1 e_2 e_3 = 0$

$\frac{\sigma_1(w)}{\sigma w} = \sqrt{\wp(w) - e_1}$

$\frac{\sigma_2(w)}{\sigma(w)} = \sqrt{\wp(w) - e_2}$

$\frac{\sigma_3(w)}{\sigma w} = \sqrt{\wp(w) - e_3}$

$\wp(w) = \frac{1}{3}\,\frac{\sigma_1^2(w) + \sigma_2^2(w) + \sigma_3^2(w)}{\sigma^2(w)}$

$4\,\frac{[\sigma_1(w)\,\sigma_2(w)\,\sigma_3(w)]^2}{\sigma^6(w)} = \wp'(w)^2$

$\wp'(w) = \pm\, \frac{2\,\sigma_1(w)\,\sigma_2(w)\,\sigma_3(w)}{\sigma^3(w)}$

...: $\wp(w) = \frac{1}{w^2} + \cdots$

$\wp'(w) = -\frac{2}{w^3} + \cdots$

$-$

$\sigma_\nu(w) = 1 + \cdots$

$\sigma(w) = 1 + \cdots$

Periode

$\sigma_1(w+\omega) = e^{\eta_1(w+\frac{\omega}{2})}\,\sigma_1(w)$

σ^2 ... dw

... $\sigma(w) = \pm\, \sigma_1(w) + \sigma(w)$

$\sigma_1'(w+\omega) = \mp\, e^{\eta(w+\frac{\omega}{2})}\,\sigma_1'(w) + \sigma_1(w)$ ...

$$\frac{\sigma(u)}{w} = \frac{u}{w} \prod^{\infty} \left[1 - \frac{u}{w} \, e^{\frac{u}{w} + \frac{u^2}{2w^2}}\right] \qquad w = \mu\omega + \nu\omega'$$

$$\frac{u}{w} \qquad \lambda u + \lambda'\omega' \qquad \lambda w$$

$$\sigma(\lambda u, \lambda\omega, \lambda\omega') = \prod 1 - \lambda u$$

$$\sigma(\lambda u, \lambda w) = \lambda \sigma(u)$$

$$\lambda = \frac{1}{\omega} \qquad = \sigma\left(\frac{u}{\omega}, \frac{\omega'}{\omega}\right)$$

$$\sigma\left(u + \frac{w}{2}\right) = \sigma\left(u - \frac{w}{2} + w\right) = -e^{\eta\left(u - \frac{w}{2} + \frac{w}{2}\right)} \sigma\left(u + \frac{w}{2}\right)$$

$$\frac{}{\sigma\frac{w}{2}} = \frac{e^{\eta u} \sigma\left(\frac{w}{2} - u\right)}{\sigma\frac{w}{2}}$$

$$\sigma_1(-u) \qquad \sigma_1 u$$

$$\sigma(u\, \omega\, \omega')$$

$$F(u+w) = F(u)$$

$$F(u+w') = e^{g(u)} \cdot F(u)$$ [illegible]

[illegible]

$$F(u+w+w') = F(u+w') = e^{g(u+w)} \underbrace{F(u+w)}_{F(u)}$$

$$e^{g(u)} F(u) =$$

$$e^{g(u+w)} - e^{g(u)} = 1$$

$$g(u+w) - g(u) = 2k\pi i$$

[illegible]

[illegible]

[illegible]

[illegible]

$$\sigma_1(u) = e^{-\eta\frac{u}{2}} \frac{\sigma(u+\frac{\omega}{2})}{\sigma(\frac{\omega}{2})}$$

$$\sigma_2(u) = e^{-(\eta+\eta')\frac{u}{2}} \frac{\sigma(u+\frac{\omega+\omega'}{2})}{\sigma(\frac{\omega+\omega'}{2})}$$

$$\sigma_3(u) = e^{-\eta'\frac{u}{2}} \frac{\sigma(u+\frac{\omega'}{2})}{\sigma(\frac{\omega'}{2})}$$

[illegible]

[illegible] $\varphi(u) = \sum_{k=0}^{n} A_k u^k$

$\chi(u) = \sum_{k=0}^{n+1} B_k u^k$ [illegible]

$\varphi(u) + \chi(u) - \chi(u+w) = 2\rho i\pi$

$\sum_{k=0}^{n} A_k u^k + \sum_{\nu=0}^{n+1} B_\nu [u^\nu - (u+w)^\nu] = 2\rho i\pi$

~~[illegible]~~

$\chi(u+w) = $ ~~B_{n+1}~~ $[$ ~~u^{n+1}~~ $+ \binom{n+1}{1} u^n w + \binom{n+1}{2} u^{n-1} w^2 + \dots$

$+ \binom{n+1}{n} u w^n + w^{n+1}]$

$+$ ~~B_n~~ $[$ ~~u^n~~ $+ \binom{n}{1} u^{n-1} w + \dots + \binom{n}{n-1} u w^{n-1} + w^n]$

$+ \dots$

$+ B_1 [u+w]$

$+$ ~~B_0~~

$-$ ~~$B_{n+1} u^{n+1}$~~ $-$ ~~$B_n u^n$~~ ~~$= \dots$~~ $\equiv \varphi(u) + 2\rho i\pi$

[illegible] $\chi(u)$ [illegible]

[illegible]

$\varphi(u) = 0$

$a\omega' - a'\omega + \mu(-\nu)\ \eta\ \pi$

$A\omega'^2 + 2B\omega\omega' + \mu-\nu)\ \frac{1}{\pi} + \eta'\omega$

$B\omega' - b'\omega = A\omega\omega'[\omega - \omega'] + (\mu-\nu)\Big[i\pi(\omega'-\omega) + \frac{\omega\omega'}{2}(\eta - \eta')\Big] - [\eta\omega' - \eta'\omega]\ 2i\pi$

$A\omega = \frac{a}{2} - (\mu-\nu)\frac{\eta}{2}$

[illegible] $\mu = \nu$ [illegible]

$A = 0$

Hermite: $B\omega' - B'\omega = \ldots(\Sigma\nu - \Sigma\ldots)\ 2i\pi$

[illegible] | [illegible]

$f(u+\omega) = e^{\varphi(u)} f$ [illegible]

$f(u+\omega') = e^{\psi(u)} f(u)$ [illegible]

$f(u) = e^{\chi(u)} F(u)$

$F(u+\omega) = F(u)$

$F(u+\omega') = e^{g u} F$

$f(u+\omega) = \sout{e^{\varphi(u)} f(u)} = e^{\chi(u+\omega)} F(u+\omega)$

$\sout{F(u+\omega) = e^{\varphi(u) - \chi(u+\omega)}}$

$= e^{\varphi(u)} f(u) = e^{\varphi(u) + \chi(u)} F(u)$

$F(u+\omega) = e^{\varphi(u) + \chi(u) - \chi(u+\omega)} F(u)$

[illegible] $= 1$

C. Pulfrich Ü. elast. Verh. v. Kautschuk ...
y constante μ Wied. Ann. XXVIII 87–107

1886
Fortschr. 473 μ = 0'458. ...

Rax p. 58 Nr 395

Lamberwark 59

Kölnbr 60

Zöllmers. Nr 2

Cirrus p. 40

Rettler 48

Cours ...

Zetuike 49

Langke ... Tiruch

12/9 Cirrus ...

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13/9 ... mit Pinsel ...

$$\text{1/}, \ \wp(u) - e_\alpha = \frac{\sigma_\alpha^2(u)}{\sigma(u)}$$

$$\wp'(u) = -2 \frac{\sigma_\alpha(u)\,\sigma_\beta(u)\,\sigma_\gamma(u)}{\sigma^3(u)}$$

$$\sigma_\alpha(u)^2 - \sigma_\beta(u)^2 + (e_\alpha - e_\beta)\,\sigma(u)^2 = 0$$

$$(e_\alpha - e_\beta)\,\sigma_\gamma(u)^2 + (e_\beta - e_\gamma)\,\sigma_\alpha(u)^2 + \qquad = 0$$

$$X_{\alpha 0} = \frac{\sigma_\alpha(u)}{\sigma(u)} \qquad X_{\beta\gamma} = \frac{\sigma_\beta(u)}{\sigma_\gamma(u)} \qquad X_{0\alpha} = \frac{\sigma(u)}{\sigma_\alpha(u)}$$

---

s 1).

$$X_{\alpha 0}^2 = \wp(u) - e_\alpha$$

diff.

$$2 X_{\alpha 0} X'_{\alpha 0} = \wp'(u) = -2\, X_{\alpha 0} X_{\beta 0} X_{\gamma 0}$$

divid.

$$\text{II).} \quad X'_{\alpha 0} = - X_{\beta 0} X_{\gamma 0}$$

$$X_{\beta\gamma} = \frac{X_{\beta 0}}{X_{\gamma 0}} \qquad X'_{\beta\gamma} = \frac{X_{\gamma 0} X'_{\beta 0} - X_{\beta 0} X'_{\gamma 0}}{X_{\gamma 0}^2}$$

$$= \frac{-X_{\alpha 0} X_{\gamma 0}^2 + X_{\beta 0}^2 X_{\alpha 0}}{X_{\gamma 0}^2}$$

$$= X_{\alpha 0} \frac{[\beta_{0 0}^2 - X_{\gamma 0}^2]}{X_{\gamma 0}^2}$$

$$\sigma_{\rho 0}^2 - \sigma_{\gamma 0}^2 + (e_\rho - e_\gamma)\,\sigma_u^2 = 0$$

$$X_{\rho 0}^2 - X_{\gamma 0}^2 = -(e_\rho - e_\gamma)$$

II $$X'_{\rho\gamma} = -(e_\rho - e_\gamma)\,X_{\alpha\gamma}\,X_{0\gamma}$$

$$X_{0\alpha} = \frac{1}{X_{\alpha 0}}$$

III $$X'_{0\alpha} = \frac{-X'_{\alpha 0}}{X_{\alpha 0}^2} = \frac{X_{\rho 0}\,X_{\gamma 0}}{X_{\alpha 0}^2} = X_{\rho\alpha}\,X_{\gamma\alpha}$$

$-\vartheta$ [illegible], $X_{\alpha 0}$ [illegible]:

$$X'^2_{\alpha 0} = X_{\rho 0}^2\,X_{\gamma 0}^2$$

$$\sigma_\alpha(u)^2 - \sigma_\rho(u)^2 + [e_\alpha - e_\rho]\,\sigma(u)^2 = 0$$

$$X_{\rho 0}^2 = X_{\alpha 0}^2 + (e_\alpha - e_\rho)$$

$$X_{\gamma 0}^2 = X_{\alpha 0}^2 + (e_\alpha - e_\gamma)$$

$$X'^2_{\alpha 0} = [e_\alpha - e_\rho + X_{\alpha 0}^2]\,[e_\alpha - e_\gamma + X_{\alpha 0}^2]$$

$$X'^2_{\rho\gamma} = (e_\rho - e_\gamma)^2\,X_{\alpha\gamma}^2\,X_{0\gamma}^2$$

$$\sigma_\beta(u)^2 - \sigma_\gamma(u)^2 + [e_\beta - e_\gamma]\,\sigma(u)^2 = 0$$

$$X_{\beta\gamma}^2 - 1 + [e_\beta - e_\gamma]\,X_{0\gamma}'^2 = 0$$

$$[e_\beta - e_\gamma]\,X_{0\gamma}^2 = 1 - X_{\beta\gamma}^2$$

$$[e_\alpha - e_\beta]\,\sigma_\gamma u^2 + [e_\beta - e_\gamma]\,\sigma_\alpha u^2 + [e_\gamma - e_\alpha]\,\sigma_\beta u^2 = 0$$

$$[e_\alpha - e_\beta] + [e_\beta - e_\gamma]\,X_{\alpha\gamma}^2 + [e_\gamma - e_\alpha]\,X_{\beta\gamma}^2 = 0$$

$$[e_\beta - e_\gamma]\,X_{\alpha\gamma}^2 = [e_\beta - e_\alpha] + [e_\alpha - e_\gamma]\,X_{\beta\gamma}^2$$

$$X_{\beta\gamma}'^2 = \left[1 - X_{\beta\gamma}^2\right]\left[(e_\beta - e_\alpha) + (e_\alpha - e_\gamma)X_{\beta\gamma}^2\right]$$

$$X_{0\alpha}' = X_{\beta\alpha}\,X_{\gamma\alpha}$$

$$X_{0\alpha}'^2 = X_{\beta\alpha}^2\,X_{\gamma\alpha}^2$$

$$\sigma_\alpha(u)^2 - \sigma_\beta(u)^2 + (e_\alpha - e_\beta)\,\sigma u^2 = 0$$

$$1 - X_{\beta\alpha}^2 + [e_\alpha - e_\beta]\,X_{0\alpha}^2 = 0$$

$$X_{\beta\alpha}^2 = 1 + (e_\alpha - e_\beta)\,X_{0\alpha}^2$$

$$X_{\gamma\alpha}^2 = 1 + (e_\alpha - e_\gamma)\,X_{0\alpha}^2$$

$$X_{0\alpha}'^2 = \left[1 - (e_\beta - e_\alpha)\,X_{0\alpha}^2\right]\left[1 - (e_\gamma - e_\alpha)\,X_{0\alpha}^2\right]$$

[illegible] 1+

$$\left(\frac{dr}{dx}\right)^2 = \left[1-(\ell\beta-\ell\alpha)\,z^2\right]\left[1-[\ell\gamma-\ell\alpha]\,z^2\right]$$

[illegible] I:

$$X_{\alpha 0} = \sqrt{\ell\beta-\ell\alpha}\,\sqrt{\ell\gamma-\ell\alpha}\;z$$

$$(\ell\alpha-\ell\beta)+X_{\alpha 0}^2 = \ell\alpha-\ell\beta+(\ell\beta-\ell\alpha)(\ell\gamma-\ell\alpha)z^2$$

$$= -(\ell\beta-\ell\alpha)\left[1-(\ell\gamma-\ell\alpha)\,z^2\right]$$

$$(\ell\alpha-\ell\gamma)+X_{\alpha 0}^2 = -(\ell\gamma-\ell\alpha)\left[1-(\ell\beta-\ell\alpha)\,z^2\right]$$

[illegible]

$$z = \frac{X_{\alpha 0}}{\sqrt{\ell\beta-\ell\alpha}\,\sqrt{\ell\gamma-\ell\alpha}}$$ [illegible]

[illegible] $X_{0\alpha}$

II $X_{\beta\gamma} = \sqrt{\ell\beta-\ell\gamma}\;z$ [illegible]

[illegible] : $z = \dfrac{X_{\beta\gamma}}{\sqrt{\ell\beta-\ell\alpha}}$

[illegible]

[illegible] 1+ [illegible] 6 [illegible]

f DH. ~ < 1 DH. e ell. fn

f $\mathscr{D}^{\mu}$ $\mathscr{D}$ 𝔑 − e 6^{μ} u = 0

$\sigma(0)$ f e $X_{0\alpha}(0) = 0$

$\sigma_{\alpha}(0) = 1$

en $X_{\beta\gamma} = \frac{\sigma_\beta(u)}{\sigma_\gamma(u)}\Big|_{u=0} = 1$

$X_{\alpha 0}\Big|_0 = \infty$

$\alpha = 3$, $\beta = 2$, $\gamma = 1$

$$\left(\frac{dz}{du}\right)^2 = \left[1-(e_2-e_1)z^2\right]\left[1-[e_1-e_3]z^2\right]$$

$z = X_{03}$

$$z = \frac{X_{30}}{\sqrt{e_2-e_3}\sqrt{e_1-e_3}}$$

$$z = \frac{X_{21}}{\sqrt{e_2-e_3}}$$

Transformation

$\xi = \sqrt{e_1-e_3}\, z$

$$\frac{1}{e_1-e_3}\left(\frac{d\xi}{du}\right)^2 = \left[1-\xi^2\right]\left[1-\overbrace{\frac{e_2-e_1}{e_1-e_3}}^{=k^2}\xi^2\right]$$

[illegible]

$$\frac{1}{(e_1-e_3)}\left(\frac{d\xi}{du}\right)^2 = [1-\xi^2][1-k^2\xi^2]$$

[illegible] $\xi = k_0$, —

[illegible]:

$\sin am\, u = \frac{\sigma(u)}{\sigma_3(u)}$

$q = e^{i\pi\frac{\omega'}{\omega}}$

$\varphi(u) = \sum a_n \, e^{\frac{m i \pi u}{\omega}}$

$\mu a_{m+k} = a_m \rho^{2m}$

$\mu^\rho a_{\rho k} = a_0 \, q^{k\rho\overline{\rho-1}}$

$\alpha < k$ $\quad m=\alpha$

$\mu \, a_{\rho k}$

$\mu a_{2k+\alpha} = a_{k+\alpha} \, q^{2k+\alpha}$

$\mu a_{\rho k+\alpha} = a_{(\rho-1)k+\alpha} \, q^{2\overline{\rho-1}\,k+\alpha}$

$\mu^\rho a_{\rho k+\alpha} = a_\alpha \, q^{2\alpha\rho + \rho\overline{\rho-1}\,k}$

$\varphi(u) = \sum_{n=0}^{k-1} \left(a_n \sum_{-\infty}^{+\infty}{}_{\rho} \frac{q^{2n\rho + \rho\overline{\rho-1}\,k}}{\mu^\rho} \, e^{(\rho k+n) i\pi \frac{u}{\omega}} \right)$

$\kappa \sigma \; \Sigma \; \theta$ fe.

$\theta_{\mu\rho}(u) = \sum e^{\left(\frac{2m+\mu}{2}\right)^2 \tau + \left(\frac{2n+\mu}{2}\right)(2u+\rho i\pi)}$

$= \sum e^{n^2\tau + \frac{\mu^2}{4}\tau + 2n\mu\tau + 2nu + \mu u + \rho n i\pi + \mu\rho\frac{i\pi}{2}}$

$$= \sum_{-\infty}^{+} e^{2\rho r \frac{i\pi\omega'}{\omega} + \rho^2 \kappa i\pi\frac{\omega'}{\omega} - \kappa\rho i\pi\frac{\omega'}{\omega}}$$

$\theta_{\rho}(z) =$

$$+ 2z\rho\kappa + 2rz$$

$$\frac{i\pi}{\omega} u = 2z$$

$$i\pi\frac{\omega'}{\omega} = \tau$$

$$= \sum e^{2\rho r\tau + \rho^2\kappa\tau - \kappa\rho\tau + 2\rho\kappa z + 2rz}$$

$$-\rho^2 y\mu$$

$\rho \kappa \sum$ [illegible]

$$\tau' = \kappa\tau$$

$$= \sum e^{n^2\tau' + \frac{2nr}{\kappa}\tau' - n\tau' + 2z(n\kappa + r) - n^2 y\mu}$$

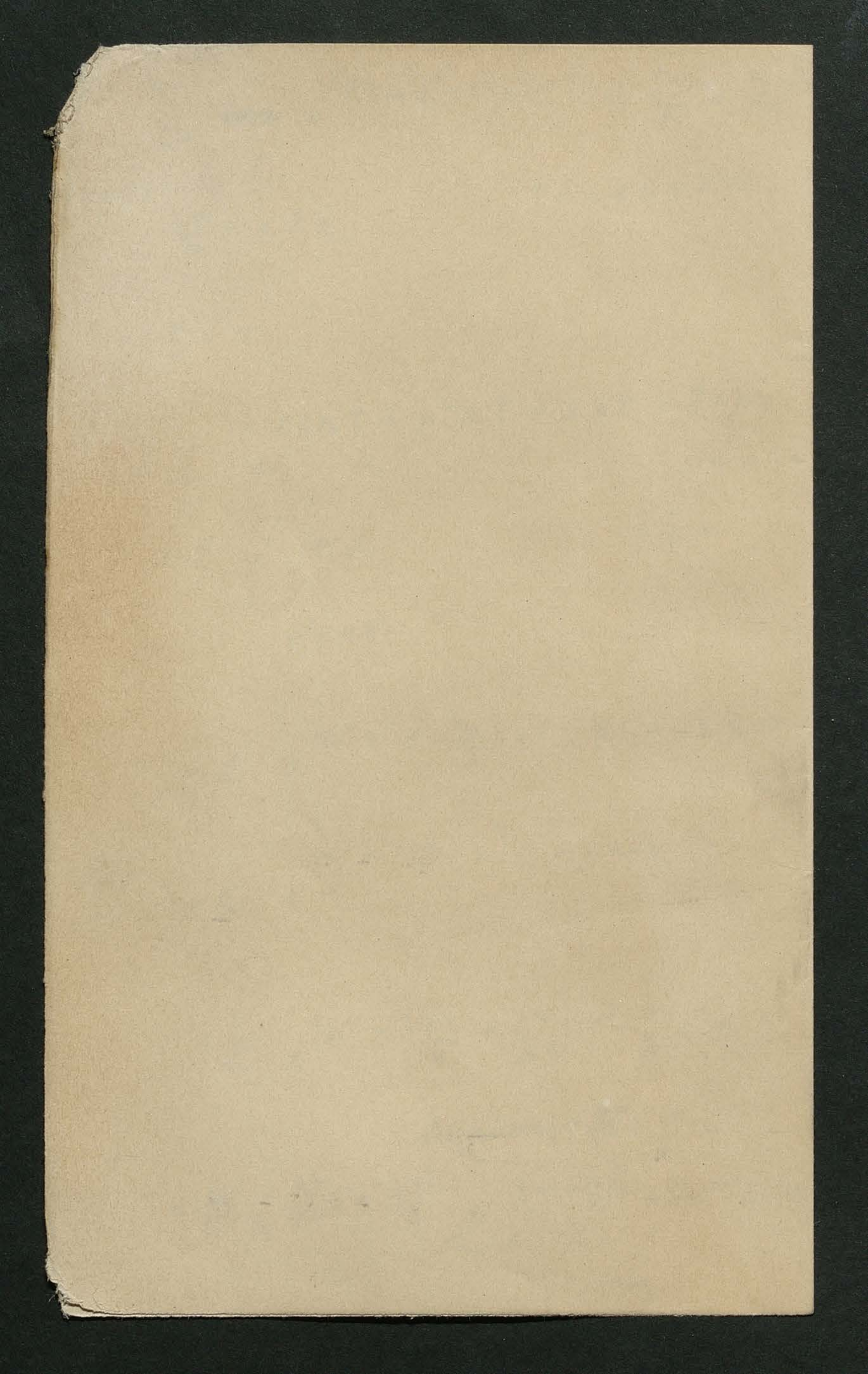
$$= \sum e^{\left[n^2 + \frac{2nr}{\kappa} - n\right]\tau' + (n\kappa + r)2z -}$$

$$\sum e^{\left(n^2 + \frac{2nr}{\kappa} + \frac{r^2}{\kappa^2}\right)\tau' - \frac{r^2}{\kappa^2}\tau' +}$$

$$~~2z(n\kappa + r)~~ + r\frac{\tau'}{\kappa} + -$$

$$= e^{-\frac{r^2}{\kappa^2}\tau' + \frac{r\tau'}{\kappa}} \sum e^{\left(n + \frac{r}{\kappa}\right)^2\tau' + 2z\left(n + \frac{r}{\kappa}\right)2\kappa - n\log\mu}$$

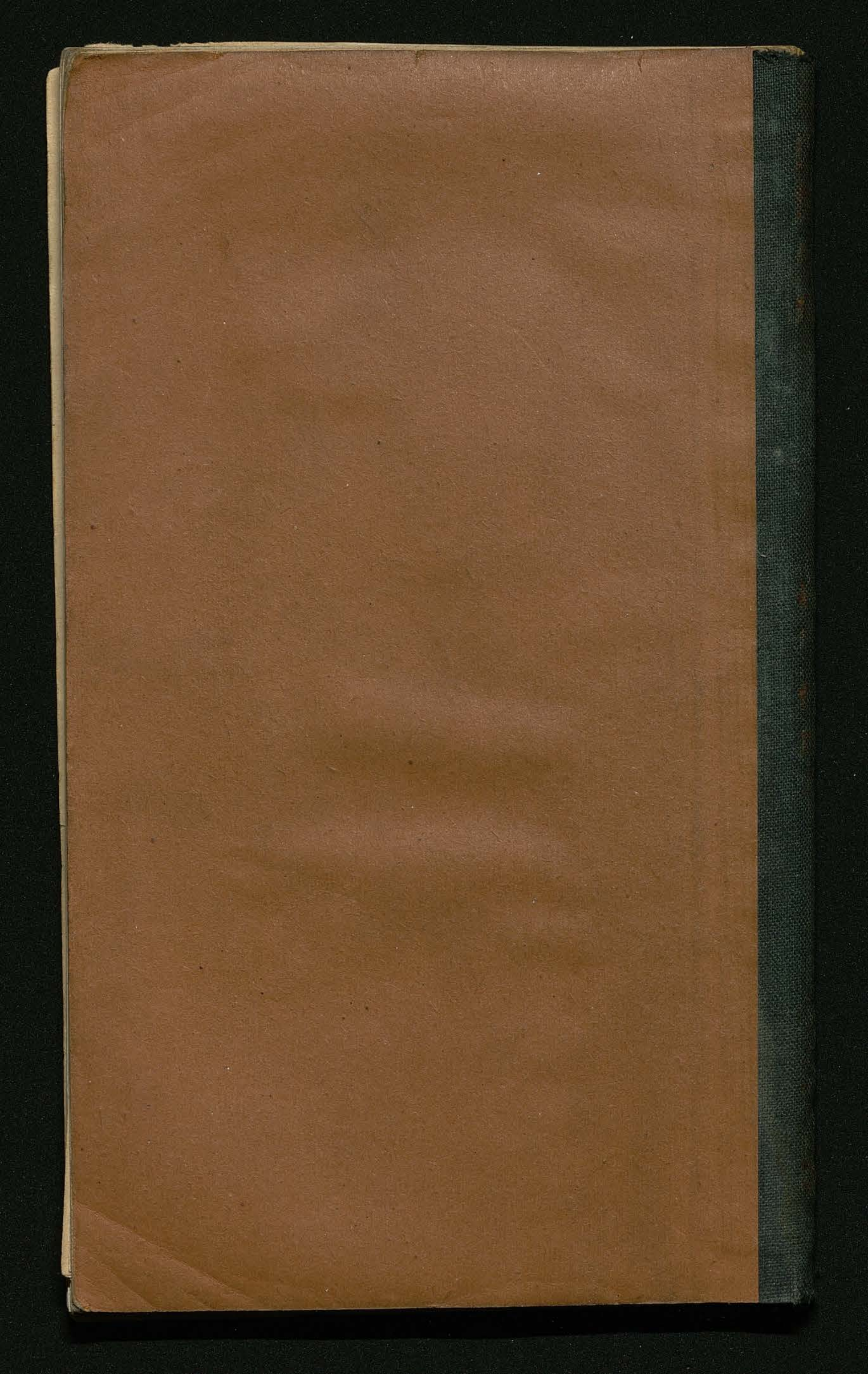
$\frac{\tau'}{\kappa^2} = \frac{\tau''}{4}$ $\quad nk+2 = 2m+p$



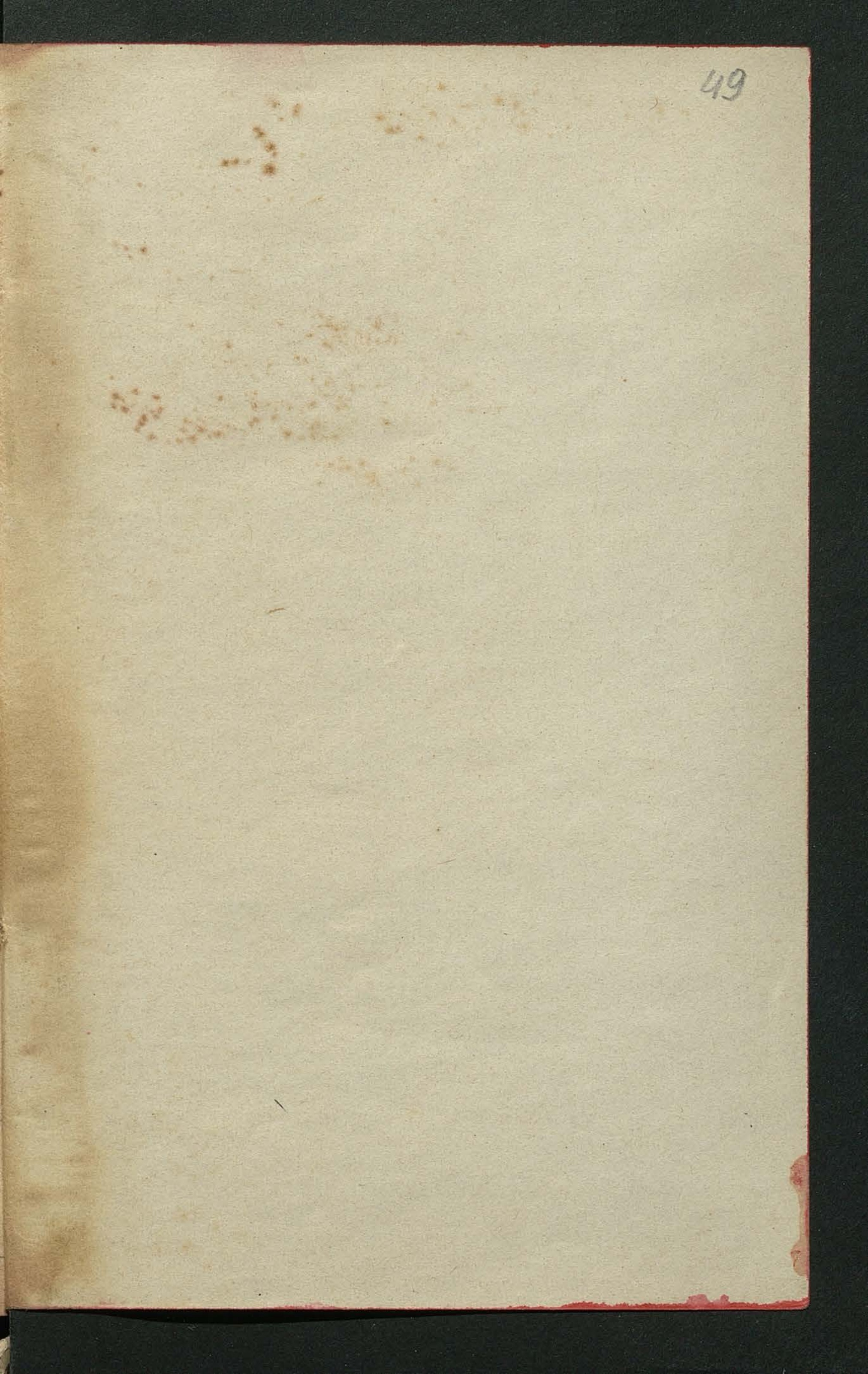
O. Reynolds On rolling friction Phil. Trans. 47
155-175 CLXVII

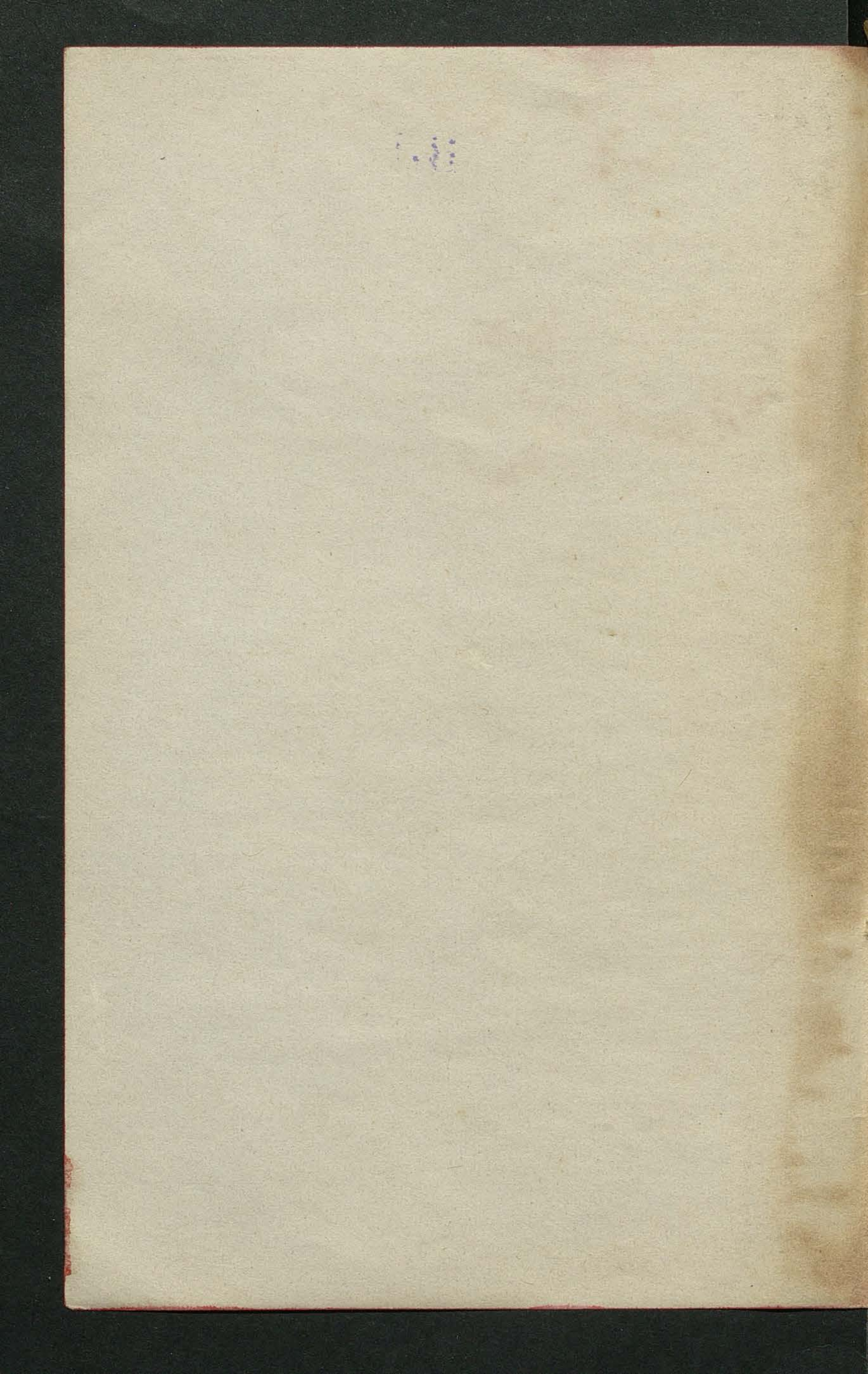
Phil. mag. (5) I. 75-77

Fortschritte in J. 1876 p. 250




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1/2

$f(x,y,z)=0$ [illegible] $\varphi(x,y,z)=0$

[illegible]-Curve

$f_1\,dx + f_2\,dy + f_3\,dz = 0$

$\varphi_1\,dx + \varphi_2\,dy + \varphi_3\,dz = 0$

$\frac{dx}{ds} = \cos\alpha$

$f_1 \cos\alpha + f_2 \cos\beta + f_3 \cos\gamma = 0$

[illegible]

$\varphi_1 \cos\alpha + \varphi_2 \quad\quad = 0$

[illegible]

$\cos\alpha : \cos\beta : \cos\gamma = \xi - x : \eta - y : \zeta - z$

$f_1(\xi - x) + f_2(\eta - y) + f_3(\zeta - z) = 0$

[illegible] Ebene [illegible]

= Tangentialebene

[illegible]

$z - f(x,y) = 0$ [illegible]

[illegible] Tg. Eb.

[illegible] $\zeta = 0$

$z = f(xy)$

$= f(0) + \quad + f_{11} x^2 + 2 f_{12} xy + 2 f_{22} y^2$

$+ (\quad)^3 + \ldots (\quad)^4$

$\omega\ z = 0$ [illegible]

[illegible]

[illegible]

sing. [illegible]

$f_{12}^2 - f_{11} f_{22} > 0$ [illegible]

< 0 [illegible]

$f(xy) - z = 0$

$\frac{\partial z}{\partial x} = p \quad \frac{\partial z}{\partial y} = q$

$\frac{\partial^2 z}{\partial x^2} = r \quad \frac{\partial^2 z}{\partial x \partial y} = s \quad \frac{\partial^2 z}{\partial y^2} = t$

$p(\xi - x) + q(\eta - y) - (\zeta - z) = 0$

$\cos X : \frac{p}{\sqrt{1+p^2+q^2}} \quad \frac{q}{\sqrt{1+p^2+q^2}} \quad \frac{-1}{\sqrt{1+p^2+q^2}}$

$dz = p\, dx + q\, dy$

$\cos \gamma = p \cos \alpha + q \cos \beta$

z. z. 26. ∡ an p H N v. Pl. N vel = θ

$$\cos\theta = \frac{p\cos\lambda + q\cos\mu - \cos\nu}{\sqrt{1+p^2+q^2}}$$

$$d\cos\gamma = p\,d\cos\alpha + q\,d\cos\beta + dp\cos\alpha + dq\cos\beta$$

$$d\sigma\cos\gamma = d\sigma(p\cos\alpha + q\cos\beta) + dp\cos\alpha + dq\cos\beta$$

$(-\cos\nu)$

$$d\sigma(p\cos\alpha + q\cos\beta) = -dp\cos\alpha + dq\cos\beta$$

$$d\sigma\cos\theta = -\frac{dp\cos\alpha + dq\cos\beta}{\sqrt{1+p^2+q^2}}$$

$$dp = r\,dx + s\,dy \qquad = ds(r\cos\alpha + s\cos\beta)$$

$$dq = s\,dx + t\,dy \qquad = \ldots$$

~~$= ds\cos\alpha + ds\cos\beta$~~

$$\underbrace{\frac{d\sigma}{ds}}_{\frac{1}{R}}\cos\theta = -\frac{r\cos^2\alpha + 2s\cos\alpha\cos\beta + t\cos^2\beta}{\sqrt{1+p^2+q^2}}$$

$$\frac{\cos\theta}{R} = \frac{r\cos^2\alpha + 2s\cos\alpha\cos\beta + t\cos^2\beta}{\pm\sqrt{1+p^2+q^2}}$$

f. W_s ~ Kr. Rad. d. 1 z. z. 2 cont.

Theorem von Meusnier:

[illegible]

$\theta = 0$ $\qquad R_1$

$$\frac{\cos\theta}{R} = \frac{1}{R_1}$$

$R_1 =$ [illegible] Rad. [illegible]

$$R_1 \cos\theta = R$$

R

θ

R_1

N

[illegible] Rad. [illegible]

[illegible] Rad. [illegible]

[illegible] R [illegible]

[illegible] R [illegible]

[illegible] Norm [illegible]

$$\frac{1}{R} = \frac{[illegible]}{V}$$

$\sin^2\alpha + \cos^2\beta = 1$

$\cos\beta = \sin\alpha$

$$\frac{1}{R} = \frac{r\cos^2\alpha + 2s\cos\alpha\sin\alpha + t\sin^2\alpha}{\sqrt{}}$$

Extreme Werte: [illegible] α diff. [illegible] $= 0$

$$-2r\cos\alpha\sin\alpha + 2s\cos 2\alpha + 2t\sin\alpha\cos\alpha$$

$$-(r-t)\,\mathrm{tg}\,2\alpha + 2s = 0$$

$$\mathrm{tg}\,2\alpha = \frac{2s}{r-t}$$

[illegible] Rad.

[illegible] Rad. [illegible]

[illegible]

[illegible] + [illegible]

[illegible] $\perp$ [illegible]

[illegible] α [illegible] $\alpha + \frac{\pi}{2}$

[illegible] N. Sch. [illegible] Haupt N. Sch.

Haupt [illegible] Rad.

[illegible] $\alpha = 0$ [illegible] $\frac{\pi}{2}$ [illegible]

$\frac{1}{R} = r_1\cos^2\alpha + t_1\sin^2\alpha$ | [illegible] $s = 0$ [illegible]

[illegible] $\alpha = 0$ [illegible] $\frac{1}{R_1} = r_1$ | $\alpha = \frac{\pi}{2}$ $\frac{1}{R_1} = t_2$

$$\frac{1}{R} = \frac{\cos^2\alpha}{R_1} + \frac{\sin^2\alpha}{R_2}$$ Euler'sche Relat.

[illegible shorthand]

$$\frac{1}{R'} = \frac{\sin^2\alpha}{R_1} + \frac{\cos^2\alpha}{R_2}$$

$$\frac{1}{R} + \frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2} = \text{const.}$$

([illegible shorthand] Rad. R_1 ; R_2

[illegible shorthand] R_1 = Rad. V. [illegible shorthand] Relat. [illegible shorthand])

$$\frac{1}{R'} = \frac{1}{R_1} + \cos^2\alpha\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

[illegible shorthand] > 0 [illegible] $\frac{1}{R'} > \frac{1}{R_1}$

$\frac{1}{R_2} - \frac{1}{R_1} > 0$ $\quad \frac{1}{R'} > \frac{1}{R_1}$

R_1 Max.

~~$R_1 - R_2 < 0$~~

$$\frac{1}{R} = \frac{1}{R_2} + \cos^2\alpha\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$R_1 - R_2 > 0$ $\quad$ R_1 Max. $\quad \frac{1}{R_1} < \frac{1}{R}$

R_2 Min. $\quad \frac{1}{R_2} > \frac{1}{R}$

$R_1 = R_2$ … R … : Nabelpunkte

R_1, R_2 …

$R = \infty$

$\frac{\cos^2\alpha}{R_1} + \frac{\sin^2\alpha}{R_2} = 0$

… R_1, R_2

… R …

… Rad.

… R_1, R_2 … R …

… $R_1 = +\infty$, $R_2 = -\infty$

$\frac{\cos^2\alpha}{R_1} + \frac{\sin^2\alpha}{R_2} = 0$

$$\operatorname{tg}\alpha = \pm\sqrt{-\frac{R_2}{R_1}}$$

hyperbolisch gekrümmt …

elliptisch " …

Haupt Tang.

$R_1 - R_2 = 0$ …

Theorem von Dupin

$$MO'^2 = OO' \cdot O'K$$

$$OK = OO' + O'K = OO' + \frac{MO'^2}{OO'}$$

$$R = \frac{1}{2} \lim \frac{MO'^2}{OO'}$$

$$= \frac{1}{2} \lim \frac{u^2}{z}$$

[illegible]

$$= \frac{1}{2h} \lim \frac{u^2 h}{z}$$

$$\lim \frac{u\sqrt{h}}{\sqrt{z}} = \rho$$

$$R = \frac{\rho^2}{2h}$$

[illegible]

[illegible] = [illegible] 2 [illegible] RV.

Indicatrix [illegible] RV [illegible] R.

[illegible] XY [illegible]

$$z = \frac{1}{2}(rx^2 + 2sxy + ty^2) + R_3$$

[illegible] Polar [illegible] RV [illegible] u = [illegible]

$$z = \frac{1}{2}u^2(r\cos^2\alpha + 2s\sin\alpha\cos\alpha + t\sin^2\alpha) + R_3$$

$$\frac{2z}{u^2} = r\cos^2\alpha + 2s\sin\alpha\cos\alpha + t\sin^2\alpha$$

$$\lim = \frac{1}{R} \qquad R = \frac{\rho^2}{2h}$$

$$2h = r\rho^2\cos^2\alpha + 2s\rho\sin\alpha\,\rho\cos\alpha + t\rho^2\sin^2\alpha$$

$$= r\xi^2 + 2s\xi\eta + t\eta^2 = 2h \text{ Indicatrix}$$

[illegible] z [illegible] centrisch

[illegible] R [illegible] RV.

[[illegible] R [illegible]

$$s^2 - rt < 0 \quad \text{Ellipse}$$

$$= 0 \quad \text{[illegible] Doppelte [illegible]}$$

$$> 0$$

$s^2 - rt > 0$ $\quad +h$

$\quad\quad\quad\quad -h$

Hyp

Asympt

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2h$$

[illegible]

[illegible]

$$\frac{\cos\theta}{R} = \frac{r\cos^2\alpha + 2s\cos\alpha\cos\beta + t\cos^2\beta}{\sqrt{1+p^2+q^2}} = \frac{1}{R}$$ [illegible]

$$\cos\gamma = p\cos\alpha + q\cos\beta$$

$$\cos^2\alpha + \cos^2\beta + (p\cos\alpha + q\cos\beta)^2 = 1$$

$$\cos^2\alpha\,(1+p^2) + \dots = 1$$

$$\frac{R}{\sqrt{1+p^2+q^2}} = \frac{(1+p^2)\cos^2\alpha + 2pq\cos\alpha\cos\beta + (1+q^2)\cos^2\beta}{r\cos^2\alpha + 2s\cos\alpha\cos\beta + t\cos^2\beta}$$

$$\frac{\cos\alpha}{\cos\beta} = \lambda$$

$$\lambda^2\left\{\frac{rR}{\sqrt{1+p^2+q^2}} - (1+p^2)\right\} +$$

$$+ 2\lambda\left\{\frac{Rs}{\sqrt{}} - pq\right\} + \left\{\frac{Rt}{\sqrt{}} - 1+q^2\right\} = 0$$

[illegible]

[illegible]

[illegible] $\lambda^2(A+BR) + 2\lambda(C+DR) + E + FR = 0$

$f(\lambda, R) = 0$

$$\frac{\partial R}{\partial \alpha} = -\frac{\frac{\partial f}{\partial \alpha}}{\frac{\partial f}{\partial R}} = 0 \qquad \frac{\partial f}{\partial \alpha} = 0 \qquad = 2 \nu \ldots$$

I II III

$$\left[\frac{Rs}{\sqrt{1+p^2+q^2}} - pq\right]^2 - \left[\frac{Rr}{\sqrt{}} - (1+p^2)\right]\left[\frac{Rt}{\sqrt{}} - (1+q^2)\right] = 0$$

Wf HKr R.

$$1/2\ \frac{R^2}{1+p^2+q^2}(s^2 - rt) + \frac{R}{\sqrt{}}\left[-2pqs + (1+p^2)t + (1+q^2)r\right]$$

$$+ p^2q^2 - (1+p^2)(1+q^2) = 0$$

$$-1 + p^2 + q^2$$

[illegible] diff

[illegible]:

$$\mathrm{I}\cos^2\alpha + \mathrm{II}\cos\alpha\cos\beta + \mathrm{III}\cos^2\beta = 0$$

$$\frac{\partial}{\partial \alpha} \quad \mathrm{I}\cos\alpha + \mathrm{II}\cos\beta = 0$$

$$\frac{\partial}{\partial \beta} \quad \mathrm{II}\cos\alpha + \mathrm{III}\cos\beta = 0$$

$$\frac{\cos\beta}{\cos\alpha} = -\frac{\mathrm{I}}{\mathrm{II}} = -\frac{\mathrm{II}}{\mathrm{III}}$$

[illegible]

[illegible]:

[illegible] R [illegible]

$$(1+p^2)\cos\alpha + pq\cos\beta - \frac{R}{\sqrt{}}\left[r\cos\alpha + s\cos\beta\right] = 0$$

$$pq\cos\alpha + (1+q^2)\cos\beta - \frac{R}{\sqrt{}}\left[s\cos\alpha + t\cos\beta\right] = 0$$

$\frac{1}{R}$ [illegible]

$$= \frac{r\cos\alpha + s\cos\beta}{(1+p^2)\cos\alpha + pq\cos\beta} = \frac{s\cos\alpha + t\cos\beta}{pq\cos\alpha + (1+q^2)\cos\beta}$$

[illegible] R

$$R_1 R_2 = -\frac{(1+p^2+q^2)^2}{s^2 - rt} \qquad \frac{1}{R_1 R_2} = \frac{rt - s^2}{(1+p^2+q^2)^2}$$

[illegible] $rt - s^2 < 0$ [illegible] ellipt. / hyperb. [illegible] ‖ [illegible]

Gesamt

Abh. [illegible]. [illegible] V Rad. 1 [illegible] Norm. 1 ‖

[illegible]

[illegible] Gesamt = Gesamtkrümmung

[illegible]

[illegible]

Gesamt [illegible] deformier,

[illegible]

= Biegungsinvariante

[illegible]

I [illegible]

II [illegible]

[illegible]

[illegible]

$$R_1 + R_2 = \frac{(1+p^2)t + (1+q^2)r - 2pqs}{rt - s^2}\sqrt{1+p^2+q^2}$$

$$\frac{1}{R_1 R_2 (R_1+R_2)}$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{(1+p^2)t + (1+q^2)r - 2pqs}{[1+p^2+q^2]^{3/2}}$$

[illegible]

[illegible] $s = 0$ [illegible]

[illegible] sattelartige [illegible]

[illegible] Minimalfläche [illegible]

[illegible] $= 0$.

Capillarität

[illegible] Lamellen [illegible] $= 0$.

Schwarz

S. Lermann

$\frac{1}{R_1^2} + \frac{1}{R_2^2}$ Casorati [illegible]

[illegible]

[illegible]

[illegible] const.

20/1 [illegible] Gaus. [illegible]

[illegible] convex. [illegible]

[illegible] Indicatrix

$r\xi^2 + t\eta^2$ [illegible] H. Kn. R.

Z

Y

X

$p=0 \quad q=0 \quad s=0$

Indic. [illegible]

[illegible]

[illegible]

[illegible] Indic.

$$\zeta - z = p(\xi - x) + q(\eta - y)$$

$$-dz = dp(\xi - x) + dq(\eta - y) - p\,dx - q\,dy$$

$\underbrace{\qquad}_{-dz}$ $= 0$ [illegible]

$$0 = dp(\xi - x) + dq(\eta - y)$$

[illegible] $= 0$

$0 = dp\,\xi + dq\,\eta.$ [illegible]

$dp \cos\alpha' + dq \sin\alpha' = 0$

$dp = r\cos\alpha\, ds$

$dq = t \sin\alpha\, ds$

$r\cos\alpha\cos\alpha' + t\sin\alpha\sin\alpha' = 0$ [illegible]

[illegible]

[illegible]

[illegible] orthogonale Paare e Involution

[illegible]

[illegible]

[illegible] Curvensystem

[illegible] = Curven [illegible]

C.)

$r\,dx\,dx' + t\,dy\,dy' = 0$

$$\frac{r\cos^2\alpha + 2s\cos\alpha\cos\beta + t\cos^2\beta}{\sqrt{1+p^2+q^2}} = \frac{1}{R}$$

$r\,dx\,dx' + s(dx\,dy' + dx'\,dy) + t\,dy\,dy' = 0$

$f(xy)=0$ ← $dx : dy : dz$

[illegible]

cong. Curven

[illegible]

[illegible] Haupt Tg. = Haupt Tang. C.

Krümmungs C.

$r\cos^2\alpha + 2s\cos\alpha\cos\beta + t\cos^2\beta = 0$ [illegible]

$r\,dx^2 + 2s\,dx\,dy + t\,dy^2 = 0$ [illegible]

$f(xy)=0$

[illegible] Tg. C.

2 Systeme

$$\frac{x^2}{a^2-\lambda^2} + \frac{y^2}{b^2-\lambda^2} = 1$$

Ell. Hyp.

[illegible] Norm [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

21/2 [illegible] N.

$$\xi = x + lu$$
$$\eta = y + lv$$
$$\zeta = z + lw$$

[illegible] $= l\sqrt{uu+vv+ww}$

[illegible]:

$$\xi = x + lu + dx + l'du$$

[illegible] l' [illegible] ξ [illegible] l

$$0 = (l'-l)u + dx + l'du$$
$$0 = (l'-l)v + dy + l'dv$$
$$0 = (l'-l)w + dz + l'dw$$

[illegible] Δ [illegible] $l'-l$ [illegible]

$$\begin{vmatrix} dx & dy & dz \\ u & v & w \\ du & dv & dw \end{vmatrix} = 0$$

= [illegible] Diff. [illegible]

= Diff. [illegible] develop. Fläche

[[illegible]]

xyz x+dx

[illegible]

= [illegible]

$du = u_1\,dx + u_2\,dy + u_3\,dz$

$$0 = dx\left(\frac{1}{l'} + u_1\right) + dy\,u_2 + dz\,u_3 + \frac{(l'-l)}{l'}\,u$$

$$0 = dx\,v_1 + dy\left(\frac{1}{l'} + v_2\right) + dz\,v_3 + \frac{l'-l}{l'}\,v$$

$$0 = dx\,w_1 + dy\,w_2 + \left(\frac{1}{l'} + w_3\right)dz + \frac{l'-l}{l'}\,w$$

[illegible]: $u\ v\ w$ [illegible] $dx\,dy\,dz$ [illegible]

$$0 = dx\,u + dy\,v + dz\,w$$

[illegible] 4 homog. [illegible] / $dx\,dy\,dz$ $\frac{l'-l}{l'}$ [illegible]

$\Delta = 0$ ~

$$\begin{vmatrix} \frac{1}{t'}+u_1 & u_2 & u_3 & u \\ v_1 & \frac{1}{t'}+v_2 & v_3 & v \\ w_1 & w_2 & \frac{1}{t'}+w_3 & w \\ u & v & w & 0 \end{vmatrix} = 0$$

[illegible] 2ten [illegible] / $\frac{1}{t'}$ [illegible]

/ $u\ v\ w$ [illegible] $p, q, -1$

$u_1\ u_2\ u_3 = r\ s\ 0$

$v_1\ v_2\ v_3 = s\ t\ 0$

$w_1\ w_2\ w_3 = 0\ 0\ 0$

[illegible] [illegible] / [illegible]

[illegible] $f(xyz) = 0$ $u = f_1$

$v = f_2$

$w = f_3$ etc.

$\int \sqrt{rt - s^2}$ [illegible]:

$$\begin{vmatrix} 0 & f_1 & f_2 & f_3 \\ f_1 & f_{11} & f_{12} & f_{13} \\ f_2 & f_{21} & f_{22} & f_{23} \\ f_3 & f_{31} & f_{32} & f_{33} \end{vmatrix}$$

[illegible] Curven [illegible] Hkr. Rad. [illegible]

[illegible] Δ [illegible] Hkr. Rad. [illegible]

$u \quad v \quad w = p \quad q \quad -1$

$$\begin{vmatrix} dx & dy & dz \\ p & q & -1 \\ dp & dq & 0 \end{vmatrix} = dq\,[p\,dz + dx] + dp\,[q\,dz + dy] = 0$$

$$\frac{dq}{q\,dz + dy} = \frac{dp}{p\,dz + dx} \qquad dz = p\,dx + q\,dy$$

$$\frac{s\,dx + t\,dy}{(1+q^2)\,dy + pq\,dx} = \frac{r\,dx + s\,dy}{(1+p^2)\,dx + pq\,dy}$$

$dx : dy$ [illegible]

$$\frac{s \cos\alpha + t \cos\beta}{(1+q^2)\cos\beta + pq\cos\alpha} = \text{———}$$

[illegible]

[illegible] Curven [illegible] – devel.

Olinde Rodriguez:

[illegible]

Rückkehr C. [illegible]

[illegible]

[illegible]

$a\ b\ c$ = [illegible]

[illegible] Coord. [illegible] $x + Ra$

ds

dR

R

[illegible] Curve

$$\frac{dx + d(Ra)}{a} = ds =$$

$$= \frac{dy + d(Rb)}{b} = \frac{dz + d(Rc)}{c}$$

$$\frac{dx + R\,da}{a} + dR$$

$$ds - dR = \frac{dx + R\,da}{a} = \frac{dy + R\,db}{b} = \frac{dz + R\,dc}{c}$$

$$dx + R\,da = a(ds - dR) \quad | \quad a \quad = 0$$
$$dy + R\,db = b(ds - dR) \quad | \quad b \quad = 0$$
$$dz + R\,dc = c(\qquad) \quad | \quad c \quad = 0$$

$$0 + 0 = ds - dR$$

$$ds = dR$$

f 3 Formeln [illegible]

F. [illegible] Rodriguez

∴ Dr. e Rückk. C. = D° ∴ 6

ω ∴ d. = d. ∫ e r e tg. N

σ ∼ f. = Evolute

e ∫ e H.Kr. Rad. = Evolute e

∫ u N < 2 H.Kr.

∴ u N 2 devel. Fläche

∴ tg. ∫ e H.Kr.

I

II

II

Fläche der Krümmungs Centra

H.Kr. L.

e Rückk. C. e devell. e ∫ e Centra

∫ e ∼ N. = tg. ~~+ 2~~ e ∫ e Kr. Centra

∫ e ∫ tg. ; 6 2

[illegible] I [illegible] II
[illegible] II

[illegible] ([illegible] I [illegible])

[illegible] Ebene [illegible] 2 [illegible]

[illegible] I [illegible] II

[illegible] Ebene [illegible] II = [illegible] II [illegible] I

[illegible] ⊥ [illegible]

[illegible] I [illegible] II ⊥ [illegible]

[illegible] Normalen [illegible] geodät. Lin.

Kr. L. [illegible] evolv. geodätische Linien

[illegible] Kr. Centra

/ Geodätische Linien

[illegible]

[illegible] Normale [illegible] 18.[illegible]

~~[illegible]~~

$$\begin{vmatrix} \xi - x & \eta - y & \zeta - z \\ dx & dy & dz \\ d^2x & d^2y & d^2z \end{vmatrix} = 0 \qquad \xi\eta\zeta$$

[illegible] Coeff. [illegible] $\xi - x$ [illegible] proj. [illegible] [illegible] 2

[illegible] N. [illegible] $=0$

$$\begin{vmatrix} u & v & w \\ dx & dy & dz \\ d^2x & d^2y & d^2z \end{vmatrix} = 0$$

= Die Gleichung [illegible] geod. Linie

[illegible] Centre

[illegible] ($\ldots$ cos)

[illegible]

const. [illegible] ; [illegible] centre

[illegible] Evolute [illegible] + 1 [illegible]

---

g'' g'

g [illegible] Evolute

$\| g' - g$

$\tau + g$

R

C

$$u = \frac{R}{\cos(\tau + g)}$$

[illegible] Evolute [illegible] C.

[illegible] const.

$\sim g' - g$

[illegible] Evolute [illegible]

[illegible] const. [illegible] Evolute

– durch Fl.

[illegible]

[illegible]

[illegible] Evolventen [illegible]

[[illegible] Centra [illegible]]

[illegible]

BH.
[illegible] Curve
Theorem v. Joachimsthal

BM.

II
I

[illegible] dev. F.

[illegible] Fl. = Evolu. [illegible] geod. Krümm.

[illegible] Centra [illegible]

[illegible] Evolu. [illegible]

[illegible] Tg. d. Evoluten [illegible]

N. [illegible] N.

[illegible] Evolute [illegible] const. ∡

[illegible] N. [illegible] const. ∡ ged.

[illegible] = Evolvente [illegible] Curven [illegible] N.

[illegible] C. [illegible] Evolute [illegible]

[illegible] const. ∡ [illegible]

[illegible]

[illegible] N. [illegible] N. [illegible]

[illegible] Evol.

[illegible] const. ∡ N_2 [illegible]

[illegible] -dev. Fl.

[illegible] Torsion [illegible]

Theorem von Lancret

[illegible] ∡ [illegible]

[illegible]

[illegible] ∡ = Torsion [illegible]

[illegible] [illegible] AN.

[illegible] AN. [illegible] N. $= \tau g$

[illegible] $= d\tau =$ Tors. [illegible]

[illegible] geodet. [illegible]

[illegible] geod. [illegible] N.

$\tau g = 0$

$d\tau = 0$

Schnitt von 3 Flächen

$f(xyz) = \lambda$

$\varphi(xyz) = \mu$

$\psi(xyz) = \nu$

[illegible]

[illegible]

$x = f_1(\lambda\mu\nu)$

$y =$

$z =$

[illegible] C. [illegible]

[illegible] prop. $\frac{\partial x}{\partial \lambda}$ $\frac{\partial y}{\partial \lambda}$ $\frac{\partial z}{\partial \lambda}$

$$\frac{\partial x}{\partial \lambda}\frac{\partial x}{\partial \nu}+\frac{\partial y}{\partial \lambda}\frac{\partial y}{\partial \nu}+\frac{\partial z}{\partial \lambda}\frac{\partial z}{\partial \nu}=0$$ [illegible]

[illegible]:

$$\frac{\partial x}{\partial \lambda}\frac{\partial x}{\partial \mu}+ \text{—} = 0$$

$$\frac{\partial x}{\partial \mu}\frac{\partial x}{\partial \nu}+ \text{—} = 0$$

[illegible]

$$\frac{\partial x}{\partial \lambda} = \rho\left(\frac{\partial y}{\partial \nu}\frac{\partial z}{\partial \mu}-\frac{\partial z}{\partial \nu}\frac{\partial y}{\partial \mu}\right)$$

$$\frac{\partial y}{\partial \lambda} = \rho\left(\frac{\partial z}{\partial \nu}\frac{\partial x}{\partial \mu}-\frac{\partial x}{\partial \nu}\frac{\partial z}{\partial \mu}\right)$$

$$\frac{\partial z}{\partial \lambda} = \rho\left(\frac{\partial x}{\partial \nu}\frac{\partial y}{\partial \mu}-\frac{\partial y}{\partial \nu}\frac{\partial x}{\partial \mu}\right)$$

$$\frac{\partial I}{\partial \mu}\quad \frac{\partial II}{\partial \nu}\quad \frac{\partial III}{\partial \lambda}:$$

$$\sum \frac{\partial^2 x}{\partial \lambda\,\partial \mu}\frac{\partial x}{\partial \nu}+\frac{\partial x}{\partial \lambda}\frac{\partial^2 x}{\partial \nu\,\partial \mu} = 0$$

$$\sum \frac{\partial^2 x}{\partial \lambda\,\partial \nu}\frac{\partial x}{\partial \mu}+\frac{\partial^2 x}{\partial \nu\,\partial \mu}\frac{\partial x}{\partial \lambda} = 0$$

$$\sum \frac{\partial^2 x}{\partial \lambda\,\partial \mu}\frac{\partial x}{\partial \nu}+\frac{\partial^2 x}{\partial \nu\,\partial \lambda}\frac{\partial x}{\partial \mu} = 0$$

$$1+2-3=0$$

[illegible]

$$\sum \frac{\partial^2 x}{\partial\lambda\,\partial\mu}\,\frac{\partial x}{\partial\nu} = 0$$

$$\sum \frac{\partial^2 x}{\partial\lambda\,\partial\nu}\,\frac{\partial x}{\partial\mu} = 0$$

$$\sum \frac{\partial^2 x}{\partial\mu\,\partial\nu}\,\frac{\partial x}{\partial\lambda} = 0$$

[illegible]

[illegible]

$$\begin{vmatrix} dx & dy & dz \\ u & v & w \\ du & dv & dw \end{vmatrix} = 0$$

[illegible]

$$\begin{vmatrix} \frac{\partial x}{\partial\mu} & \frac{\partial y}{\partial\mu} & \frac{\partial z}{\partial\mu} \\ \frac{\partial x}{\partial\nu} & \frac{\partial y}{\partial\nu} & \frac{\partial z}{\partial\nu} \\ \frac{\partial^2 x}{\partial\mu\,\partial\nu} & \frac{\partial^2 y}{\partial\mu\,\partial\nu} & \frac{\partial^2 z}{\partial\mu\,\partial\nu} \end{vmatrix} = 0$$

$du =$ [illegible]

[illegible] Relat. [illegible]

[illegible] ed.

[illegible]

Dupin: [illegible] orthog. [illegible]

Rotations

$$\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} + \frac{z^2}{c^2+\lambda} = 1$$

$-\infty \quad \lambda \quad +\infty$

xyz

$$\frac{x}{a^2-\lambda} \qquad \frac{y}{b^2-\lambda} \qquad \frac{z}{c^2-\lambda}$$

$$\frac{x}{a^2-\mu} \qquad \frac{y}{b^2-\mu} \qquad \frac{z}{c^2-\mu}$$

$$\frac{x^2}{(a^2-\mu)(a^2-\lambda)} + \frac{y^2}{(b^2-\lambda)(b^2-\mu)} + \frac{z^2}{(c^2-\lambda)(c^2-\mu)} = 0$$

[illegible]: $\frac{x^2}{a^2-\mu} + \frac{y^2}{b^2-\mu} + \frac{z^2}{c^2-\mu} = 1$

$-$

$$(\lambda-\mu)\left[\frac{x^2}{(a^2-\lambda)(a^2-\mu)} + \frac{y^2}{\quad} + \frac{z^2}{\quad}\right] = 0$$

[illegible] $\perp$ [illegible]

$f(\lambda) =$

$$(a^2-\lambda)(b^2-\lambda)(c^2-\lambda) - (b^2-\lambda)(c^2-\lambda)x^2 - (a^2-\lambda)(c^2-\lambda)y^2$$
$$- (a^2-\lambda)(b^2-\lambda)z^2 = 0$$

[illegible]:

[illegible] $\quad -\infty \; v_1 \; c^2$

$c^2 \; v_2 \; b^2$

$b^2 \; v_3 \; a^2$

$\lambda \qquad c^2 \qquad b^2 \qquad a^2$

[illegible]

$$\frac{x^2}{a^2-\lambda} + \frac{y^2}{b^2-\lambda} + \frac{z^2}{c^2-\lambda} = 1$$

[illegible] Inter. [illegible] 36 [illegible]

λ_1 — λ_1 | c^2 | λ_2 | b^2 | λ_3 | a^2

λ_1 – Ellipsoide

λ_2 einschaliges Hyperboloid

λ_3 zweischalige Hyperboloide

[$\lambda > \lambda_3$ [illegible] reell. Fläche [illegible]]

[illegible] $\lambda_1 = c^2$ [illegible]

[illegible]

[illegible] Ellipsoide [illegible] $\lambda = c^2$ [illegible] $z^2 = 0$

[illegible] XY Ebene

$$\frac{x^2}{a^2-c^2} + \frac{y^2}{b^2-c^2} = 1$$ — [illegible] Ellipsoid 16

Focal-Ellipse

[illegible] $\lambda_2 = c^2$ [illegible]

$\frac{x^2}{a^2-c^2} + \frac{y^2}{b^2-c^2} = 1$ [illegible]

Focalell.

λ_2 b^2

$$\frac{x^2}{a^2-b^2} + \frac{z^2}{c^2-b^2} = 1$$

= Hyperbel

λ_3 b^2

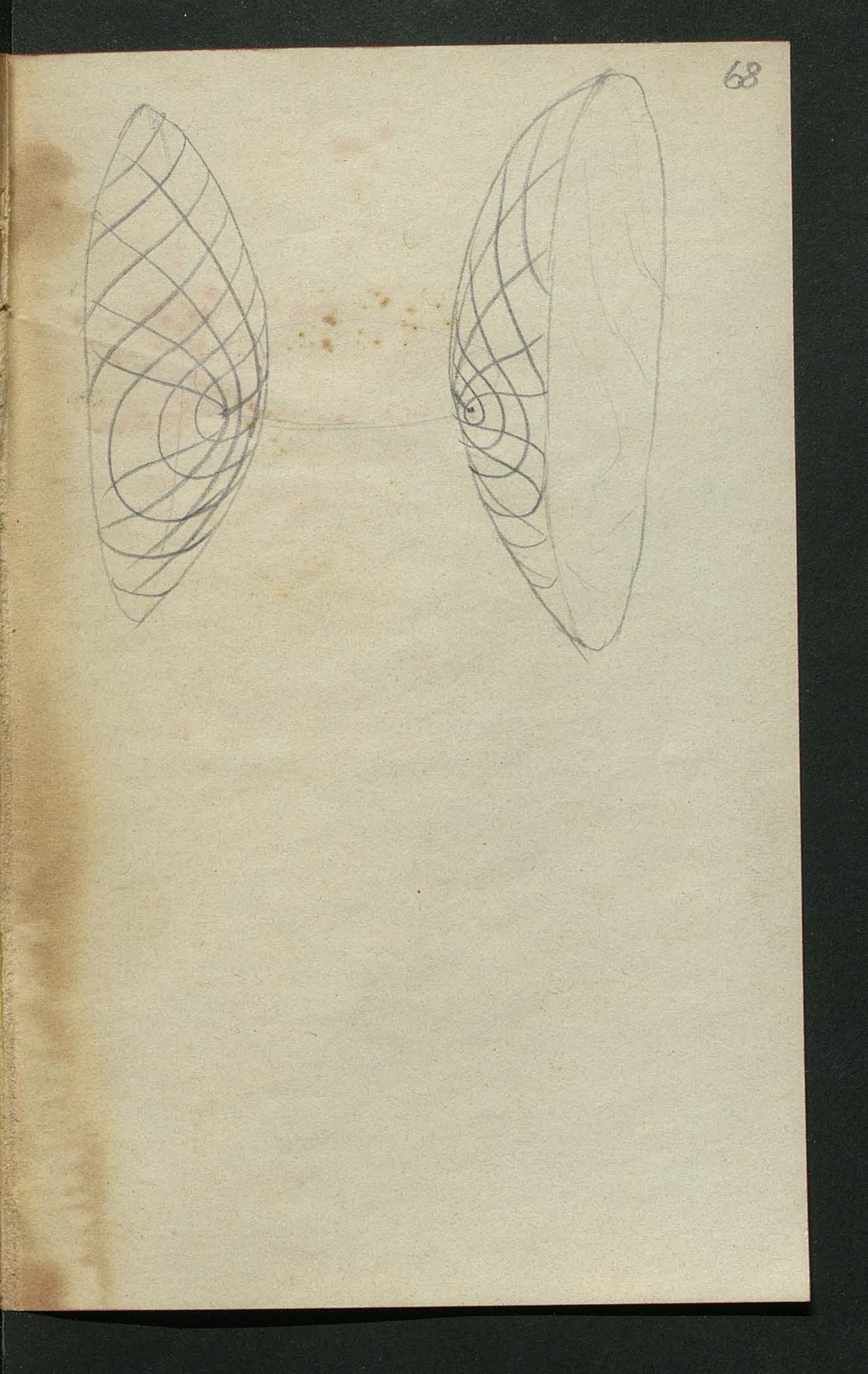
Hyp

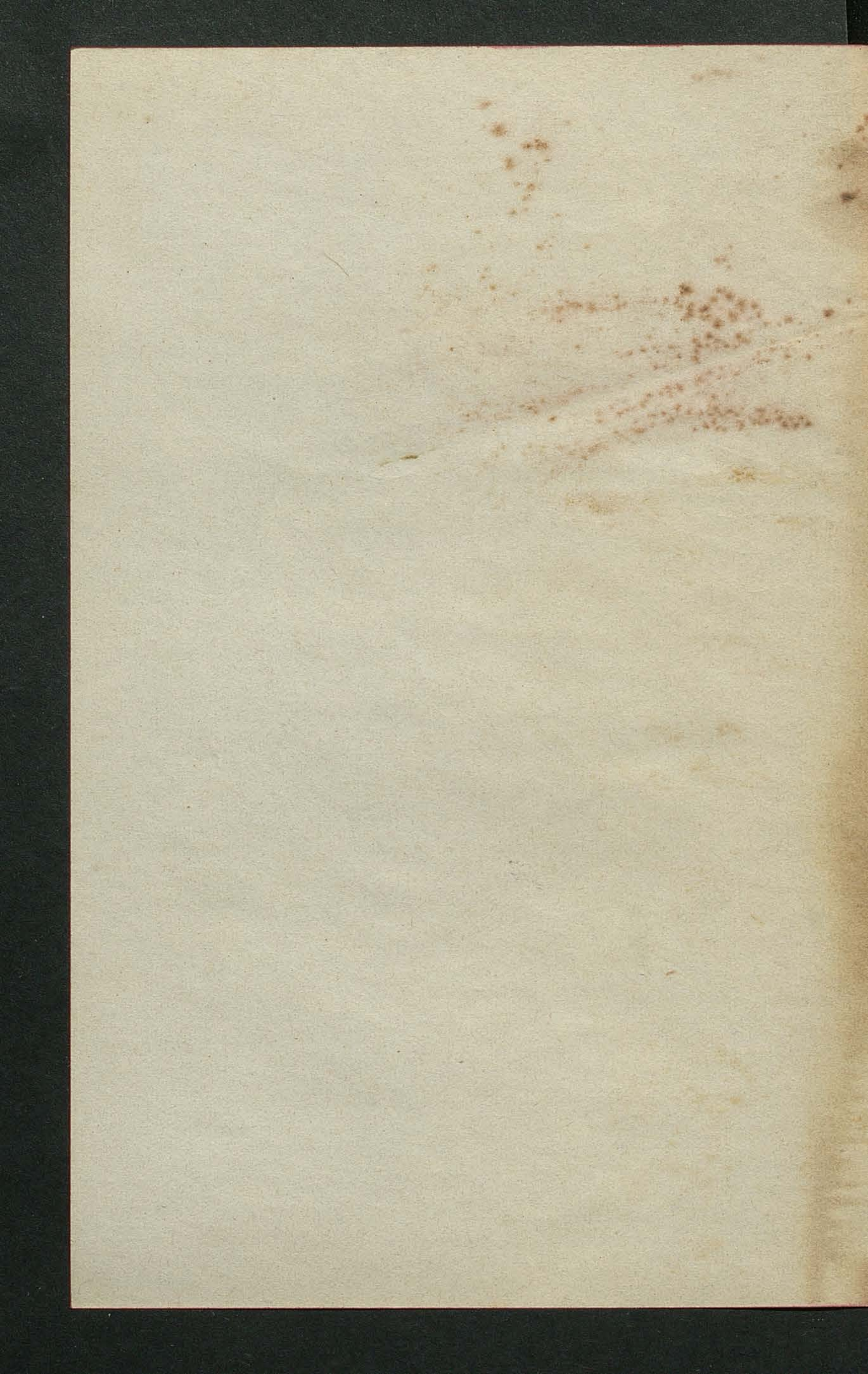
Focalhyperbel

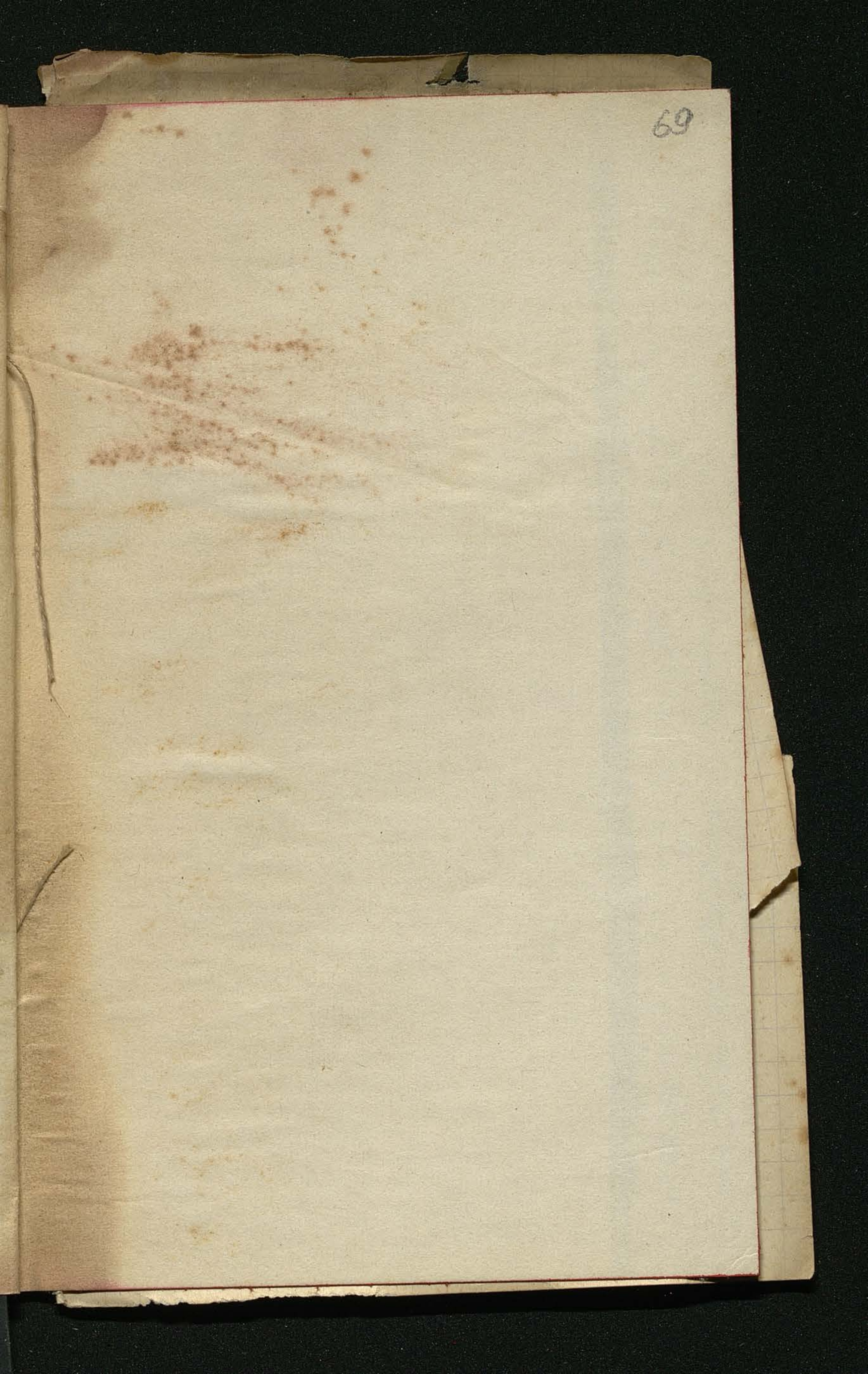
λ_3 a^2

YZ

Nebel.







C. Neumann:

[illegible] [Mech.] [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$\int_{t_0}^{t_1} (\delta H + \Sigma P \delta p)\, dt = 0 \qquad \frac{\partial}{\partial t}\delta p_1$

$\frac{\partial H}{\partial p_1} \delta p_1 \Big|_{t_0} \quad + \frac{\partial H}{\partial \dots}$

$\therefore \frac{\partial}{\partial t}\left(\frac{\partial H}{\dots}\right) \dots P_1$ } allg. Lagrange'sche G.

$T = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2) \qquad U = mgy + \text{const}$

$H = T - U \qquad H =$ kinet. Potential

$\therefore \frac{\partial}{\partial t}(m\dot{x}) = X \qquad \frac{\partial}{\partial t}(m\dot{y}) = -mg + Y$

Ableitg. des Prinzipes der Energie

$$p_1 \frac{\partial}{\partial t}\left(\frac{\partial H}{\partial \dot{p}_1}\right) - p_1 \frac{\partial H}{\partial p_1} = p_1 P_1$$ [illegible]

$$\sum\left[\frac{dp_1}{dt}\frac{d}{dt}\left(\frac{\partial H}{\partial \dot{p}_1}\right) - \frac{dp_1}{dt}\frac{\partial H}{\partial p_1}\right] = \sum -$$

$$\frac{dH}{dt} = \sum \frac{\partial H}{\partial p_1}\frac{\partial p_1}{\partial t} + \sum \frac{\partial H}{\partial \dot{p}_1}\frac{\partial \dot{p}_1}{\partial t}$$

$$\sum \frac{dp_1}{dt}\frac{d}{dt}\left(\frac{\partial H}{\partial \dot{p}_1}\right) + \frac{\partial H}{\partial \dot{p}_1}\frac{\partial \dot{p}_1}{\partial t} - \frac{dH}{dt} = \sum p_1 P_1$$

$$\sum\left[\frac{d}{dt}\left(\dot{p}_1 \frac{\partial H}{\partial \dot{p}_1}\right) - \frac{dH}{dt}\right] = -$$

$$= d\left[\sum \dot{p}_1 \frac{\partial H}{\partial \dot{p}_1} - H\right] = \sum P_1 dp_1$$

$$= \Sigma \, [illegible]$$ [illegible]

T = homog. 2 fc. $\therefore \sum x \frac{\partial T}{\partial x} = 2T$

[illegible] p_1 etc. [illegible]

$$E = 2T - H = 2T - T + U = T + U$$

$= \Sigma$ [illegible] pot. Energie

[illegible] E [illegible] H [illegible]

[illegible] E [illegible] H [illegible]

t. [shorthand]

$\delta\int(T-U+\sum P\delta p)\,dt=0$

[shorthand] $\delta T=0$ [shorthand] $\delta U=\sum P\delta p$ [Ord. v. 1]

Einführung einer neuen Veränderlichen in Lagr. Gl.

$t' = \text{const} - t$

$\therefore dt' = -dt$ $\qquad \dot p' = -\dot p$

$H' = H$ [shorthand]

$+\frac{d}{dt'}\left(\frac{dH}{d\dot p'}\right) = \frac{dH}{dp}$ [shorthand] $-t$

[shorthand]

[shorthand] $t' = t_0 t_1 - t$ [shorthand]

[shorthand] reversible [shorthand] Ham[shorthand]

[shorthand] Irrev. [shorthand] C [shorthand] H

[shorthand] dt [shorthand] Rot. [shorthand] [[shorthand], C etc]

[shorthand] rev. sim. [shorthand]

[shorthand] $\frac{d^2}{dt^2}$ [shorthand]

[shorthand] $\frac{d}{dt}$ [shorthand]

[shorthand]

[shorthand]

[illegible] [C [illegible]]

[illegible] P [illegible]

$$\frac{d}{dt}\left(\frac{\partial H}{\partial \dot p}\right) = \frac{\partial H}{\partial p} + P$$ [illegible] v. Helmholtz

[illegible] Dispersion [illegible]

[illegible]

[illegible]

[illegible]

Elastische Vorgänge

Ableitung der Bewegungsgl. [illegible] Krystalle

[illegible] contin. Medium, [illegible] Cond. [illegible]

[illegible]: $\int_{t_0}^{t_1} (\delta H + \Sigma P \, \delta p)\, dt = 0$

[illegible]

$H = T - U$ $T = \Sigma$ [illegible]

x, y, z; u, v, w } [illegible] $\dot u$, $\dot v$, $\dot w$ [illegible]

$$T = \int \frac{\mu}{2}(\dot u^2 + \dot v^2 + \dot w^2)\, d\tau$$

U = pot. En., [illegible]

[illegible] Kommensurable [illegible]

$x_x \quad y_y \quad z_z$

$\frac{\partial u}{\partial x} \quad \frac{\partial v}{\partial y} \quad \frac{\partial w}{\partial z}$

$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = x_y = y_x$

$\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = y_z = z_y$

$\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = z_x = x_z$

[illegible]

$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \quad \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} \quad \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}$

[illegible] U

[illegible] $F d\tau$ [illegible] F [illegible]

F [illegible] > 0 [illegible]

[illegible] $= 0$

[illegible] homog. [illegible] $F < 0$

[illegible]

$F = a x_x^2 + \ldots\ldots$ etc.

[illegible] ν = Norm.

X_ν

Y_ν

Z_ν

$\Sigma P \delta_r = \int X_\nu \delta x + Y_\nu \delta y + Z_\nu \delta z$

$\Sigma P \delta r = \int d\sigma (X_\nu \delta u + Y \delta v + Z \delta w)$

$$\int_{t_0}^{t} dt \left\{ \int d\tau \, k (\dot{u} \delta \dot{u} + \dot{v} \delta \dot{v} + \dot{w} \delta \dot{w}) - \int d\tau \, \delta f + \right.$$

$\delta(d\tau f) = d\tau \, \delta f + \delta d\tau + f$

[illegible shorthand]

$\delta d\tau$ [illegible shorthand] δf [illegible]

$$\left. + \int d\sigma \{ X_\nu \delta u + Y_\nu \delta v + Z_\nu \delta w \} \right\} = 0$$

$\dot{u} \delta \dot{u}$ part. int.

$$\int_{t_0}^{t_1} dt \left\{ \int d\tau \, k (\ddot{u} \delta u + \ddot{v} \delta v + \ddot{w} \delta w) - \int d\tau \left[\frac{\partial f}{\partial x_x} \delta x_x + \frac{\partial f}{\partial y_y} \delta y_y + \frac{\partial f}{\partial z_z} \delta z_z + \frac{\partial f}{\partial x_y} \delta x_y + \text{---} \right] \right.$$

$$+ \int d\sigma [\;] = 0$$

$x_x = \frac{\partial u}{\partial x}$ zur Abkürzg.: $- \frac{\partial f}{\partial x_x}$ etc. $= X_x$ etc

$$\int d\tau \left[X_x \, \delta \frac{\partial u}{\partial x} + \cdots \right] = - \int \left[\frac{\partial X_x}{\partial x} \delta u \cdots \right.$$

$$\left. + \left(\frac{\partial X_y}{\partial y} \delta u + \frac{\partial X_y}{\partial x} \delta v \right) + (\;\cdots\;) + (\;\;) \right] d\tau +$$

$$- \int d\sigma \left[X_x \cos \nu_x \delta u + Y_y \cos \nu_y \delta v + Z_z \cos \nu_z \delta w + \right.$$

$$\left. + (X_y \cos \nu_y \delta u + X_y \cos \nu_x \delta v) + (\;) + (\;) \right].$$

[illegible]

$$k\ddot{u} + \frac{\partial X_x}{\partial x} + \frac{\partial X_y}{\partial y} + \frac{\partial X_z}{\partial z} = 0$$

$$k\ddot{v} + \frac{\partial Y_x}{\partial x} + \frac{\partial Y_y}{\partial y} + \frac{\partial Y_z}{\partial z} = 0$$

$$k\ddot{w} + \text{———} = 0$$

[illegible] Elast. [illegible]

$$X_\nu = X_x \cos\nu_x + X_y \cos\nu_y + X_z \cos\nu_z$$

$$Y_\nu = Y_x \cos\nu_y + Y_y \cos\nu_y + Y_z \cos\nu_y$$

$$Z_\nu = \text{———}$$

[illegible] X_x etc. [illegible]

[illegible] X_ν [illegible]

[illegible] homog. lin. [illegible]

Sem.

[illegible]

[illegible] t_0 $x_0 y_0$ [illegible]

t_1 $x_1 y_1$

$$H = \frac{m}{2}(\dot{x}^2 + \dot{y}^2 - mgy)$$

$\int dt\, H$ [illegible]

[illegible] $t_0 = 0$ $t_1 = 1$ $x_0 = y_0 = 0$

$x_1 = 1$ $y_1 = 0$

[illegible] Parabel ; [illegible]

$x = t \qquad y = \frac{1}{2} g t (1 - t)$

[illegible]

[illegible] $x = \sqrt{t}$

$y = [illegible]\, t$ [illegible]

[illegible]

[illegible] Pot. [illegible] virtuelle [illegible]

Bewegung eines Rotations Körpers

[illegible] Cardanischer Suspension

1 [illegible] vertical $x\ \alpha$

1 [illegible] horizontal $x\ \beta$

[illegible] $x\ \gamma$

U [illegible] $= 0$ [illegible]

$$H = T = \frac{1}{2}\left[P p^2 + Q q^2 + R r^2\right]$$ [illegible] Rot. [illegible]

([illegible])

$$p = \sin\beta \sin\gamma\, \dot\alpha + \cos\gamma\, \dot\beta$$

$$q = \sin\beta \cos\gamma\, \dot\alpha - \sin\gamma\, \dot\beta$$

$$r = \cos\beta\, \dot\alpha + \dot\gamma$$

[illegible] $P = Q$ [illegible] Rot. [illegible]

$$H = \frac{1}{2} Q (\sin^2\beta\, \dot\alpha^2 + \dot\beta^2) + R (\cos^2\beta\, \dot\alpha^2 + \dot\gamma^2 + 2\cos\beta\, \dot\alpha \dot\gamma)$$

[illegible]

$$\frac{d}{dt}\left(\frac{\partial W}{\partial p}\right) - \frac{\partial W}{\partial p} = P$$

$$\begin{cases} \frac{d}{dt}\left[\varphi \sin^2\beta\, \dot\alpha + R\cos\beta(\cos\beta\,\dot\alpha + \dot\gamma)\right] = A \\ \frac{d}{dt}[\varphi\,\dot\beta] - \{\sin\beta\cos\beta\,\varphi\,\dot\alpha^2 - R(\cos\beta\,\dot\alpha + \dot\gamma)\sin\beta\,\dot\alpha\} = B \\ \frac{d}{dt}\left[R(\cos\beta\,\dot\alpha + \dot\gamma)\right] = \Gamma \end{cases}$$

Euler [illegible]

[illegible] A [illegible] B [illegible]

[illegible]

$\Gamma = 0$

$$R(\cos\beta\,\dot\alpha + \dot\gamma) = \text{const} = c$$

$$\dot\gamma = \frac{c}{R} - \cos\beta\,\dot\alpha$$

$$\begin{cases} \frac{d}{dt}\left[\varphi\sin^2\beta\,\dot\alpha + c\cos\beta\right] = A \\ \frac{d}{dt}[\varphi\,\dot\beta] - \left[\varphi\,\dot\alpha^2\sin\beta\cos\beta - c\sin\beta\,\dot\alpha\right] = B \end{cases}$$

[illegible]

[illegible]

[illegible] $\dot\gamma = 0$ $\quad c = R\cos\beta\,\dot\alpha$

$$\begin{cases} A = \frac{d}{dt}\left[\varphi\sin^2\beta\,\dot\alpha + R\cos^2\beta\,\dot\alpha\right] \\ B = \frac{d}{dt}[\varphi\,\dot\beta] - \sin\beta\cos\beta\left[\varphi - R\right]\dot\alpha^2 \end{cases}$$

[illegible] Coord. [illegible]

[illegible] Coord. [illegible]

[illegible] Foucault [illegible] Pendel

[illegible] Rot. [illegible]

Allgem. Methode [illegible] Helmholtz [illegible] Coord [illegible]

[illegible] Coord [illegible] $p_1 \quad p_2 \quad p_3$ ([illegible] Coord.)

$p_1' \quad p_2' \quad p_3'$ ([illegible] Coord.)

P_1' etc. [illegible] $= 0$ [illegible]

[illegible] Pot. H [illegible] p' $\quad H = f_n(p, \dot{p}, \dot{p}')$

$$\frac{\partial H}{\partial p'} = 0 \quad [illegible] \quad \frac{\partial H}{\partial \dot{p}'} \; [illegible] \lessgtr 0$$

$$P = \frac{\partial}{\partial t}\left(\frac{\partial H}{\partial \dot{p}}\right) - \frac{\partial H}{\partial p} \quad \Bigg| \quad P' = \frac{\partial}{\partial t}\left(\frac{\partial H}{\partial \dot{p}'}\right) - \frac{\partial H}{\partial p'}$$

$$\frac{\partial H}{\partial \dot{p}_1'} = c_1$$

$$\frac{\partial H}{\partial \dot{p}_2'} = c_2 \quad \text{etc.}$$

[illegible] lin. homog. fn. [illegible]

[illegible] $\dot{p}_1'$ $\dot{p}_2'$ etc. [illegible]

[illegible] Coord. [illegible] H [illegible] redu.

[illegible] $-$ fn. ($\mathcal{L}$) [illegible]

[illegible] $\mathcal{L}$ / $<$ H

"secondary" "primaire" [illegible] K. P.

$$\frac{\partial \mathcal{L}}{\partial p} = \frac{\partial H}{\partial p} + \sum \frac{\partial H}{\partial p_1'} \frac{\partial p_1'}{\partial p}$$

"c_1 etc.

$$= \frac{\partial}{\partial p}\left[H + \Sigma c_1 p_1'\right]$$

$$\frac{\partial \mathcal{L}}{\partial \dot{p}} = \frac{\partial H}{\partial \dot{p}} + \sum \frac{\partial H}{\partial p_1'} \frac{\partial p_1'}{\partial \dot{p}}$$

$$= \frac{\partial}{\partial \dot{p}}\left[H + \Sigma c_1 p_1'\right]$$

$$H - \Sigma c_1 p_1' = K = \text{fn}(p, \dot{p})$$

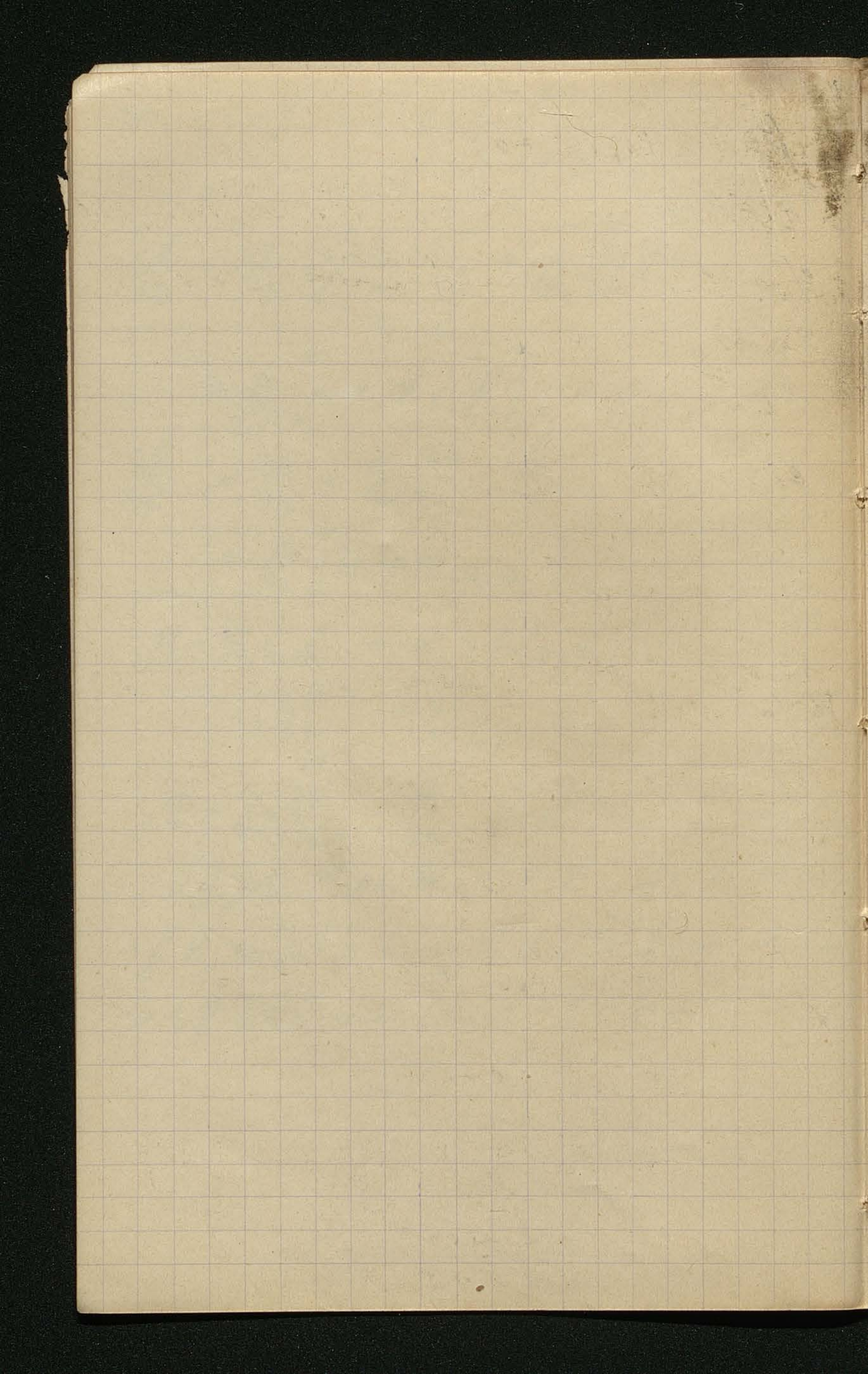
$$\left(\frac{\partial H}{\partial p}\right)_{p_1 \dot{p}} = \left(\frac{\partial K}{\partial p}\right)_{p \dot{p}}$$

$$P = \frac{\partial}{\partial t}\left(\frac{\partial K}{\partial \dot{p}}\right) - \frac{\partial K}{\partial p}$$ [illegible] K. coord.

[illegible] K [illegible]

[illegible] K [illegible] homog. [illegible] linear [illegible]

[illegible] irrev. [illegible] 8 [illegible] [Force [illegible] irrev.]



14/5

$$\int_{t_0}^{t_1} (\delta A + \Sigma P \delta p)\, dt = 0$$

[illegible]

[illegible] $u\ v\ w$ [illegible] XYZ [illegible] LMN

[illegible] $\dot u\ \dot v\ \dot w$ [illegible]

$$L = \dot u \qquad M = \dot v \qquad N = \dot w$$

[illegible]
$$X = \frac{c}{K}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$
$$Y = \frac{c}{K}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$
$$Z = \cdot\ -\ -\ -$$

[illegible] $u\ v\ w$ [illegible]
[illegible]

$$\delta A = \delta T - \delta U \qquad \Sigma P \delta p = \Sigma U \delta u + \Sigma V \delta v + -$$

[illegible]

$$U = U_0$$

$$\int_{t_0}^{t_1} \delta T\, dt = \int_{t_0}^{t_1} \frac{\mu}{4\pi}\left[L \delta L + M \delta M + N \delta N\right] =$$

$$= \int_{t_0}^{t_1} \frac{\mu}{4\pi}\left[\dot u \delta \dot u + -\right.$$

$$= -\int_{t_0}^{t_1} \frac{\mu}{4\pi}\left[\frac{\partial L}{\partial t}\delta u + \frac{\partial M}{\partial t}\delta v + \frac{\partial N}{\partial t}\delta w\right]$$

$$+ \int_{t_0}^{t_1}\left[L \delta u + M \delta v + \quad\right]$$

$\delta u = \delta v = \delta w = 0$

$$\int_{t_0}^{t_1} \delta U \, dt = -\int_{t_0}^{t_1} \frac{K}{4\pi}\left[X\delta X + \ldots\right] dt$$

$$= -\frac{c}{4\pi}\int_{t_0}^{t_1}\iint\left[X\,\delta\left(\frac{\partial w}{\partial y} - \ldots\right) + \ldots\right] d\tau\, dt$$

$$= \frac{c}{4\pi}\iint\left[\left(\frac{\partial X}{\partial y} - \frac{\partial Y}{\partial x}\right)\delta w + \ldots\right] dt$$

$$+ \frac{c}{4\pi}\int_{t_0}^{t_1}\int\left[X(\delta w \cos \nu y - \delta v \cos \nu z) + \ldots\right] dS$$

$$\therefore \quad \mu \frac{\partial L}{\partial t} = c\left(\frac{\partial Y}{\partial z} - \frac{\partial Z}{\partial y}\right)$$

$$\mu \frac{\partial M}{\partial t} = c\left(\frac{\partial Z}{\partial x} - \frac{\partial X}{\partial z}\right)$$

$$\mu \ldots$$

[illegible] Cond.

[illegible] $XYZ = \ldots$ [illegible] Energy [illegible]

[illegible]

[illegible] $T = \ldots (\dot{u}^2 + \dot{v}^2 + \dot{w}^2)$

[illegible] $U = \ldots \left[\left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right)^2 + \ldots\right]$

[illegible] Medium [illegible] Elastic

[illegible]: $U_r = -\frac{c}{4\pi}\left[Y\cos(\nu z) - Z\cos(\nu y)\right]$

$V_r =$

$W_r =$

[illegible shorthand] $u\,v\,w$ [illegible shorthand]

Energie [illegible] $(U_r\,\delta u + V_r\,\delta v + W_r\,\delta w)\,d\sigma\,dt$

$\frac{c}{4\pi}\,d\sigma\;dt\,[U_r\,\dot u + \cdots$

$= \dot u\,(Z\cos\nu y - Y\cos\nu z)$

$+ \dot v\,(\quad - \quad) + \dot w\,(\;-\;)$

$= L\,(Z\cos\nu y - Y\cos\nu z) + \cdots$

$=$ Energiefluss ([illegible shorthand])

[illegible shorthand] spec. Interpr. [illegible] $u\,v\,w$ [illegible shorthand]; [illegible] Äther / elast. [illegible]

[illegible] $\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}$ etc. [illegible shorthand]

17/5 Spezielles Beispiel: Endliche Anzahl von linearen Leitern, welche [illegible shorthand] (etc.)

[illegible] Var. [illegible shorthand] $p_1\ p_2\ p_3$

[illegible shorthand] $\xi_1\ \xi_2\ \xi_3$ [[illegible shorthand]]

$H = T - U$

$$\underbrace{\tfrac{1}{2}\left[L_1\,\dot\xi_1^{\,2} + L_2\,\dot\xi_2^{\,2} + \cdots\right] + L_{12}\,\dot\xi_1\,\dot\xi_2 + \cdots}_{=E} + T$$

$L_1 = \frac{1}{c^2}\iint \frac{ds_1\,ds_1'}{r}\cos\Theta$

[illegible shorthand] Energie [illegible]

[illegible] P_1, P_2 ... E_1, E_2 ... [illegible] ; [illegible]$^c = 0$

$$P = \frac{\partial}{\partial t}\left(\frac{\partial H}{\partial \dot p}\right) - \frac{\partial H}{\partial p}$$

$$= \frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot p}\right) - \frac{\partial T}{\partial p} - \frac{1}{2}\left[L_1 \dot i_1^2 \frac{dL_1}{dp} + \cdots\right] +$$

[illegible]

$$E_1 = \frac{\partial}{\partial t}\left[L_1 \dot i_1 + L_{12} \dot i_2 + \cdots\right]$$

= Induktionsgesetz

[illegible] E_1 [illegible]

elektromot. K

Bewegungsgleichg. [illegible] rotierenden Kugel

Z, Y Osten, X Süden; x', y', z'

[illegible] 4te Coord. [illegible]

[illegible] Z [illegible]

$$\frac{1}{2}\mathfrak{C}\dot\omega^2 + \frac{m}{2}\left[\dot x'^2 + \dot y'^2 + \dot z'^2\right]$$

$$\dot x' = \dot x - y \sin\varphi\, \dot\omega$$

$$\dot y' = \dot y - [(R+x)\sin\varphi + z\cos\varphi]\,\dot\omega$$

$$\dot z' = \dot z - y\cos\varphi\, \dot\omega$$

$$= \frac{1}{2} m$$

$$\Omega = \frac{\partial}{\partial t}\left(\frac{\partial H}{\partial \dot\omega}\right)$$

const.

$$\Omega = x'Y' - y'X' = (xY - yX)\sin\varphi + [(R+z)Y - yZ]\cos\varphi$$

$$X = m\left[\ddot x - \frac{\partial}{\partial t}(y\sin\varphi)\dot\omega\right] = m\ddot x - 2\dot y\sin\varphi\,\dot\omega - \rho\dot\omega^2\sin\varphi$$

$$Y = m\,\ddot y - \frac{\partial}{\partial t}(\rho) - \ldots = m\ddot y - 2\dot x\sin\varphi\,\dot\omega + 2\dot z\cos\varphi\,\dot\omega - y\dot\omega^2$$

$$Z = \qquad = m\ddot z - 2\dot y\cos\varphi\,\dot\omega - \rho\dot\omega^2\cos\varphi$$

eliminieren

$$\frac{\partial \dot\Omega}{\partial t} = 0 \qquad \dot\Omega = \text{const}$$

$$Z = g \qquad X = Y = 0$$

$$\ddot x - R\dot\omega^2\sin\varphi = 0$$
$$\ddot y - y\dot\omega^2 = 0$$
$$\ddot z - R\dot\omega^2\cos\varphi = g$$

$$g = \ddot x - 2\dot y\sin\varphi\,\dot\omega$$
$$0 = \ddot y$$
$$-g = \ddot z -$$

$\dot y$ $\dot x$ $\dot z$

$$\therefore\ x = 0$$
$$z = h - g\frac{t^2}{2}$$
$$y = \frac{1}{3}\dot\omega\cos\varphi\, g t^3$$

$$dE = dQ - P\,dv$$

$$\frac{dQ}{\theta} = dS$$

[illegible] Vol

$\frac{It}{\varepsilon} = N + N'$ [illegible] Concentr. [illegible]

[illegible] Ag Cl [illegible] Conc. [illegible] ; [illegible] NO_3 [illegible]

[illegible]

$\frac{N'}{N+N'}$ [illegible] Hittorf = [illegible] = n'

U = [illegible] Ionen P = Intensit. [illegible] elect. Feldes

ε gr. mol. NO_3 [illegible] $1 cm^3$ [illegible] φ = [illegible]

$CUt\varphi = \frac{It}{\varepsilon} n$ $CVt\varphi = \frac{It}{\varepsilon} n'$

$U = \frac{In}{\varepsilon \varphi C}$ $V = \frac{In'}{\varepsilon \varphi C}$

$\varepsilon = \frac{El. n}{[illegible]}$ = [illegible] gr. Ion.

[illegible] HCl [illegible]

[illegible] 5% Lösung

1 gr. – 20 cm³ $\quad \varepsilon = 9653 \quad J = 10^{-1}$

36·5 gr. 730 cm $\quad q = 10$

$C = \frac{1}{730} \quad n = 0·81$

$U = 0·00061$ cm $\quad V = 0·00014$ cm p. sec.

Potentialgefälle P, κ = [illegible]

$$U = \frac{\kappa P n}{C \varepsilon} \qquad V = \frac{\kappa P n'}{C \varepsilon} \qquad U + V = \frac{\kappa P}{C \varepsilon}$$

$\frac{\kappa}{C}$ = molecularen [illegible] (λ)

[illegible] $\frac{\lambda n}{\varepsilon}$ [illegible] Jons (5° [illegible])

[illegible] $\lambda n'$

[illegible] ; [illegible]

[illegible] Kation [illegible] Anion [illegible]

[illegible] (Kohlrausch : [illegible])

$\lambda = u_0 + v_0 \qquad u_0 = \frac{U \varepsilon}{P}$ [illegible]

[illegible] P = 1 [illegible]

	K	Na	Li	Ag	NH_4	H			ca 18°C
$u_0 \cdot 10^7$	60	41	33	52	60	290			80
$v_0 \cdot 10^7$	63	63	58	52	54	165	44	33	
	Cl	J	NO_3	ClO_3	ClO_4	HO	CHO_2	$C_2H_3O_2$	

[dabei ist $\lambda = \frac{\kappa}{m}$ [illegible] 1 Liter]

$m = 1000 . C$

[[illegible] Ag]

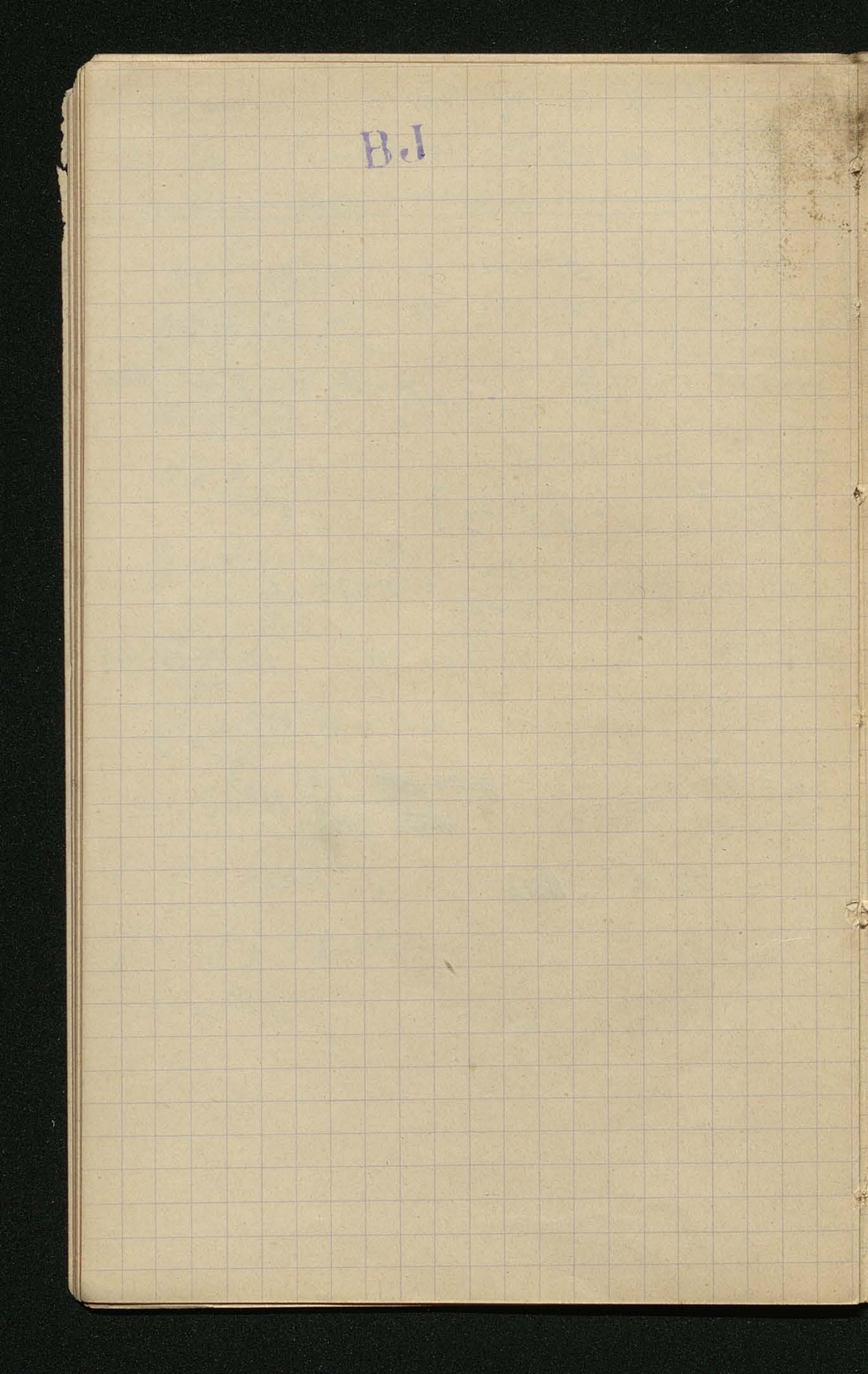
[illegible] $AgNO_3$ $\frac{52}{52+58} = 0.473$, Ag

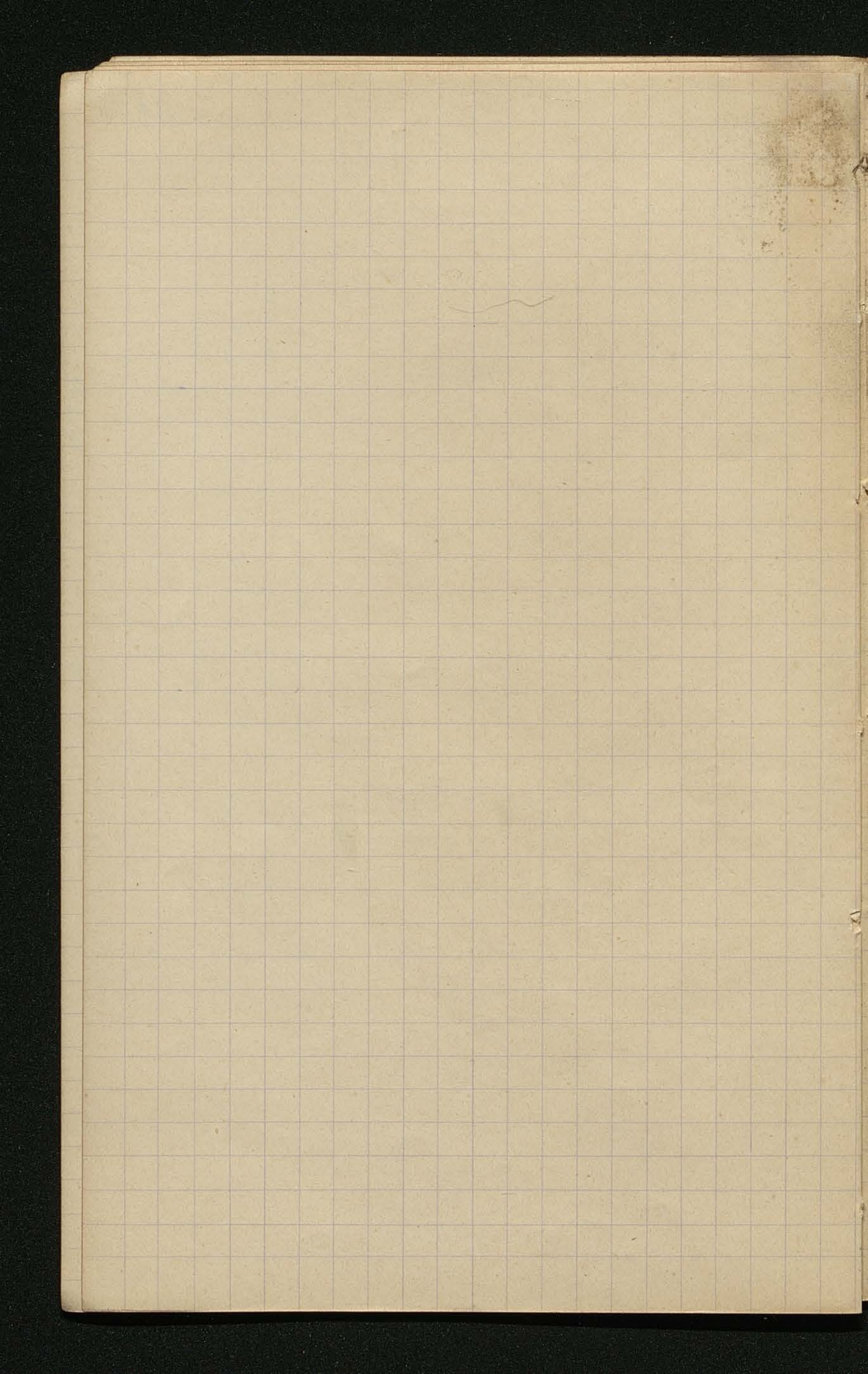
[illegible] 1 Volt pro cm $\quad$ 1 Ohm = 1.063 [illegible] $= 10^9$

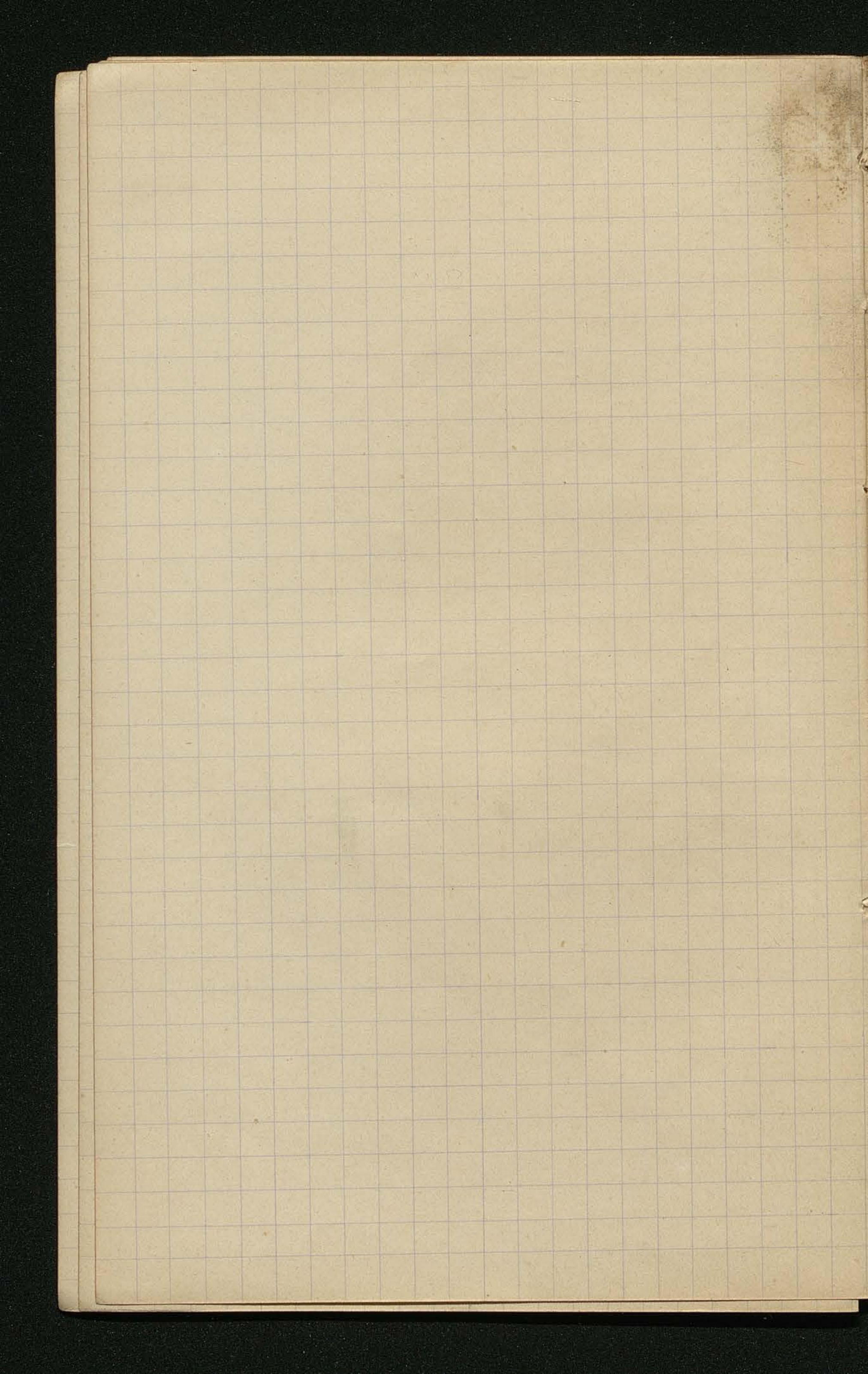
$P = 10^{+8} \qquad \varepsilon = 9653 \qquad u_0(abs) = 1000 . u_0 . \lambda_{Ag}$

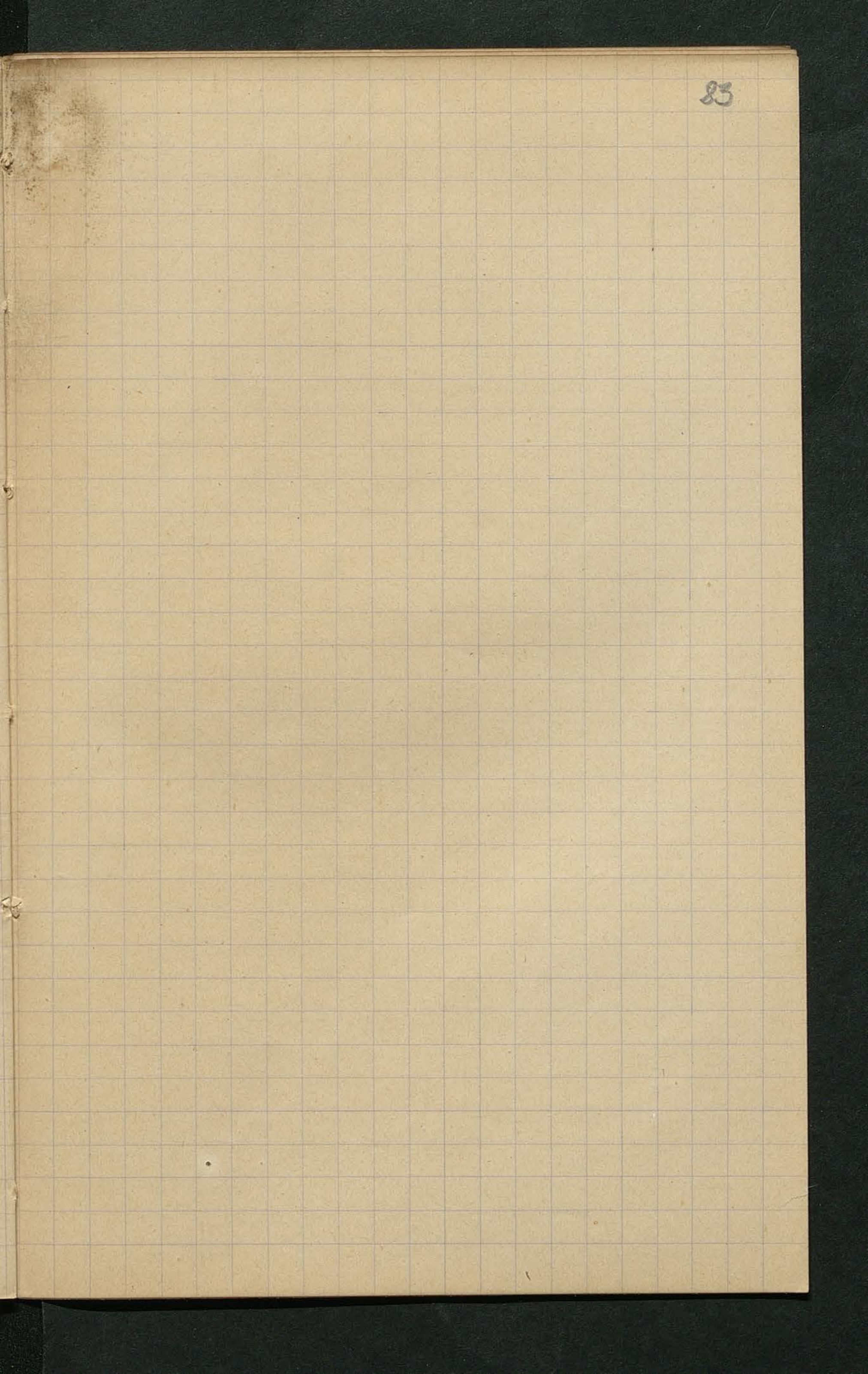
$U = \frac{10^{+8} . 10^3 . 10^{-7} . 52 . \frac{1.063}{10^9} . 10^{4}}{9653 .}$

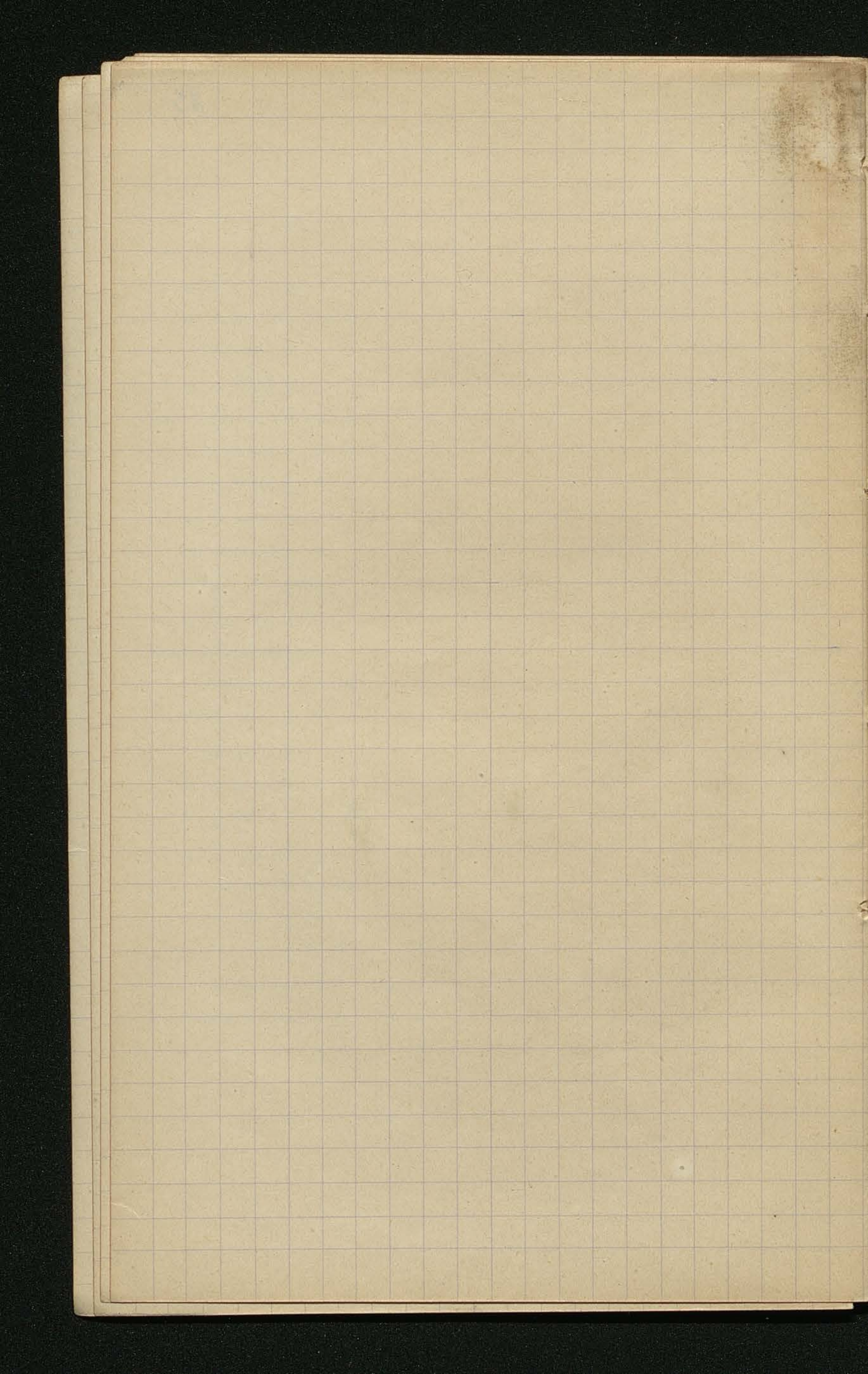
$= 52 . 10^{-7} . 2$

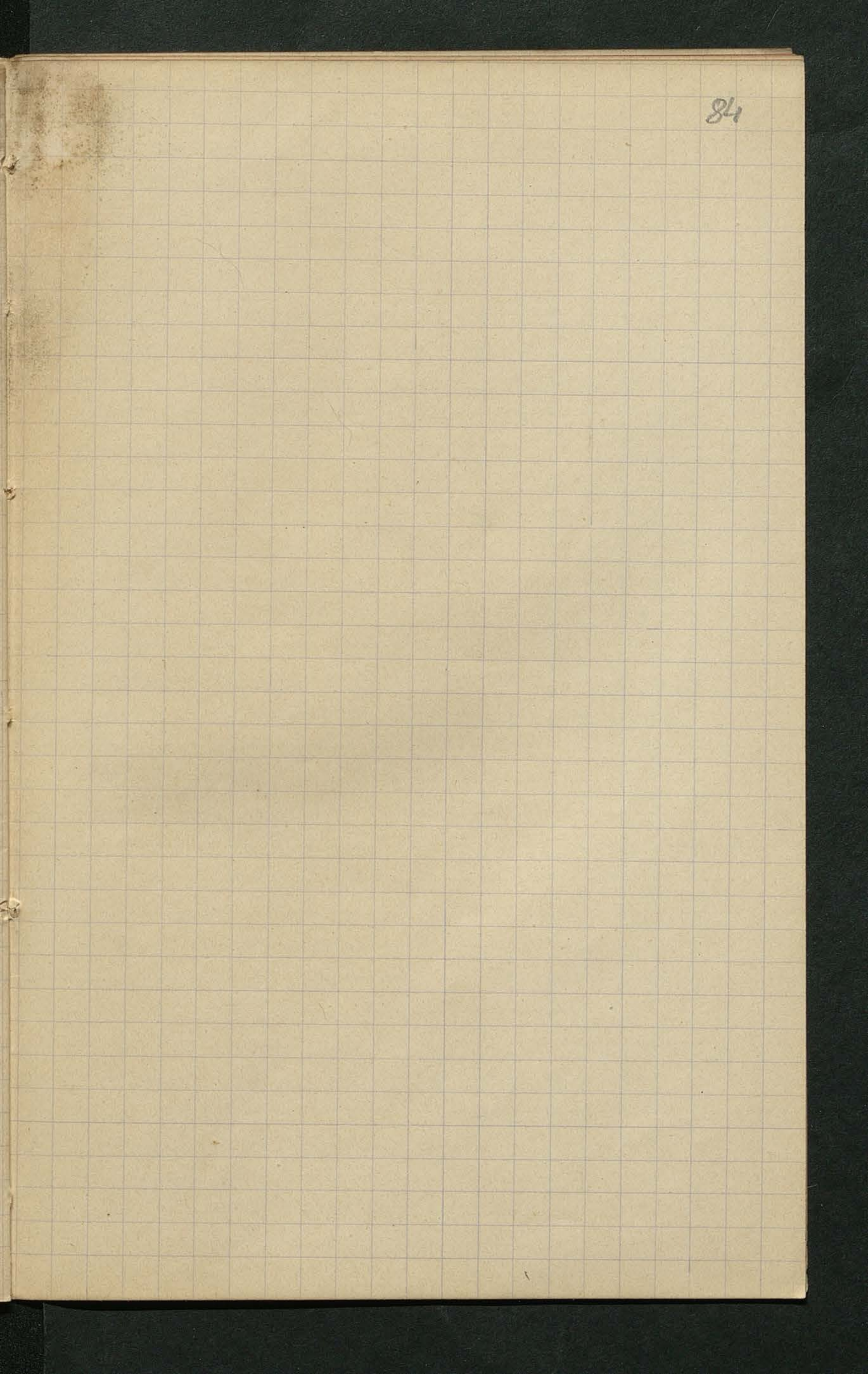


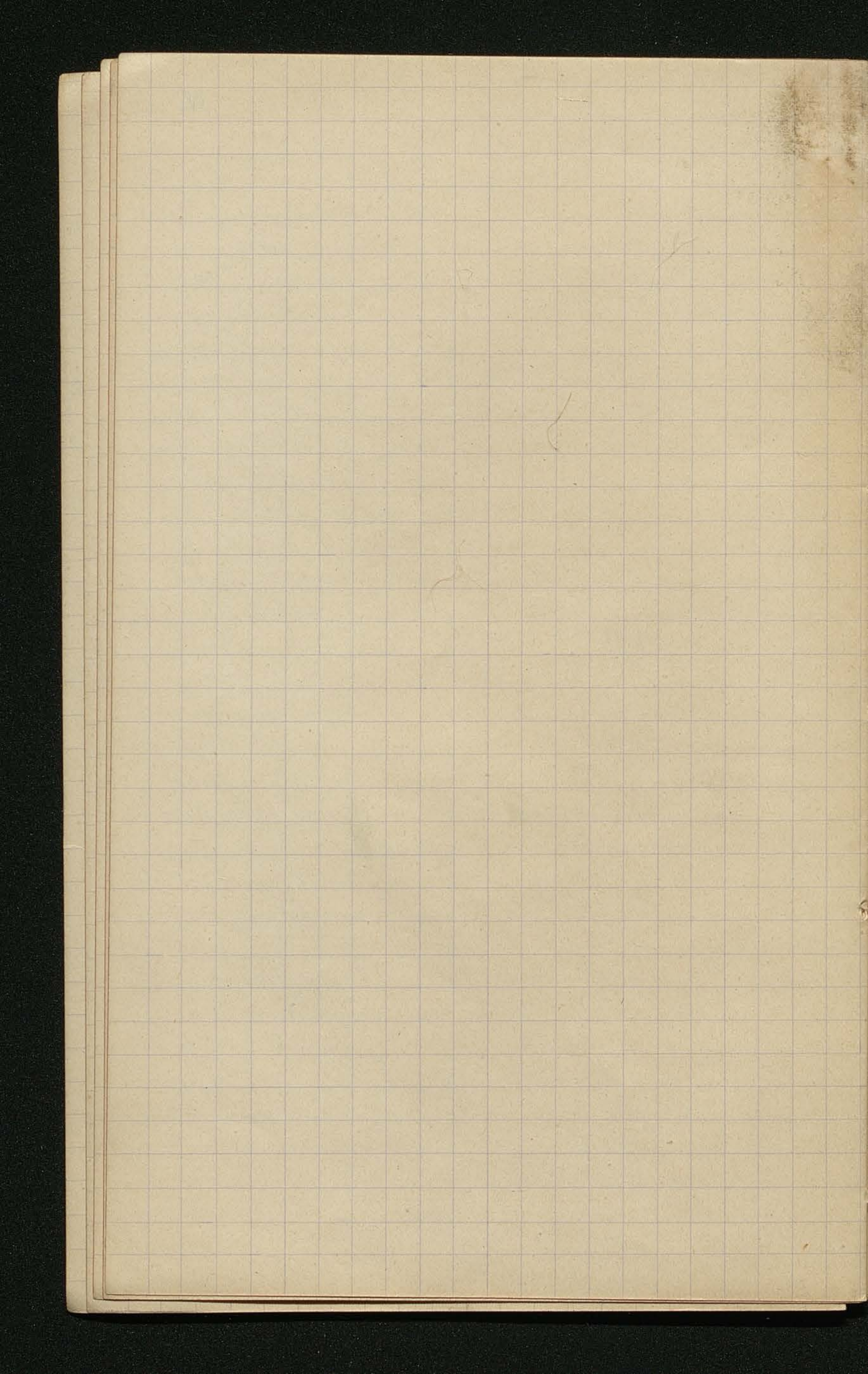


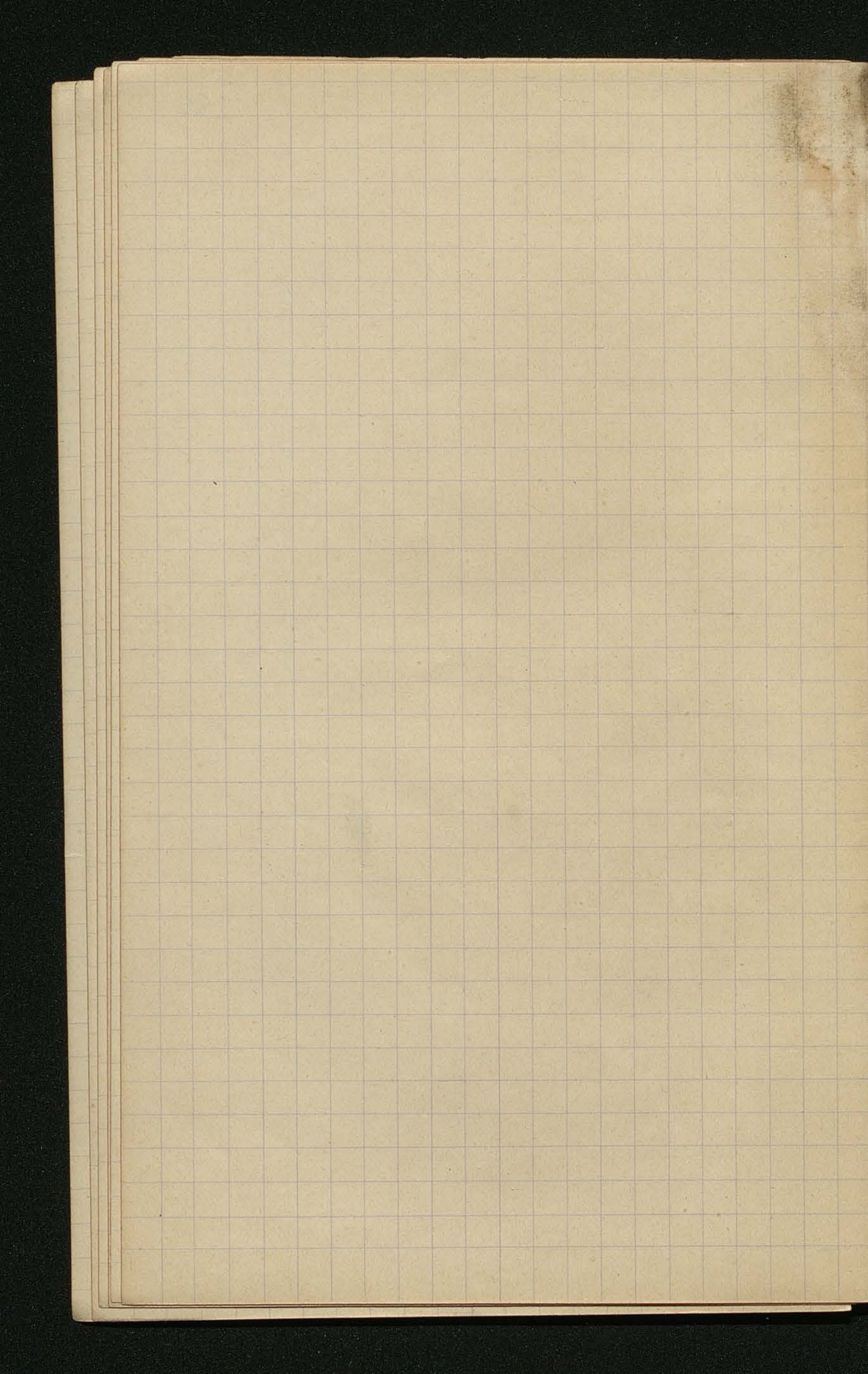


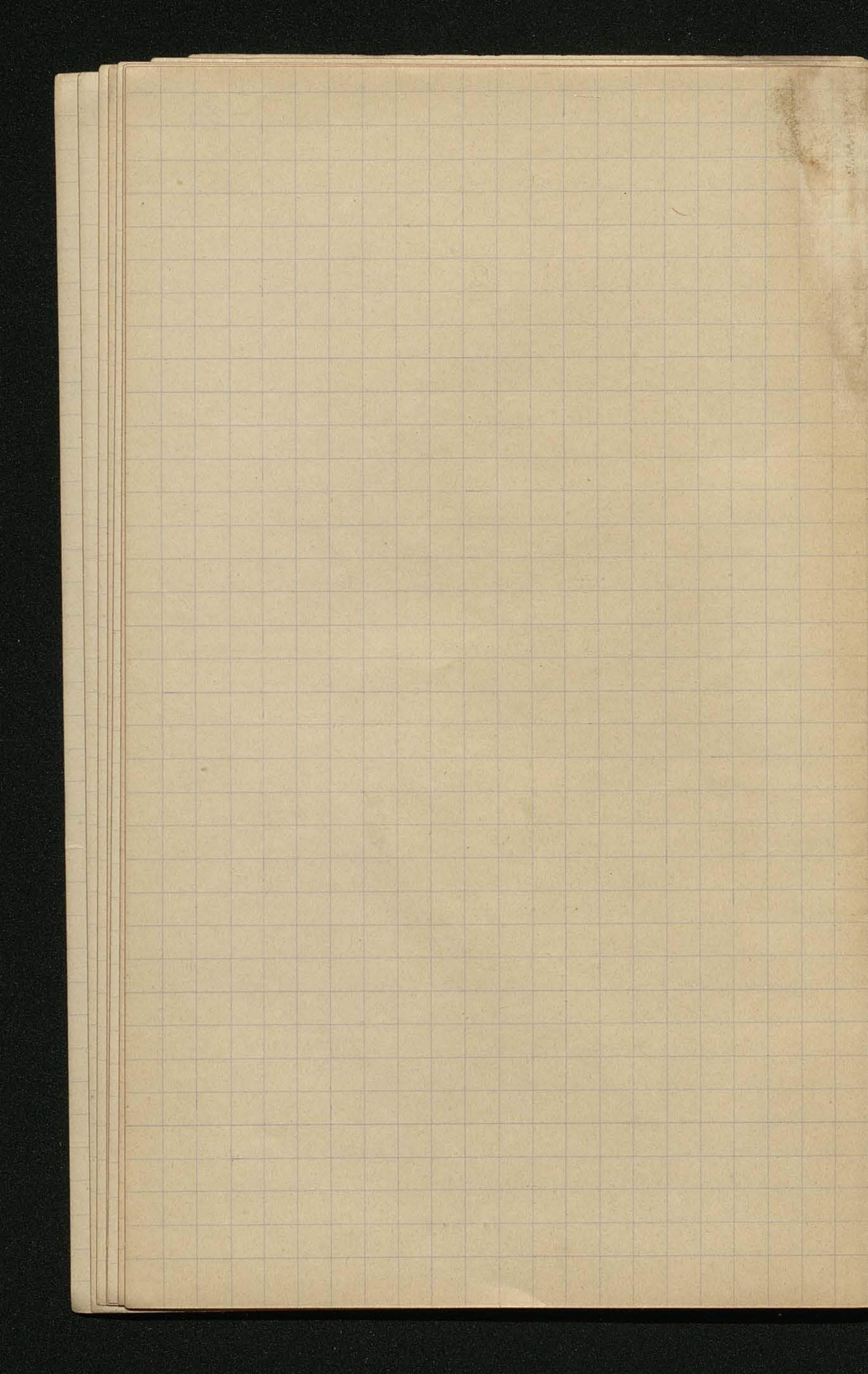


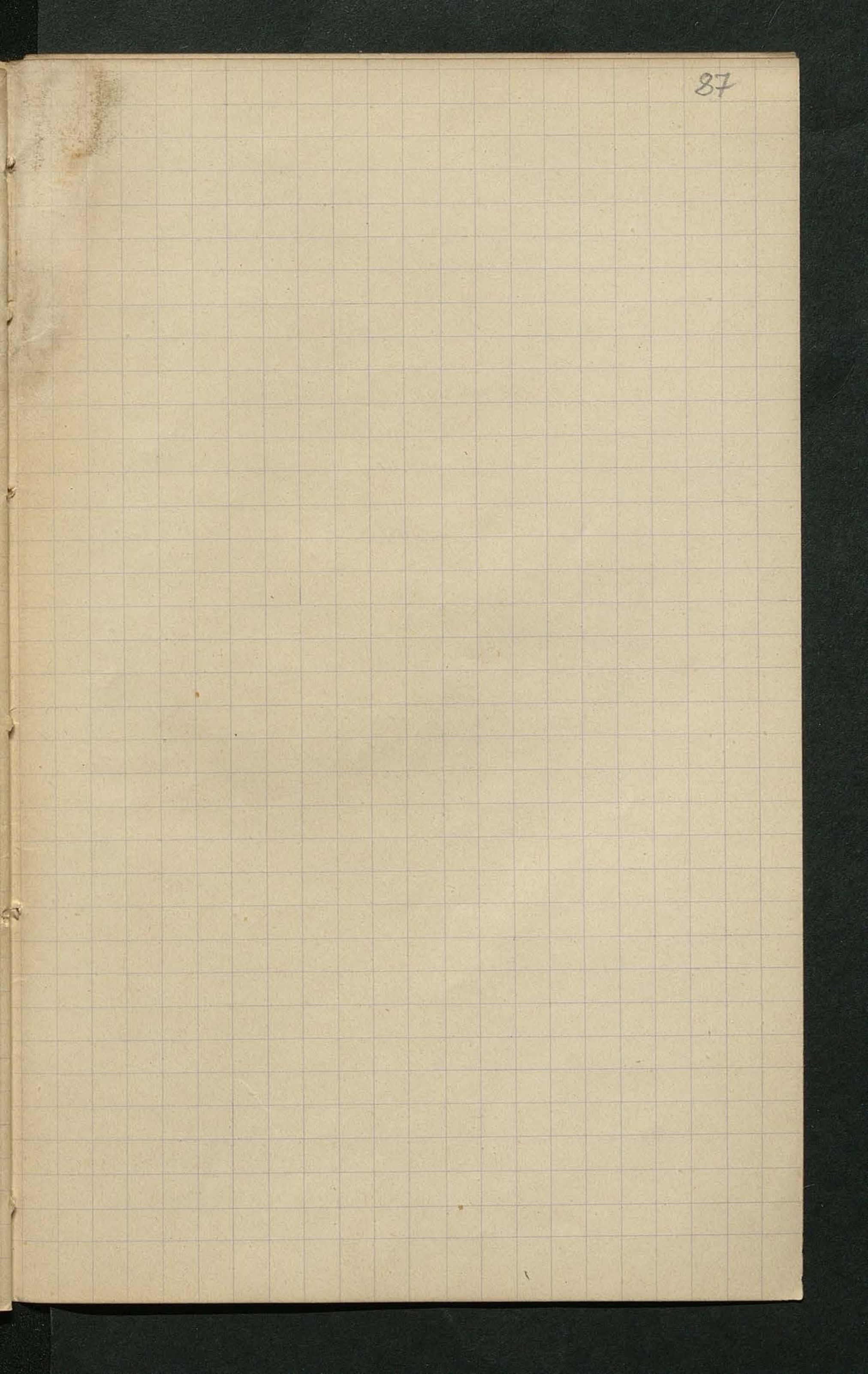


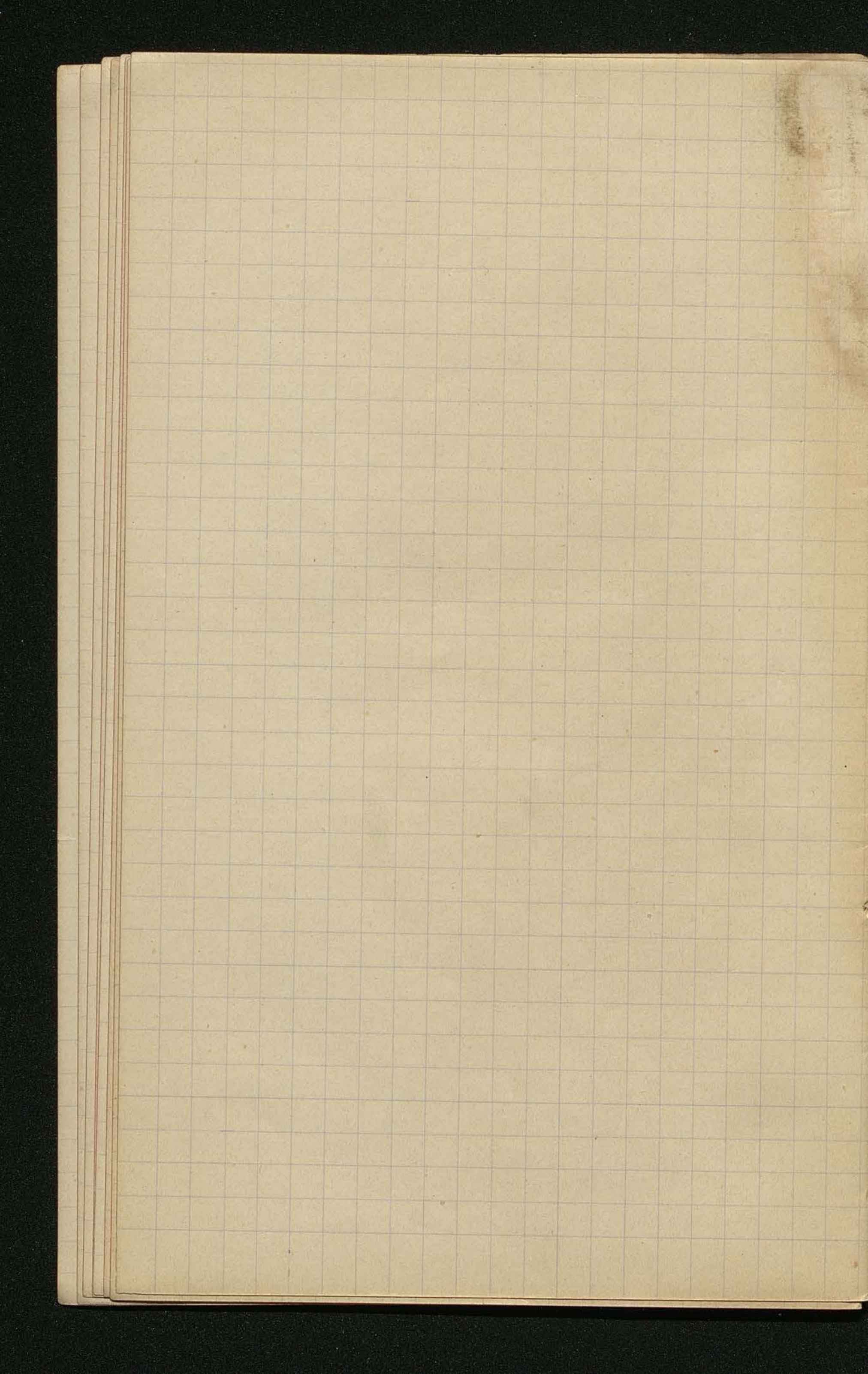


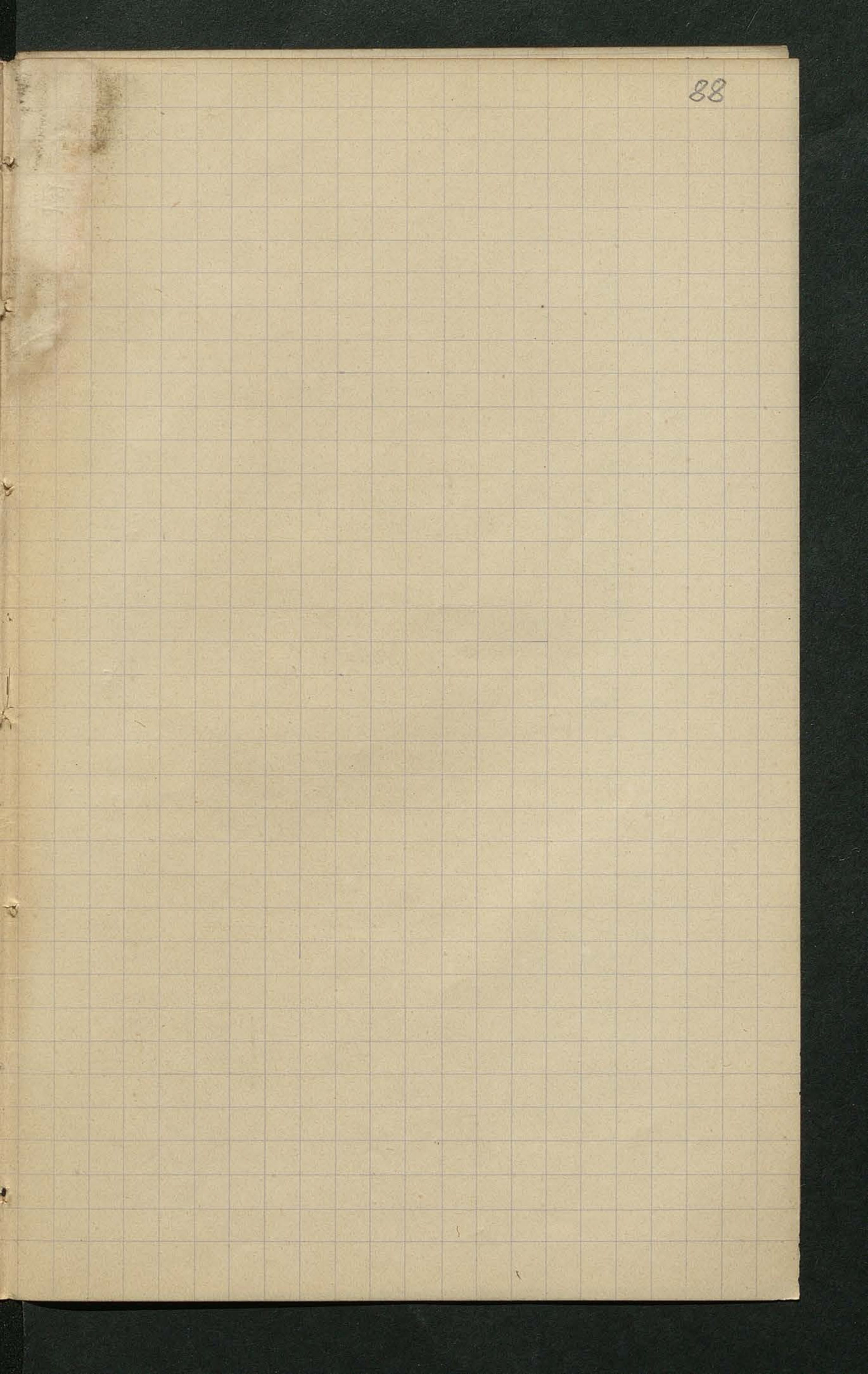


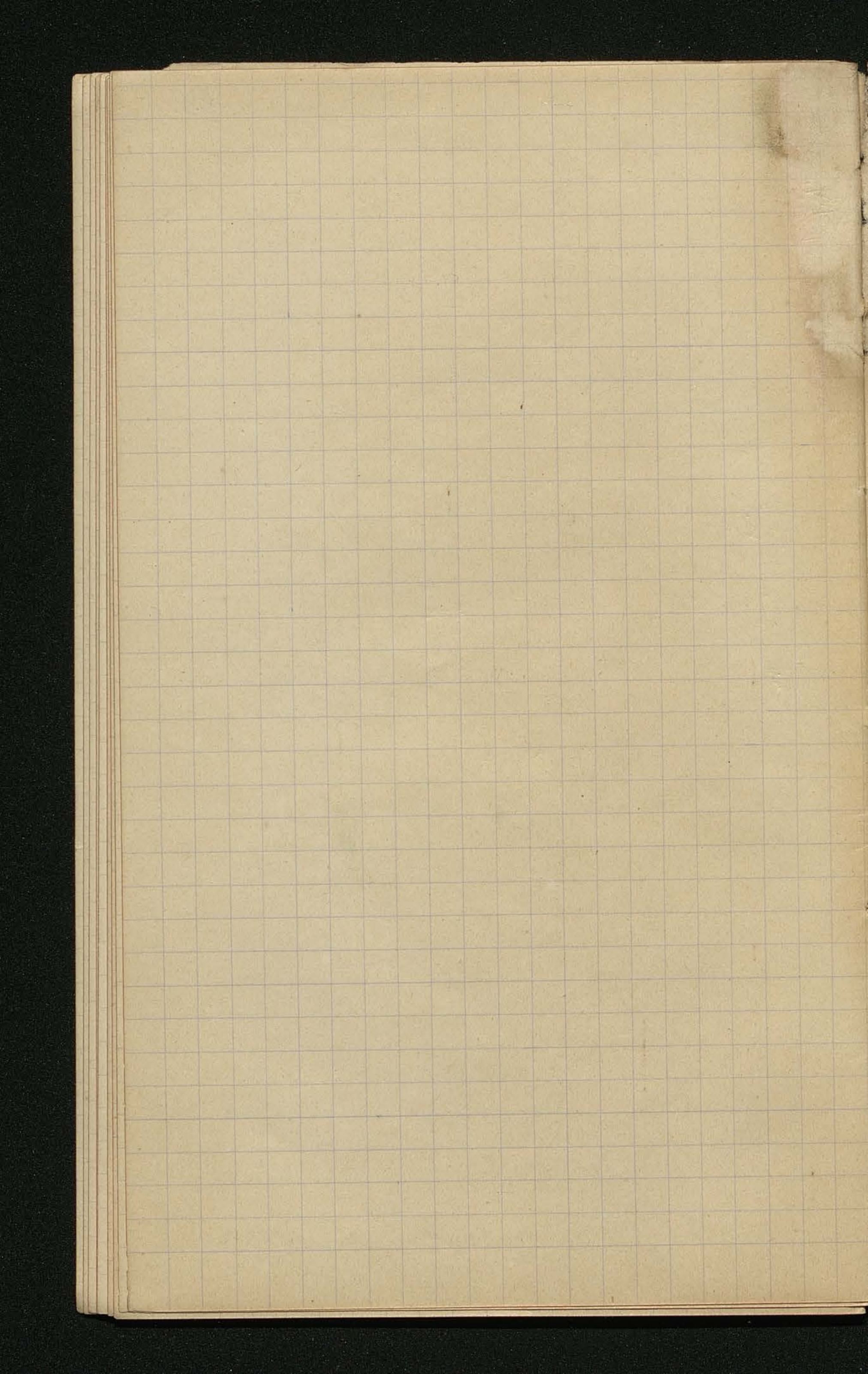


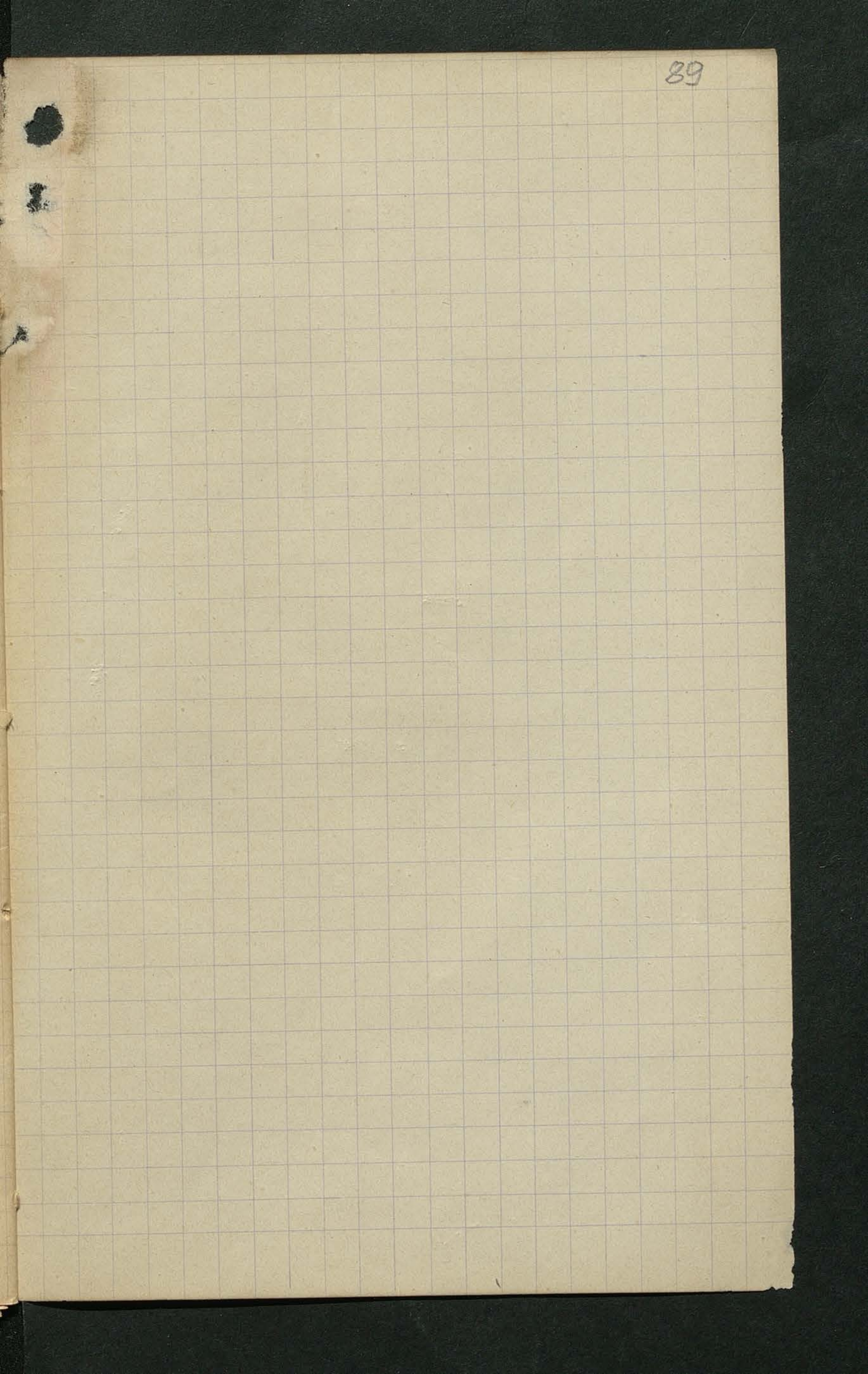


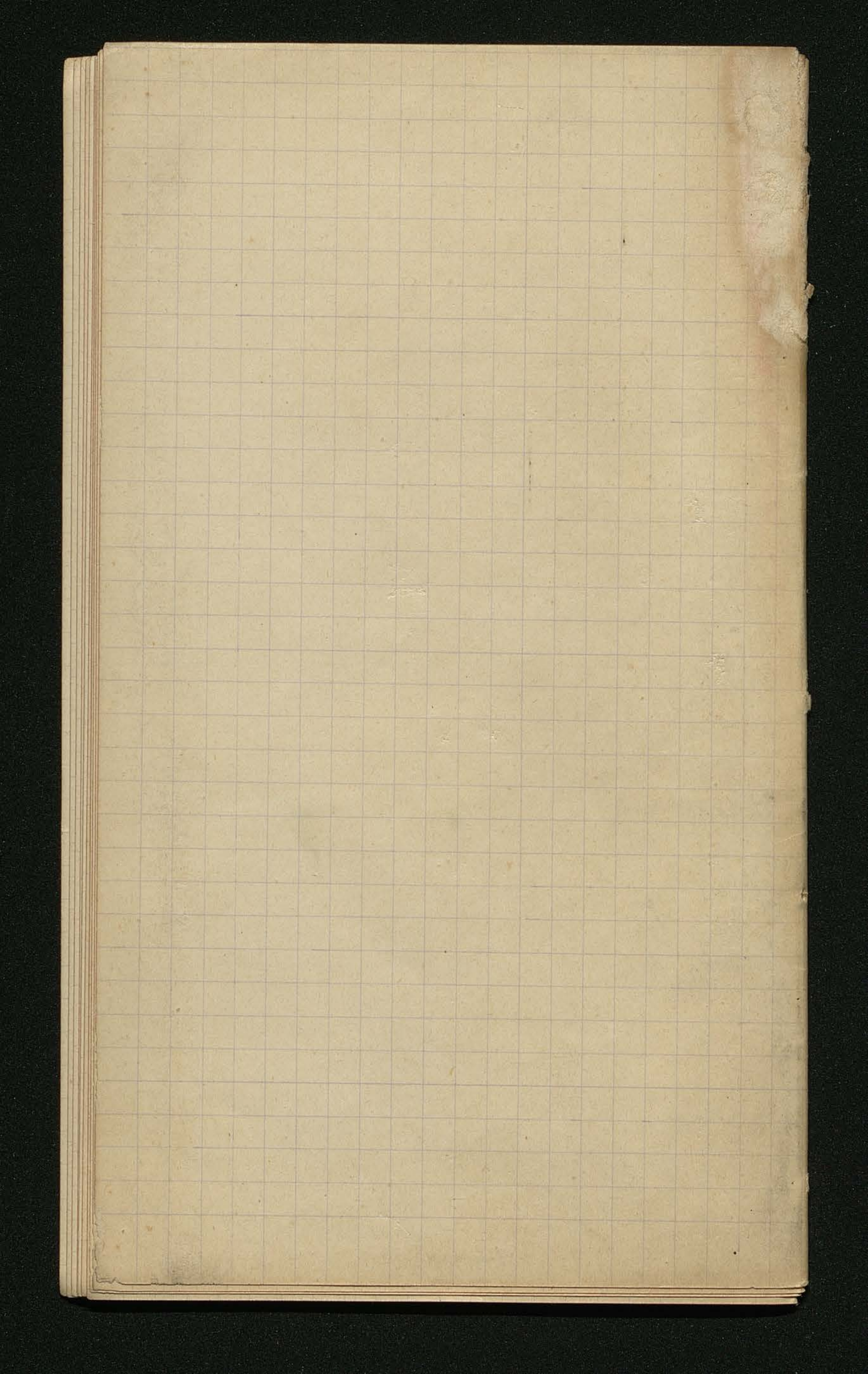


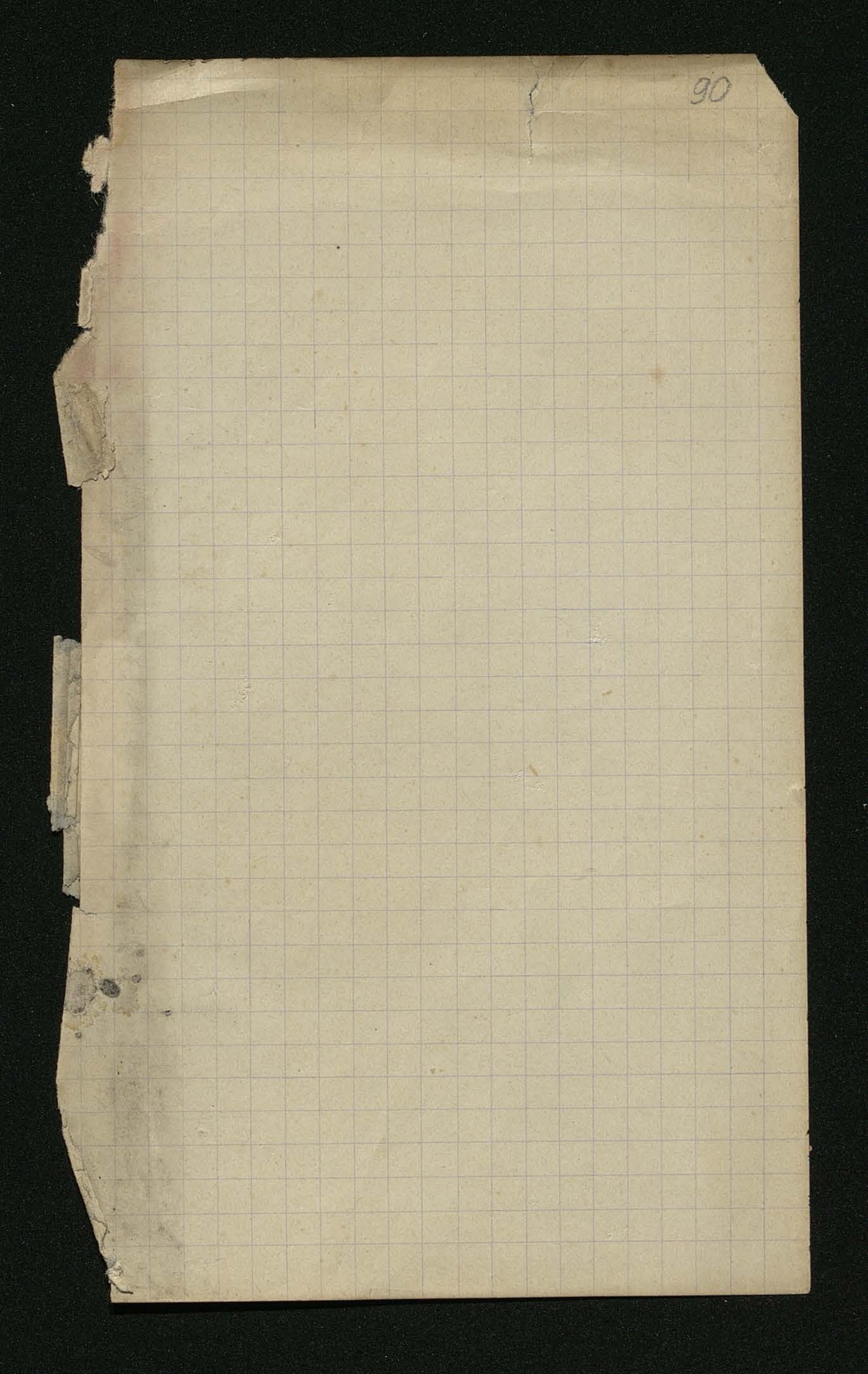


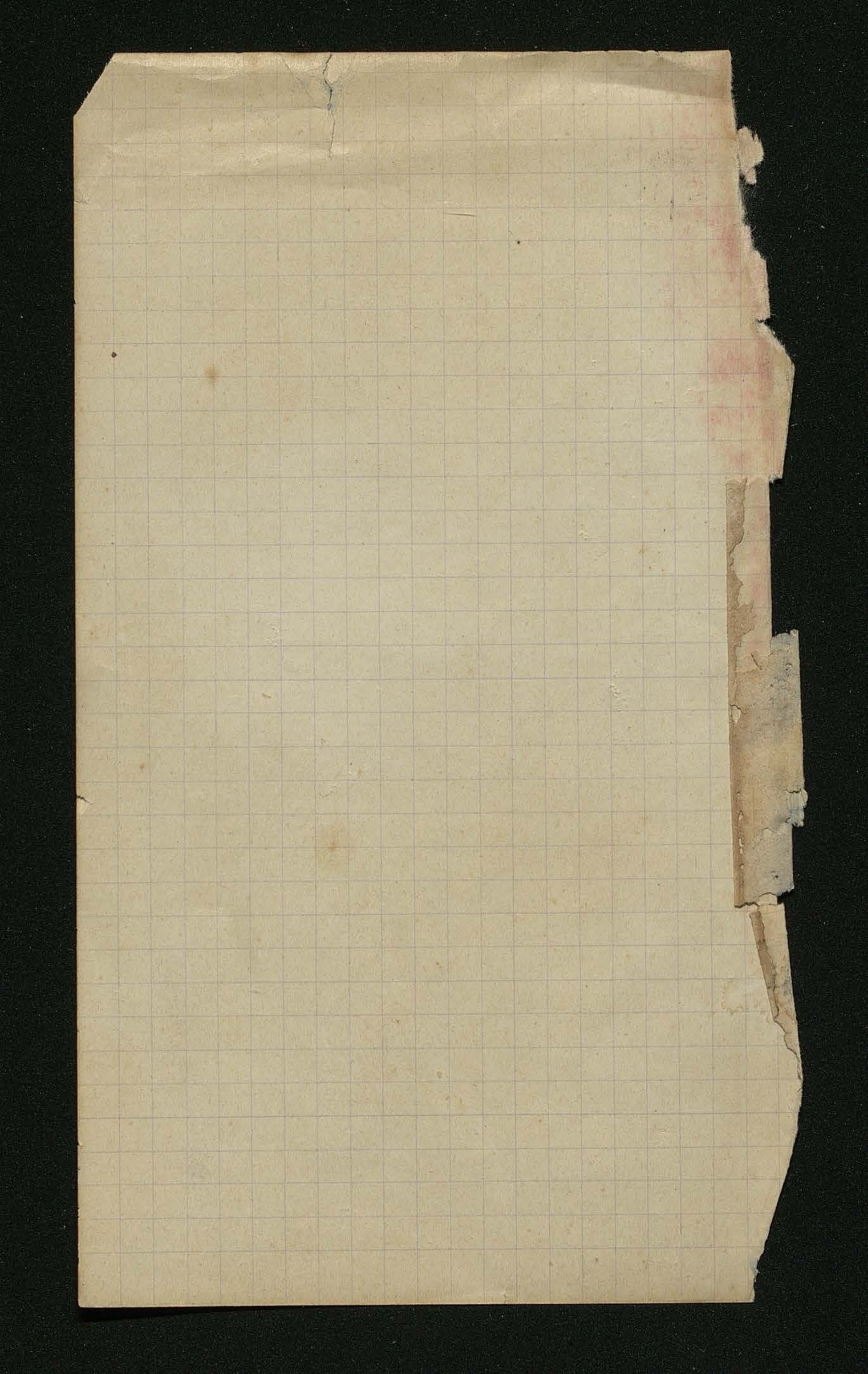






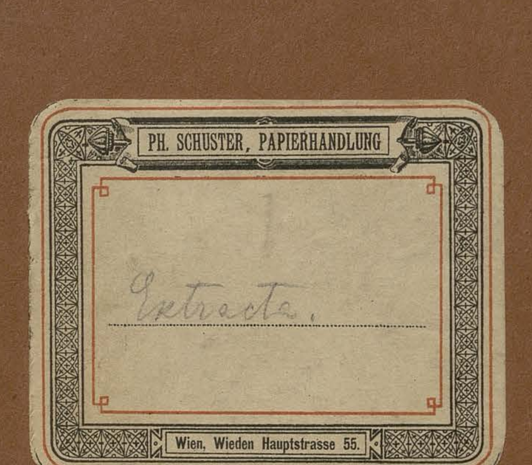


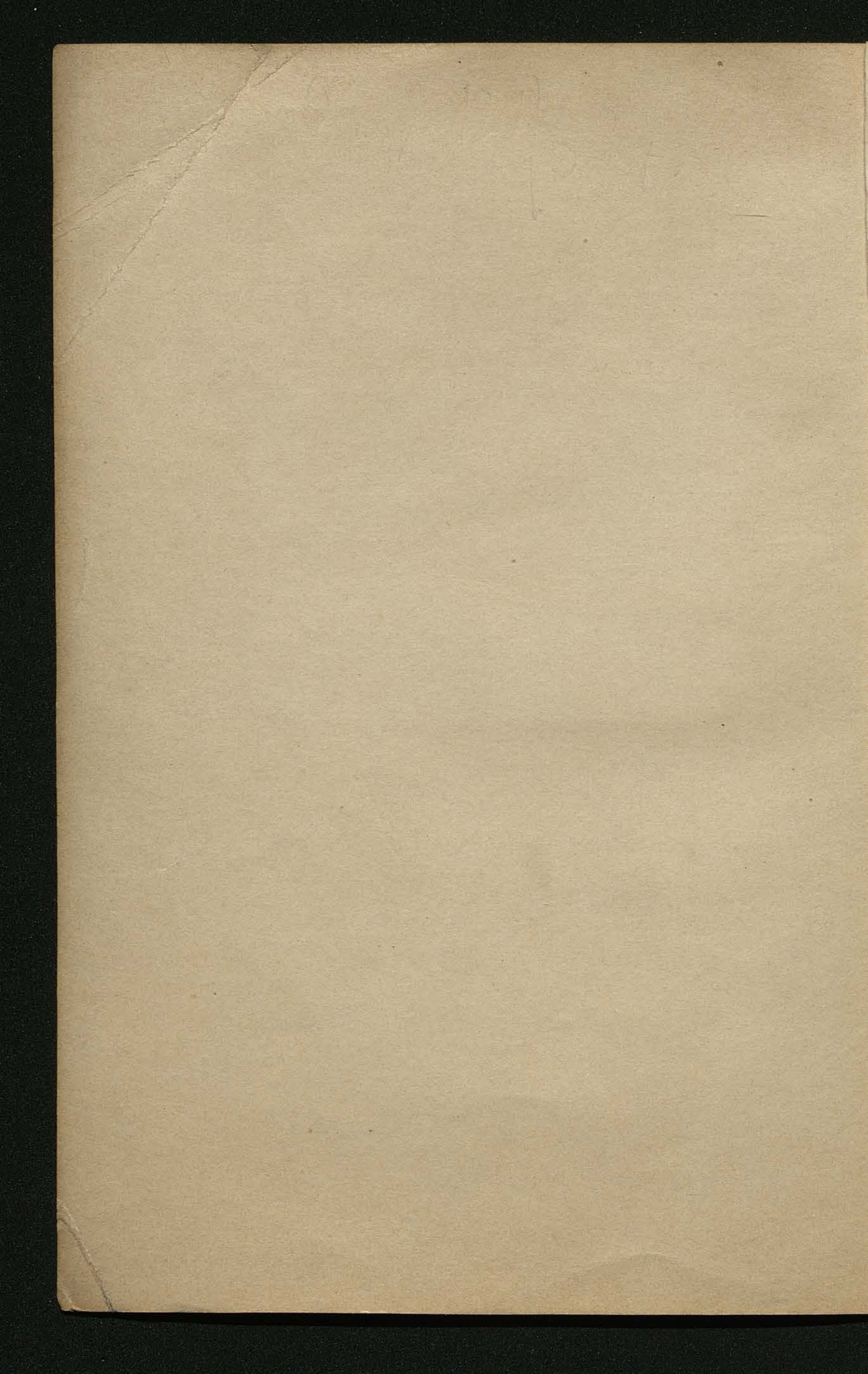


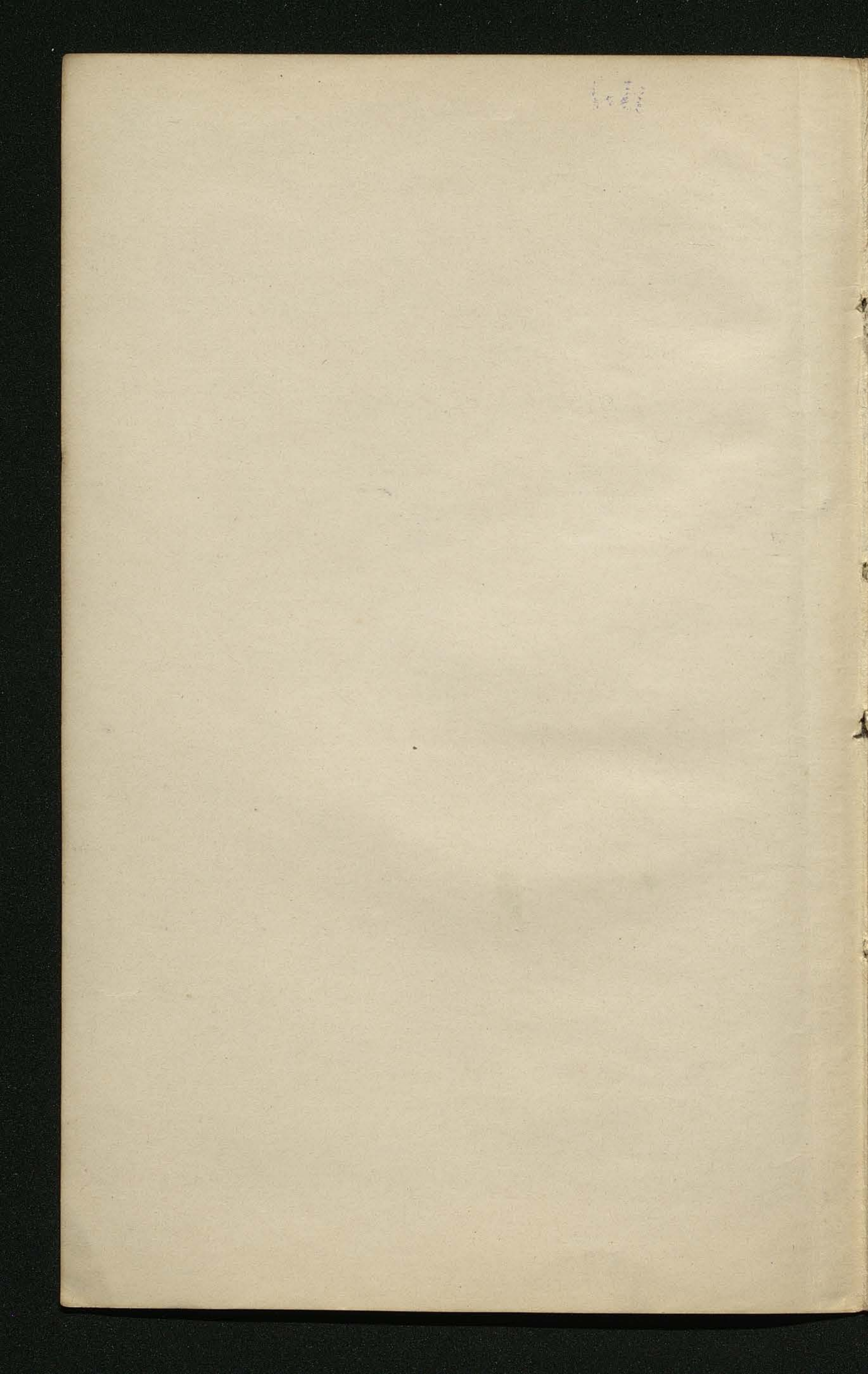


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PH. SCHUSTER, PAPIERHANDLUNG
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Extracta.
Wien, Wieden Hauptstrasse 55.





Riemann: Partielle Differential-Gl. und Anwendungen auf Physik.

Braunschweig 1869
[illegible]

Newtons Principia philosophiae nat. math. 1687

Erste [illegible] p. D. G. [illegible] d'Alembert [illegible] 1747

Fourier's [illegible] 1807

I Bestimmte $\int$

a ξ_1 x_1 ξ_2 x_2 ξ_3 x_3 b

[illegible]

$$S = (x_1 - a) f(\xi_1) + (x_2 - x_1) f(\xi_2) + \text{———} (b - x_{n-1}) f(\xi_n)$$

[illegible] M [illegible], m [illegible] $< \delta$,

[illegible] 1). M [illegible]
2). m [illegible] } [illegible]
3). $M - m$ [illegible]

BW (3): $M = (x'_1 - a) f(\xi'_1) + (x'_2 - x'_1) f(\xi'_2) + \text{———} (b - x'_{n'-1}) f(\xi'_{n'})$

$m = (x''_1 - a) f(\xi''_1) + (x''_2 - x''_1) f(\xi''_2) + \text{———} (b - x''_{n''-1}) f(\xi''_{n''})$

[illegible]: $r_1, r_2 \text{ — } r_{\mu-1}$

$$M = (r_1 - a) f(\rho'_1) + (r_2 - r_1) f(\rho'_2) + \text{ — } (b - r_{\mu-1}) f(\rho'_\mu)$$

[[illegible] ρ' [illegible]]

$$m = (r_1 - a) f(\rho''_1) + (r_2 - r_1) f(\rho''_2) + \text{ — } (b - r_{\mu-1}) f(\rho''_\mu)$$

$$M - m = (r_1 - a)\left[f(\rho'_1) - f(\rho''_1)\right] + (r_2 - r_1)\left[f(\rho'_2) - f(\rho''_2)\right] + \text{ — }$$
$$+ (b - r_{\mu-1})\left[f(\rho'_\mu) - f(\rho''_\mu)\right]$$

ρ'_k [illegible] ρ''_k [illegible] $r_k - r_{k+1}$

resp. —

[illegible] $d =$ [illegible] $f(x)$ [illegible] x [illegible] $[f(\rho'_k) - f(\rho''_k)] < \delta$

[illegible]:

$$M - m < d\,[r_1 - a + r_2 - r_1 + r_3 - r_2 + \text{ — } b - r_{\mu-1})$$

$$M - m < d\,(b - a)$$

[illegible] d [illegible] etc.

$$\int_a^b f(x)\,dx = \lim \left\{(x - a) f(a) + (x_2 - x_1) f(x_1) + \text{ — } (b - x_{n-1}) f(x_{n-1})\right\}$$

$$= \lim \left\{(x_1 - a) f(x_1) + (x_2 - x_1) f(x_2) + \text{ — } \right\}$$

Wallis [illegible] C. [illegible]: $y = m x^k$ [[illegible] Parab.]

[illegible]: $a\ x_1\ x_2 \text{ — } x_{n-1}\ b$ [illegible] – geom. Progr. [illegible]

$$\sqrt[n]{\frac{b}{a}} = \sigma \qquad \begin{aligned} x_1 &= a\sigma \\ x_2 &= a\sigma^2 \\ x_\nu &= a\sigma^\nu \end{aligned}$$

$$\Sigma = \sum_{\nu=0}^{n-1} f(a\sigma^\nu)\, a\sigma^\nu(\sigma-1)$$

z.B. $f(x) = x^k$

$$\Sigma = a^{k+1}(\sigma-1)\sum_0^{n-1}\sigma^{(k+1)\nu}$$

$$\sum_0^{n-1}\sigma^{(k+1)\nu} = 1+\sigma^{k+1}+\sigma^{2(k+1)}+ \text{———} \ \sigma^{(n-1)(k+1)}$$

$$= \frac{\sigma^{n(k+1)}-1}{\sigma^{k+1}-1}$$

$$\Sigma = a^{k+1}\left[\left(\frac{b}{a}\right)^{k+1}-1\right]\frac{\sigma-1}{\sigma^{k+1}-1} = \left(b^{k+1}-a^{k+1}\right)\frac{\sigma-1}{\sigma^{k+1}-1}$$

$$\lim \frac{\sigma-1}{\sigma^{k+1}-1} = \lim\frac{1}{1+\sigma+\sigma^2+ \text{—} \sigma^k} = \frac{1}{k+1}$$

$$\Sigma = \left[b^{k+1}-a^{k+1}\right]\frac{1}{k+1}$$

Doppel-$\int$

$$\sum_{\nu=0}^{n-1}(x_{\nu+1}-x_\nu)\left\{\sum_{\mu=0}^{m-1} f(x_\nu, y_\mu)\,(y_{\mu+1}-y_\mu)\right\}$$

$$= \sum_{\mu=0}^{m-1}(y_{\mu+1}-y_\mu)\left\{\sum_{\nu=0}^{n-1} f(x_\nu, y_\mu)\,(x_{\nu+1}-x_\nu)\right\}$$

$$\int_a^b dx\left\{\int_g^h f(x,y)\,dy\right\} = \int_g^h dy\left\{\int_a^b f(x,y)\,dx\right\}$$

$$\iint x^{y-1}\,dx\,dy = \int_g^h dy \int_a^b x^{y-1}\,dx = \int_g^h \frac{b^y - a^y}{y}\,dy$$

$$= \int_a^b dx \int_g^h x^{y-1}\,dy = \int_a^b \frac{x^{h-1} - x^{g-1}}{\log x}\,dx$$

$\begin{cases} a = 0 \\ b = 1 \end{cases}$ $$\int_0^1 \frac{x^{h-1} - x^{g-1}}{\log x}\,dx = \int_g^h \frac{dy}{y} = \log \frac{h}{g}$$

$h > 0$
$g > 0$

$$\int e^{-ax}\,dx = \frac{e^{-ax}}{-a} + C$$

$$\int e^{-(a+bi)x}\,dx = \frac{e^{-(a+bi)x}}{-(a+bi)} + C$$

$$\int_0^\infty \text{———} = \frac{1}{a+bi} \qquad a > 0$$

$$e^{-(a+bi)x} = e^{-ax}(\cos bx - i \sin bx)$$

$$\int_0^\infty e^{-ax}\cos bx\,dx = \frac{a}{a^2+b^2} \qquad a > 0$$

$$\int_0^\infty e^{-ax}\sin bx\,dx = \frac{b}{a^2+b^2} \qquad a > 0$$

$$\frac{1}{y} = \int_0^\infty e^{-yx}\,dx$$

$$\int_g^h \frac{\sin \beta y}{y}\, dy = \int_g^h \sin \beta y \, dy \int_0^\infty e^{-yx}\, dx$$

$$= \int_0^\infty dx \int_g^h e^{-yx} \sin \beta y \, dy$$

$\begin{cases} g = 0 \\ h = \infty \end{cases}$

$$\int_0^\infty \frac{\sin \beta y}{y}\, dy = \begin{cases} \frac{\pi}{2} & \beta > 0 \\ 0 & \beta = 0 \\ -\frac{\pi}{2} & \beta < 0 \end{cases}$$

$$\int_0^\infty \frac{\sin y}{y} \cos \mu y \, dy = \frac{1}{2} \int_0^\infty \frac{\sin (1+\mu) y}{y}\, dy + \frac{1}{2} \int_0^\infty \frac{\sin (1-\mu) y}{y}\, dy$$

$$\int_0^\infty \frac{\sin y}{y} \cos \mu y \, dy = \begin{cases} 0 & \mu > 0 \\ \frac{\pi}{4} & \mu = 1 \\ \frac{\pi}{2} & 1 > \mu > -1 \\ \frac{\pi}{4} & \mu = -1 \\ 0 & -1 > \mu \end{cases}$$

$$\int_0^\infty \frac{y \sin k y}{y^2 + b^2}\, dy = \int_0^\infty \cos bx \, dx \int_0^\infty e^{-xy} \sin ky \, dy$$

$$= \int_0^\infty \frac{k \cos bx}{k^2 + x^2}\, dx$$

$y = \frac{b}{k} x$

$$\underbrace{\int_0^\infty \frac{k \cos bx}{k^2+x^2}\, dx}_{= ku} = \underbrace{\int_0^\infty \frac{x \sin bx}{k^2+x^2}\, dx}_{= -\frac{du}{db}} \qquad \frac{b}{k} > 0$$

$k\,b = -\log u + \log c$

$u = c\,e^{-kb}$

$b=0 \Big| \quad u = c = \int_0^\infty \frac{dx}{k^2+x^2} = \frac{1}{k} \operatorname{arctg} \frac{x}{k} + c$

$c = \pm \frac{\pi}{2k}$

$$\int_0^\infty \frac{x \sin bx}{k^2+x^2}\,dx = \begin{cases} \frac{\pi}{2} e^{-bk} & b>0,\ k>0 \\ -\frac{\pi}{2} e^{-bk} & b<0,\ k<0 \end{cases}$$

[illegible] k:

$$= \begin{cases} \frac{\pi}{2} e^{-b\sqrt{k^2}} & b>0 \\ -\frac{\pi}{2} e^{-b\sqrt{k^2}} & b<0 \end{cases}$$

[illegible]:

$$\int_0^\infty \frac{\cos bx}{k^2+x^2}\,dx = \begin{cases} \frac{\pi}{2\sqrt{k^2}} e^{-b\sqrt{k^2}} & b>0 \\ \frac{\pi}{2\sqrt{k^2}} e^{b\sqrt{k^2}} & b<0 \end{cases}$$

$k>0$

$$\int_0^\infty \frac{x \sin bx + k \cos bx}{k^2+x^2}\,dx = \begin{cases} \pi e^{-bk} & b>0 \\ \frac{\pi}{2} & b=0 \\ 0 & b<0 \end{cases}$$

$$\int_0^\infty e^{-ax^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{a}} \qquad \frac{\partial}{\partial a}$$

$$\int_0^\infty e^{-ax^2} x^{2n}dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}\frac{(2n)!}{n!}\frac{1}{(4a)^n}$$

$$\int_0^\infty e^{-ax^2}\cos\beta x\,dx = \int_0^\infty dx\, e^{-ax^2}\sum_{n=0}^\infty \frac{(-\beta^2x^2)^n}{(2n)!}$$

$$= \sum_0^\infty \frac{(-\beta^2)^n}{(2n)!}\frac{1}{2}\sqrt{\frac{\pi}{a}}\frac{(2n)!}{n!}\frac{1}{(4a)^n}$$

$$= \frac{1}{2}\sqrt{\frac{\pi}{a}}\, e^{-\frac{\beta^2}{4a}}$$

Unendliche Reihen.

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+ \text{——} \qquad = \int_0^1 \frac{1-x^n}{1-x}dx$$

$$\int_0^{1-k\varepsilon}\frac{dx}{1-x} = -\log\left[1-(1-k\varepsilon)\right] = -\log k\varepsilon$$

$$\lim \text{——} = \infty$$

$$\lim\left[\underbrace{1+\frac{1}{2}-1}+\underbrace{\frac{1}{3}+\frac{1}{4}-\frac{1}{2}}+\text{——}\right] = \lim\sum_{n=m+1}^{2m}\frac{1}{n}$$

$$= \lim\frac{1}{m}\left[\frac{1}{1+\frac{1}{m}}+\frac{1}{1+\frac{2}{m}}+\text{——}\right] = \int_1^2\frac{dx}{x} = \log 2$$

$$\lim\left[\underbrace{1+\frac{1}{2}+\text{——}\frac{1}{k}-1}_{k+1}+\underbrace{\frac{1}{k+1}+\frac{1}{k+2}+\text{---}\frac{1}{2k}}_{k+1}-\frac{1}{2}+\text{-}\right] = \lim\sum_{m+1}^{km}\frac{1}{n}$$

$$= \lim\frac{1}{n}\left\{\frac{1}{1+\frac{1}{n}}+\text{——}\frac{1}{1+\frac{(k-1)m}{m}}\right\} = \int_1^k\frac{dx}{x} = \log k$$

$$f(x) = a_1 \sin x + a_2 \sin 2x + \cdots\cdots a_{n-1} \sin(n-1)x$$

$$= 0 \;\Big|\; \begin{matrix} x = 0 \\ x = \pi \end{matrix}$$

$$0 < x < \pi$$

$(n-1)$ [illegible] Ord. [illegible]

[illegible] x: $\frac{\pi}{n}, \frac{2\pi}{n}, \frac{3\pi}{n}, \cdots \frac{m\pi}{n} \cdots \frac{(n-1)\pi}{n}$.

$\qquad y_1 \quad y_2 \quad y_3 \quad \cdot \quad y_m \quad y_{n-1}$

$$f\left(\frac{\pi}{n}\right) = a_1 \sin\frac{\pi}{n} + a_2 \sin 2\frac{\pi}{n} + \cdots\cdots a_{n-1} \sin(n-1)\frac{\pi}{n} \qquad \Big|\; 2\sin m\frac{\pi}{n}$$

$$f\left(\frac{2\pi}{n}\right) = a_1 \sin\frac{2\pi}{n} + a_2 \sin 2\frac{2\pi}{n} + \cdots\cdots a_{n-1} \sin(n-1)\frac{2\pi}{n} \qquad \Big|\; 2\sin m\frac{2\pi}{n}$$

$$f\left(\frac{(n-1)\pi}{n}\right) = a_1 \sin\frac{(n-1)\pi}{n} + \cdots\cdots a_{n-1} \sin(n-1)\frac{(n-1)\pi}{n} \qquad \Big|\; 2\sin m\frac{(n-1)\pi}{n}$$

[illegible] Lagrange [illegible] Coeff. a_m:

[illegible] a_k: $A_k = 2\left(\sin k\frac{\pi}{n}\sin m\frac{\pi}{n} + \cdots\cdots \sin k\frac{(n-1)\pi}{n}\sin m\frac{(n-1)\pi}{n}\right)$

$$= \cos(k-m)\frac{\pi}{n} + \cos(k-m)\frac{2\pi}{n} + \cdots\cdots \cos(k-m)\frac{(n-1)\pi}{n}$$

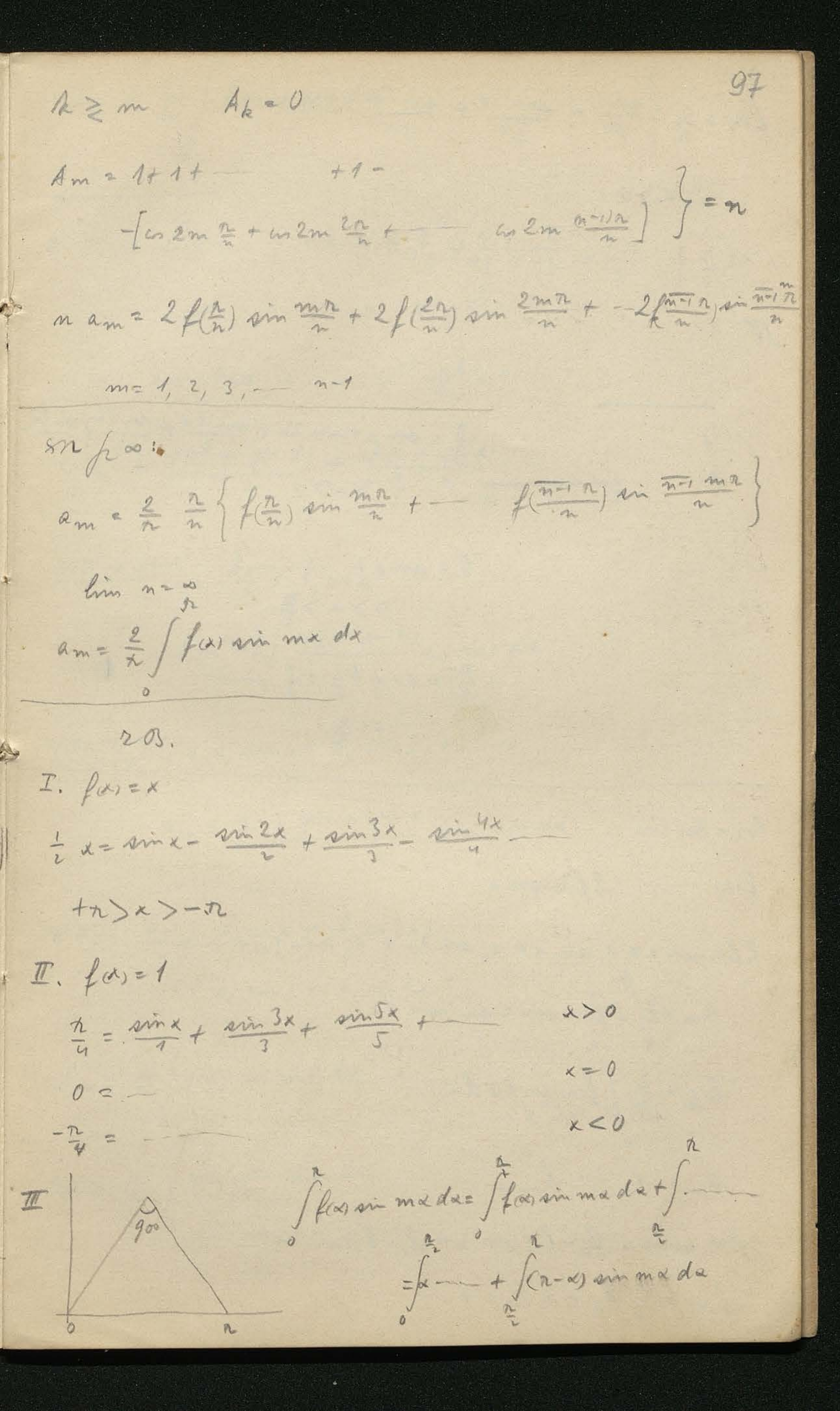
$$-\left[\cos(k+m)\frac{\pi}{n} \cdots\cdots \cos(k+m)\frac{(n-1)\pi}{n}\right]$$

$$s = \cos\theta + \cos 2\theta + \cdots\cdots \cos(n-1)\theta \qquad \Big|\; 2\cos\theta$$

$$2s\cos\theta = 1 + \cos\theta + \cos 2\theta + \cdots\cdots \cos(n-2)\theta$$

$$+ \cos 2\theta + \cdots\cdots \cos n\theta \;\Big\}\; = \begin{matrix} 1 - \cos(n-1)\theta + s \\ \cos n\theta - \cos\theta + s \end{matrix}$$

$$s = -\frac{1}{2} + \frac{\sin(2n-1)\frac{\theta}{2}}{2\sin\frac{\theta}{2}}$$



$k \gtrless m \qquad A_k = 0$

$$A_m = 1 + 1 + \cdots + 1 - \left[\cos 2m\frac{\pi}{n} + \cos 2m\frac{2\pi}{n} + \cdots \cos 2m\frac{(n-1)\pi}{n}\right] \Bigg\} = n$$

$$n\, a_m = 2f\left(\frac{\pi}{n}\right)\sin\frac{m\pi}{n} + 2f\left(\frac{2\pi}{n}\right)\sin\frac{2m\pi}{n} + \cdots 2f\left(\frac{\overline{n-1}\,\pi}{n}\right)\sin\frac{\overline{n-1}\,m\pi}{n}$$

$m = 1, 2, 3, \ldots n-1$

---

n $\not\sim \infty$:

$$a_m = \frac{2}{\pi}\,\frac{\pi}{n}\left\{f\left(\frac{\pi}{n}\right)\sin\frac{m\pi}{n} + \cdots f\left(\frac{\overline{n-1}\,\pi}{n}\right)\sin\frac{\overline{n-1}\,m\pi}{n}\right\}$$

$\lim n = \infty$

$$a_m = \frac{2}{\pi}\int_0^\pi f(x)\sin mx\, dx$$

---

z.B.

I. $f(x) = x$

$$\frac{1}{2}x = \sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \frac{\sin 4x}{4} \cdots$$

$+\pi > x > -\pi$

II. $f(x) = 1$

$$\frac{\pi}{4} = \frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \cdots \qquad x > 0$$

$$0 = \cdots \qquad x = 0$$

$$-\frac{\pi}{4} = \cdots \qquad x < 0$$

III

$$\int_0^\pi f(x)\sin mx\,dx = \int_0^{\frac{\pi}{2}} f(x)\sin mx\,dx + \int_{\frac{\pi}{2}}^{\pi} \cdots$$

$$= \int_0^{\frac{\pi}{2}} x \cdots + \int_{\frac{\pi}{2}}^{\pi} (\pi - x)\sin mx\,dx$$

$$f(x) = \frac{4}{\pi}\left\{\frac{\sin x}{1^2} - \frac{\sin 3x}{3^2} + \frac{\sin 5x}{5^2} + \cdots\right\}$$

$\pi > x > 0$

$x = \frac{\pi}{2}$ | $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

IV.

1

0 $\frac{\pi}{2}$ π

$$\frac{\pi}{2} = \sin x + \sin 2x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \frac{\sin 6x}{3} + \cdots$$

$\frac{\pi}{2} > x > 0$

$0 = \sin x + \cdots$

$\pi > x > \frac{\pi}{2}$

$$\left(\frac{\pi}{4}\right) = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \cdots$$

$x = \frac{\pi}{2}$

Cos – Reihen

$f(x)$ —— $2f(x)\sin x$

$$2f(x)\sin x = a_1 \sin x + a_2 \sin 2x + a_3 \sin 3x + \cdots$$

$$a_m = \frac{2}{\pi}\int_0^\pi 2f(x)\sin x \sin mx \, dx$$

$$b_k = \frac{2}{\pi}\int_0^\pi f(x)\cos kx \, dx$$

$$a_m = b_{m-1} - b_{m+1}$$

$$2f(x)\sin x = (b_0 - b_2)\sin x + (b_1 - b_3)\sin 2x + \cdots$$

$$f(x) = \frac{1}{2}b_0 + b_1 \cos x + \cdots$$

$$b_m = \frac{2}{\pi}\int_0^{\pi} f(\alpha)\cos m\alpha\, d\alpha$$

---

x — [illegible] sin [illegible] $-x$ [illegible] $+x$

" " cos " " " [illegible] " " "

" " sin + cos " " [illegible] " " "

$$\varphi(x) = \frac{\varphi(x)+\varphi(-x)}{2} + \frac{\varphi(x)-\varphi(-x)}{2}$$

$$\frac{\varphi(x)+\varphi(-x)}{2} = \frac{b_0}{2} + b_1\cos x + \text{———}$$

$$b_m = \frac{2}{\pi}\int_0^{\pi} \frac{\varphi(\alpha)+\varphi(-\alpha)}{2}\cos m\alpha\, d\alpha$$

$$\frac{\varphi(x)-\varphi(-x)}{2} = a_1\sin x + a_2\sin 2x + \text{———}$$

$$a_m = \frac{2}{\pi}\int_0^{\pi} \frac{\varphi(\alpha)-\varphi(-\alpha)}{2}\sin m\alpha\, d\alpha$$

$$\varphi(x) = \frac{b_0}{2} + b_1\cos x + b_2\cos 2x + \text{———}$$
$$+ a_1\sin x + a_2\sin 2x + \text{———}$$

$$b_m = \frac{1}{\pi}\int_{-\pi}^{\pi} \varphi(\alpha)\cos m\alpha\, d\alpha \qquad \pi > x > -\pi$$

$$a_m = \frac{1}{\pi}\int_{-\pi}^{\pi} \varphi(\alpha)\sin m\alpha\, d\alpha$$

Convergenz-Beweis [n. Dirichlet (Crelle IV, Dove I)

$$S_{2n+1} = \frac{1}{2} b_0 + b_1 \cos x + b_2 \cos 2x + \text{---} \; b_n \cos nx$$

$$+ a_1 \sin x + a_2 \sin 2x + \text{---} \; a_n \sin nx$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} \varphi(\alpha) \frac{\sin (2n+1) \frac{\alpha - x}{2}}{2 \sin \frac{\alpha - x}{2}} d\alpha$$

$$\frac{1}{2} + \cos 2\beta + \cos 4\beta + \text{---} \; \cos 2n\beta = \frac{\sin (2n+1)\beta}{2 \sin \beta}$$

$$\int \text{--} \, d\beta$$

$$\frac{\pi}{2} = \int_0^{\frac{\pi}{2}} \frac{\sin (2n+1)\beta}{\sin \beta} d\beta$$

$$= \int_0^{\frac{\pi}{h}} + \int_{\frac{\pi}{h}}^{\frac{2\pi}{h}} + \text{---} + \int_{\frac{\mu\pi}{h}}^{\frac{\pi}{2}}$$

$$= \rho_0 - \rho_1 + \rho_2 - \text{---} + (-1)^n \rho_n$$

$$\rho_{\nu-1} = (-1)^{\nu-1} \int_{\frac{(\nu-1)\pi}{h}}^{\frac{\nu\pi}{h}} \frac{\sin h\beta}{\sin \beta} d\beta$$

$$\frac{(-1)^{\nu-1}}{\sin \frac{(\nu-1)\pi}{h}} \int_{\frac{(\nu-1)\pi}{h}}^{\frac{\nu\pi}{h}} \sin h\beta \, d\beta > \rho_{\nu-1} > \frac{(-1)^{\nu-1}}{\sin \frac{\nu\pi}{h}} \int_{\frac{(\nu-1)\pi}{h}}^{\frac{\nu\pi}{h}}$$

$$\frac{2}{h \sin \frac{(\nu-1)\pi}{h}} > \rho_{\nu-1} > \frac{2}{h \sin \frac{\nu\pi}{h}} > \rho_\nu > \text{---}$$

$\frac{\pi}{2} < \rho_0 - \rho_1 + \rho_2 - \cdots + \rho_{2\mu}$

$\frac{\pi}{2} > \rho_0 - \rho_1 + \rho_2 - \cdots - \rho_{2\mu-1} \qquad 2\mu < n$

$$T = \int_0^b f(\beta) \frac{\sin h\beta}{\sin\beta} d\beta = \int_0^{\frac{\pi}{h}} + \int_{\frac{\pi}{h}} \cdots\cdots \int_{\frac{m\pi}{h}}^b .$$

f nur abnehmend

$$= r_0 - r_1 + r_2 - \cdots + (-1)^m r_m$$

$$r_m = (-1)^m \int_{\frac{m\pi}{h}}^b f(\beta) \frac{\sin h\beta}{\sin\beta} d\beta$$

$$\rho_{\nu-1} f\left(\frac{\nu-1}{h}\pi\right) \geqq r_{\nu-1} \geqq \rho_{\nu-1} f\left(\frac{\nu\pi}{h}\right) > \rho_\nu f\left(\frac{\nu\pi}{h}\right) \geqq r_\nu$$

$$T > \rho_0 f\left(\frac{\pi}{h}\right) - \rho_1 f\left(\frac{\pi}{h}\right) + \rho_2 f\left(\frac{3\pi}{h}\right) - \cdots - \rho_{2\mu-1} f\left(\frac{2\mu-1}{h}\pi\right)$$

$$> f\left(\frac{2\mu\pi}{h}\right)\left[\rho_0 - \rho_1 + \rho_2 - \cdots - \rho_{2\mu-1}\right]$$

$$T < \rho_0 f(0) - f\left(\frac{2\mu\pi}{h}\right)\left[\rho_1 - \rho_2 + \rho_3 - \cdots - \rho_{2\mu}\right]$$

$$T > f\left(\frac{2\mu\pi}{h}\right)\left(\frac{\pi}{2} - \rho_{2\mu}\right)$$

$$T < \rho_0 \left\{ f(0) - f\left(\frac{2\mu\pi}{h}\right) \right\} + \left(\frac{\pi}{2} + \rho_{2\mu}\right) f\left(\frac{2\mu\pi}{h}\right)$$

$\mu = \infty$
$h = \infty \qquad \lim \frac{\mu}{h} = 0 \qquad \lim \rho_{2\mu} = 0$
$m = \infty$

$$\lim T = \frac{\pi}{2} f(0)$$

$$\lim \int_0^b (c + f(\beta)) \frac{\sin h\beta}{\sin\beta} d\beta = \frac{\pi}{2}(c + f(0))$$

$$\lim \int_0^b c \text{ ------ } = \frac{\pi}{2} c$$

$$\lim \int_0^b f(\beta) \frac{\sin h\beta}{\sin\beta} d\beta = \frac{\pi}{2} f(0)$$

$$\lim \int (-f\beta) \text{ ------ } = \frac{\pi}{2}[-f(0)]$$

[illegible]

[illegible] $\int_0^c + \int_c^b = \frac{\pi}{2} f(0)$

[illegible]

[illegible] $f(\beta) = \infty$ [illegible] $\int_\beta^{c_1} f(\beta)\, d\beta$ [illegible] $\int_{c_1}^{\beta}$ [illegible] $\beta = c_1$ $= 0$

[illegible] Σ.

$$S_{2n+1} = \frac{1}{2\pi} \int \text{ ...... } \qquad \alpha - x = 2\beta$$

$$= \frac{1}{\pi} \int_{-\frac{\pi+x}{2}}^{\frac{\pi-x}{2}} \varphi(x + 2\beta) \frac{\sin h\beta}{\sin\beta} d\beta = \frac{1}{\pi}\left[\int^0 + \int_0\right]$$

etc. etc.

$$\lim S_{2n+1} = \frac{1}{2}\left[\varphi(x+0) + \varphi(x-0)\right] \qquad \pi > x > -\pi$$

$$= \frac{1}{2}\left[\varphi(-\pi+0) + \varphi(\pi-0)\right] \qquad x = \pm\pi$$

$$\varphi\left(\frac{cz}{\pi}\right) = \frac{1}{2}b_0 + b_1\cos z + b_2\cos 2z + \text{———}$$

$$+ a_1 \sin z + a_2 \sin 2z + \text{———}$$

$$\begin{cases} b_m = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} \varphi\left(\frac{c\alpha}{\pi}\right)\cos m\alpha \, d\alpha \\ a_m = \text{— — — — —} \end{cases} \qquad \pi > z > -\pi$$

$$\varphi(x) = \frac{1}{2}b_0 + b_1 \cos\frac{\pi x}{c} + \text{———}$$

$$+ a_1 \sin\frac{\pi x}{c} + \text{———} \qquad c > x > -c$$

$$\begin{cases} b_m = \dfrac{1}{c}\displaystyle\int_{-c}^{c} \varphi(\lambda)\cos\frac{m\pi\lambda}{c}\, d\lambda \\ a_m = \text{— — — — —} \end{cases}$$

$$\varphi(x) = \frac{1}{c}\left\{-\frac{1}{2}\int_{-c}^{c}\varphi(\lambda)\,d\lambda + \sum_{0}^{\infty}\int_{-c}^{c} d\lambda\, \varphi(\lambda)\cos\frac{m\pi}{c}(\lambda - x)\right.$$

$$\frac{m\pi}{c} = \alpha \qquad c = \infty \qquad d\alpha = \frac{\pi}{c}$$

$$\varphi(x) = \frac{1}{\pi}\int_{0}^{\infty} d\alpha \left\{\int_{-\infty}^{\infty} d\lambda\, \varphi(\lambda)\cos\alpha(\lambda - x)\right\}$$

Methoden zur Zurückführung einer lin. D. Gl. mit zw. Gl.

1). $a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \text{———} + a_n y = X$

$\int X z \, dx$

$\int a_n z y \, dx = \int \text{—}$

$\int a_{n-1} z \frac{dy}{dx} dx = a_{n-1} z y - \int y \frac{d(a_{n-1} z)}{dx} dx$

———

$\frac{d^n (a_0 z)}{dx^n} - \frac{d^{n-1}(a_1 z)}{dx^{n-1}} \text{———} + (-1)^n a_n z = 0$

$\int X z \, dx = y (a_{n-1} z + \text{——})$

$+ \frac{dy}{dx} (\quad)$

$+ \text{———}$

$+ \frac{d^{n-1} y}{dx^{n-1}} a_0 z$

---

2). $y = u \cdot v$

$X = u \{ a_0 \frac{d^n v}{dx^n} + a_1 \frac{d^{n-1} v}{dx^{n-1}} + \text{——} a_n v \} = 0$

$+ \frac{du}{dx} \{ n_1 a_0 \frac{d^{n-1} v}{dx^{n-1}} + \text{———} \}$

$+ \text{——}$

$+ \frac{d^n u}{dx^n} a_0 v$

$$b_0 \frac{d^{n-1}w'}{dx^{n-1}} + b_1 \frac{d^{n-2}w'}{dx^{n-2}} + \cdots + b_{n-1} u' = X$$

3). Variation der Constanten.

Bewegung der Wärme in festen Körpern

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$$

$t = 0 \quad u = 0$

$x = 0 \quad u = \varphi t$

$u = \varphi(0) \qquad \theta \geqq t > 0$

$u = \varphi(\theta) \qquad 2\theta \geqq t > \theta$

$u = \varphi(\overline{n-1}\,\theta) \qquad n\theta \geqq t > \overline{n-1}\,\theta \qquad u = u_1 + u_2 + \cdots u_n$

$$\chi(x\,t) = \frac{2}{\sqrt{\pi}} \int_{\frac{x}{2a\sqrt{t}}}^{\infty} e^{-\beta^2} d\beta$$

$$u_m = \varphi(\overline{m-1}\,\theta) \left\{ \chi(x, t - \overline{m-1}\,\theta) - \chi(x, t - m\theta) \right\}$$

$$u = \sum_{m=1}^{n} \varphi(\overline{m-1}\,\theta) \frac{\chi(x, t - \overline{m-1}\,\theta) - \chi(x, t - m\theta)}{\theta} \theta$$

$$= \int_0^t \varphi(\lambda) \frac{\partial \chi(x, t-\lambda)}{\partial t} d\lambda$$

$$u = \frac{x}{2a\sqrt{\pi}} \int_0^t \varphi(\lambda)\, e^{-\frac{x^2}{4a^2(t-\lambda)}} (t-\lambda)^{-\frac{3}{2}} d\lambda$$

$$\begin{cases} \dfrac{\partial u}{\partial t} = a^2 \dfrac{\partial^2 u}{\partial x^2} \\ t=0 \quad u=0 \\ x=0 \quad u=0 \\ x=c \quad u=\psi(t) \end{cases}$$

$$u = \psi(t) \left\{ \frac{x}{c} + \frac{2}{\pi} \sum_1^\infty \frac{(-1)^n}{n} \sin \frac{n\pi x}{c} \right\} +$$

$$+ \frac{2a^2\pi}{c^2} \sum_1^\infty (-1)^{n-1} n \sin \frac{n\pi x}{c} \int_0^t \psi(\lambda) e^{-a^2\left(\frac{n\pi}{c}\right)^2 (t-\lambda)} d\lambda$$

Für Kugel mit Berücks. der Ausstrahlung (Kugelfn)

Schwingungen elastischer Körper

Bewegung der Flüssigkeiten.

Riemann: Schwere, Elektricität und Magnetismus.

hg. v. Hattendorf   Hannover: C. Rümpler 76.

$$X = k\mu \sum_{i=1}^{n} \frac{m_i (a_i - x)}{r_i^3}$$

$$Y = k\mu \sum \frac{m_i (b_i - y)}{r_i^3}$$

$$Z = -$$

Lagrange hat [illegible] $\frac{\partial}{\partial}$ [illegible]:

$$X = \sum m \frac{\partial(\frac{1}{r})}{\partial x} = \frac{\partial V}{\partial x} \qquad V = \sum \frac{m}{r}$$

$$Y =$$

$$Z =$$

[illegible]

$$X = \iiint \frac{a-x}{r^3} \rho \, da \, db \, dc$$

$$Y =$$

$$Z =$$

$$V = \iiint \rho \frac{da\,db\,dc}{r}$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \quad \text{[illegible]}$$

208. Anziehung einer Kugelschale

$$s^2 = x^2 + y^2 + z^2$$

$$V = F(s) \qquad \frac{\partial V}{\partial x} = \frac{dV}{ds}\frac{\partial s}{\partial x}$$

$$\frac{\partial^2 V}{\partial x^2} = \frac{d^2 V}{ds^2}\left(\frac{\partial s}{\partial x}\right)^2 + \frac{dV}{ds}\frac{\partial^2 s}{\partial x^2}$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{d^2 V}{ds^2} + \frac{2}{s}\frac{dV}{ds} = 0$$

$$\lg \frac{dV}{ds} + 2 \log s = c.$$

$$V = \frac{\alpha}{s} + \beta$$

I. [illegible] : $s < r$

[illegible]: $V_0 = 4\pi \int_r^{\varrho} \rho s \, ds$

$\alpha = 0$ [illegible]

$$V = 2\pi\rho\left[\varrho^2 - r^2\right]$$

II. $s > \varrho$

[illegible] $s = \infty$: $V = 0$

$\therefore \beta = 0$

$$\frac{1}{s-\varrho} > \frac{1}{r} > \frac{1}{s+\varrho} \quad \Big| \; \rho \, da \, db \, dc \int$$

$$\frac{M}{s-\varrho} > V > \frac{M}{s+\varrho}$$

$$\frac{1}{1-\frac{\varrho}{s}} > \frac{\alpha}{M} > \frac{1}{1+\frac{\varrho}{s}} \Bigg|^{s=\infty} \qquad \frac{\alpha}{M} = 1$$

$$V = \frac{M}{s}$$

A. einer homog. Kugel

$s < \varrho \qquad V_1 = \frac{4}{3}\pi\rho s^2 + 2\pi\rho(\varrho^2 - s^2)$

$\qquad\qquad = 2\pi\rho\varrho^2 - \frac{2}{3}\pi\rho s^2$

$s > \varrho \qquad V_2 = \frac{4\pi\rho\varrho^3}{3s}$

$$\frac{\partial V_1}{\partial x}\Bigg|^{s=\varrho} = \frac{\partial V_2}{\partial x}\Bigg|^{s=\varrho}$$

$$\frac{\partial^2 V_1}{\partial x^2}\Bigg|^{s=\varrho} \neq \frac{\partial^2 V_2}{\partial x^2}\Bigg|^{s=\varrho}$$

[illegible]

$$\frac{\partial^2 V}{\partial x^2} + \cdots = -4\pi\rho$$

Allgem.: V und ihre Deriv. für ein. inneren Punkt

$$V = \iiint \rho \frac{da\,db\,dc}{r}$$ [illegible] ∞

$a = x + r \sin\vartheta \cos\varphi$

$b = y + r \sin\vartheta \sin\varphi$

$c = z + r \cos\vartheta$

$$V = \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\vartheta\, d\vartheta \int_0^{R} \rho\, r\, dr$$

ebenso [illegible] C. Transf.:

$$X = \int_0^{2\pi} \cos\varphi\, d\varphi \int_0^{\pi} \sin^2\vartheta\, d\vartheta \int_0^{R} \rho\, dr$$

$$Y = \int_0^{2\pi} \sin\varphi\, d\varphi \int_0^{\pi} \text{———}$$

$$Z = \int_0^{2\pi} d\varphi \int_0^{\pi} \cos\vartheta \sin\vartheta\, d\vartheta \int_0^{R} \rho\, dr$$

[illegible] III [illegible] $X\,Y\,Z$ [illegible] $f(x\,y\,z)$

[illegible] $X = \frac{\partial V}{\partial x}$ etc. [illegible]

[illegible] $\frac{\partial^2 V}{\partial x^2}$ etc. [illegible] C. [illegible], 6 [illegible]:

$$\frac{\partial^2 V}{\partial x^2} = \iiint \rho \sin\vartheta\, d\varphi\, d\vartheta\, dr \left\{ \frac{-1 + 3\sin^2\vartheta\cos^2\varphi}{r} \right\}$$

etc. [illegible] $r = 0$ [illegible] Radi. [illegible]

[illegible] $\int_{\varepsilon}^{R} \rho \frac{dr}{r}$ [illegible], $\lim \varepsilon = 0$

[illegible] Transformation:

$$\frac{\partial V}{\partial x} = \iiint \rho \, \frac{\partial(-\frac{1}{r})}{\partial a} \, da\, db\, dc$$

$$\int \rho \, \frac{\partial(-\frac{1}{r})}{\partial a} \, da = -\frac{\rho}{r} + \int \frac{1}{r} \frac{\partial \rho}{\partial a} \, da$$ [illegible] $\frac{\rho}{r}$ [illegible]

$$\frac{\partial V}{\partial x} = \int \frac{\rho}{r} \cos\alpha \, d\sigma + \iiint \frac{1}{r} \frac{\partial \rho}{\partial a} \, da\, db\, dc$$

Voraussetzg. ρ [illegible]

[illegible] Transf. [illegible]

[illegible] Rad. ε [illegible]

ρ [illegible] $\int\int = < \rho_1 \int \frac{d\sigma}{r}$

$$< \frac{\rho_1}{\varepsilon} \int d\sigma = < 4\pi \rho_1 \varepsilon \Big| \begin{matrix} \lim \varepsilon = 0 \\ = 0 \end{matrix}$$

ρ [illegible] $\iiint = 0$, $\varepsilon = 0$ [illegible 100]

[illegible] $\frac{\partial V}{\partial x} = \iiint \rho \, \frac{\partial(-\frac{1}{r})}{\partial a} \, da\, db\, dc$ [illegible]

2. Diff.:

$$\frac{\partial^2 V}{\partial x^2} = \int \frac{\partial\left(\frac{1}{r}\right)}{\partial x}\,\rho\cos\alpha\, d\sigma + \iiint \frac{\partial\left(\frac{1}{r}\right)}{\partial x}\,\frac{\partial \rho}{\partial a}\, da\, db\, dc$$

$$\frac{\partial^2 V}{\partial y^2} =$$

$$\frac{\partial^2 V}{\partial z^2} =$$

[illegible] Diff. [illegible] [illegible]

[illegible]

[illegible]

[illegible] V [illegible] fu. V [illegible] Deriv.

[illegible] V [illegible] $\frac{\partial V}{\partial x}$ [illegible] $\frac{\partial^2 V}{\partial x^2}$ [illegible]

[illegible] [illegible].

[illegible] V [illegible] = [illegible] fu. (xyz), $\frac{\partial V}{\partial x}$ [illegible] (xyz) [illegible]

[illegible]

[illegible] $\frac{\partial V}{\partial x}$ [illegible]:

T_1 T_2

$- \ldots R = \varepsilon$ [illegible]

$V = V_1 + V_2$

[illegible] T_2 [illegible] $\frac{\partial V_2}{\partial x}$ [illegible] fu. [illegible] $\lim d\left(\frac{\partial V_2}{\partial x}\right) = 0$

$$\frac{\partial V_1}{\partial x} = \iiint \frac{a-x}{r^3}\,\rho\, da\, db\, dc$$ [illegible]

[illegible] $< \delta$ [illegible]

[illegible] $< \delta$ [illegible]

$$\lim d\left(\frac{\partial V_1}{\partial x}\right) = 0$$

$$\lim d\left(\frac{\partial V}{\partial x}\right) = \lim d\left(\frac{\partial V_1}{\partial x}\right) + \lim d\left(\frac{\partial V_2}{\partial x}\right) = 0$$

[illegible] Deriv. [illegible]

V:

$\frac{\partial V}{\partial x}$:

$\frac{\partial^2 V}{\partial x^2}$:

$$\int \frac{1}{r^2}\cos(rn)\,d\sigma \; ; \text{ Satz von Gauss}$$

xyz [illegible]; [illegible]

Normale [illegible]: $N = \frac{1}{r^2}\cos(rn)$ [illegible $=1$]

$$r^2 = (a-x)^2 + (b-y)^2 + (c-z)^2$$

$$\frac{\partial x}{\partial n} = \cos(xn) \quad \frac{\partial y}{\partial n} = \cos(yn), \quad \frac{\partial z}{\partial n} = \cos(zn)$$

$$\cos(rn) = \frac{\partial r}{\partial x}\frac{\partial x}{\partial n} + \frac{\partial r}{\partial y}\frac{\partial y}{\partial n} + \frac{\partial r}{\partial z}\frac{\partial z}{\partial n} = \frac{\partial r}{\partial n}$$

$$\int N\,d\sigma = \int \frac{1}{r^2}\cos(rn)\,d\sigma$$

$$= 4\pi$$ [illegible]

$$= 0$$ [illegible]

$$= 2\pi$$ [illegible]

[illegible] $= dm$

$$N = \int \frac{1}{r^2}\cos(rn)\,dm$$ $=$ [illegible] (xyz) [illegible]

$$\int N\,d\sigma = \int N\,d\sigma \int \frac{1}{r^2}\cos rn\,dm =$$

[illegible] I

$$= \int dm \left\{ \int \frac{1}{r^2}\cos(rn)\,d\sigma \right\} = 4\pi M$$

[illegible] T [illegible]

[illegible]

[illegible] T [illegible]

$\int N\,d\sigma = 2\pi M$

---

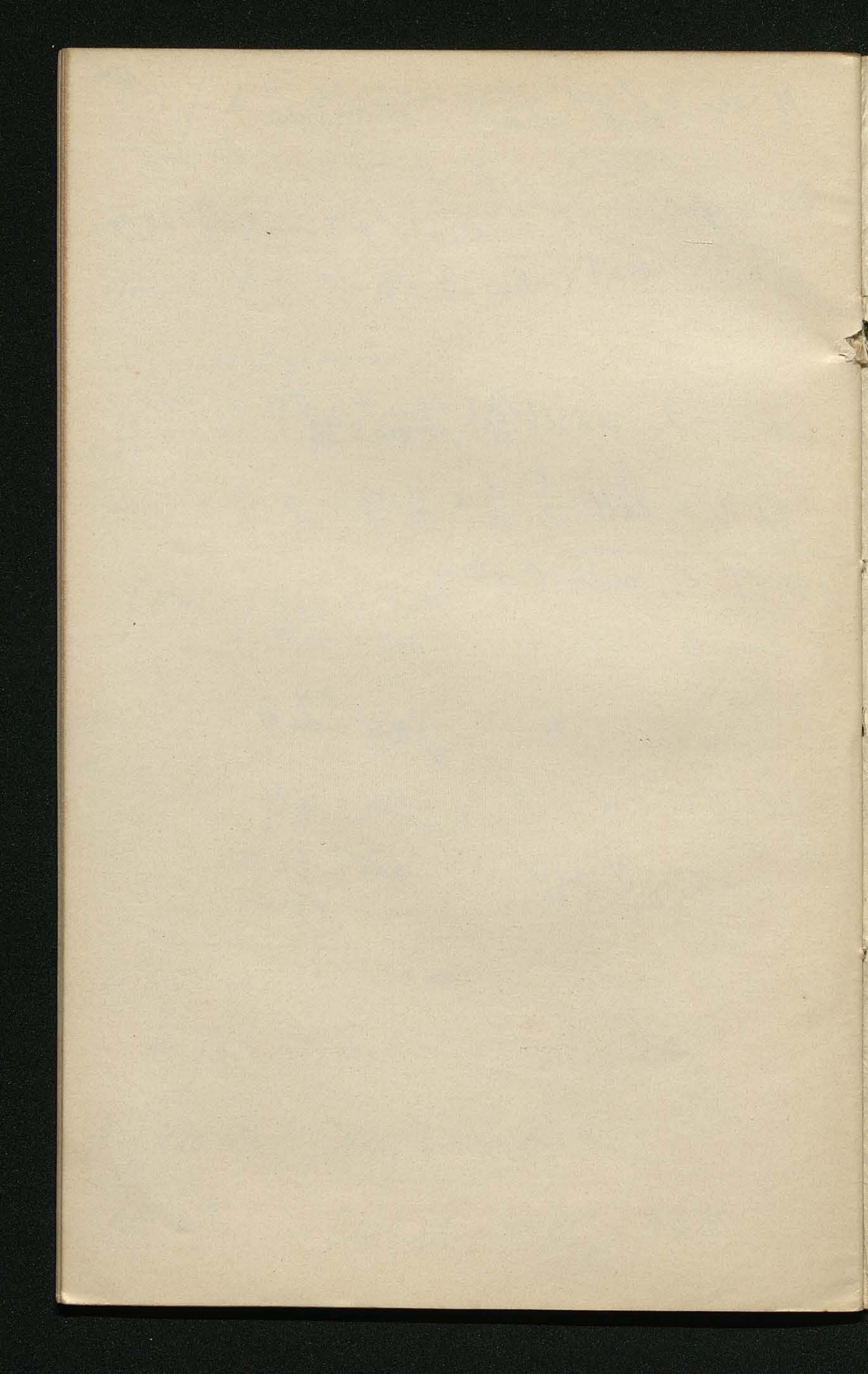
z.B. T [illegible] rechtw. Parallelep.

[illegible] M, [illegible] $\int N\,d\sigma = 4\pi M$

[illegible] M $\int N\,d\sigma = 2\pi M$

[illegible] M [illegible] $\int N\,d\sigma = \pi M$

[illegible] $\int N\,d\sigma = \frac{1}{2}\pi M$



Physik 91/92

Verdichtung einer gasförmigen Materie um einen Massenmittelpunkt herum.

$$D = f(r) \quad D' = f(r+\Delta r)$$

$$D : D' = P : P'$$

$$P = \frac{(r+\Delta r)^2 4\pi . P' + \frac{Mm}{r^2} k}{4 r^2 \pi}$$

$$m = 4 r^2 \pi \, dr . \frac{D'}{g}$$

$$P = \frac{(r+\Delta r)^2 P' + M r^2 \Delta r \frac{D' k}{g}}{r^2} \quad \Bigg| \quad P' = P - M \Delta r \frac{D k}{g}$$

$$\frac{D' - D}{\Delta r} : D = \frac{P' - P}{\Delta r} : P$$

$$D : D_0 = P : P_0$$

$$\frac{D' - D}{\Delta r} = \frac{P' - P}{\Delta r} \frac{D}{P}$$

$$\frac{D}{P} = \frac{D_0}{P_0}$$

$$\frac{dD}{dr} = \frac{dP}{dr} \frac{D_0}{P_0}$$

$$\frac{P - P}{\Delta r} = \frac{P' - M \Delta r \frac{D' k}{g} - P'}{\Delta r} = - \frac{M D' k}{g}$$

$$\frac{dD}{dr} = - \frac{D_0}{P_0} \frac{M D k}{g}$$

$$\frac{D_0}{P_0} \frac{M k}{g} = a$$

$$\frac{dP}{dr} = \frac{MD' k}{g} + \frac{2P}{r}$$

$$\frac{dD}{D} = -a\,dr$$

$$l D = -ar + b$$

$$l D_0 = -a r_0 + b$$

$$l \frac{D}{D_0} = -a(r - r_0)$$

$$D = D_0\, e^{-a(r-r_0)} = D_0 \frac{e^{-ar}}{e^{-ar_0}} = c\, e^{-ar}$$

für $M = 0$: $a = 0$
$D = D_0$

$$c = \frac{D_0}{e^{-ar_0}} = D_0\, e^{ar_0}$$

$$P = \frac{P_0}{D_0} D = P_0 \frac{e^{-ar}}{e^{-ar_0}}$$

$$G = \int_{\rho}^{R} 4 r^2 \pi D\, dr$$

$$= 4c\pi \int_{\rho}^{R} e^{-ar} r^2\, dr$$

$$\int e^{-ax} x^2 dx = -\frac{e^{-ax} x^2}{a} + \frac{2}{a} \int e^{-ax} x\, dx$$

$$= -\frac{e^{-ax} x^2}{a} + \frac{2}{a} \left\{ -\frac{e^{-ax} x}{a} + \frac{1}{a} \int e^{-ax} dx \right\}$$

$$= -\frac{e^{-ax} x^2}{a} + \frac{2}{a} \left\{ -\frac{e^{-ax} x}{a} - \frac{1}{a^2} e^{-ax} \right\} = -e^{-ax} \left[\frac{x^2}{a} + \frac{2x}{a^2} + \frac{2}{a^3} \right]$$

$$S = -4c\pi\, e^{-ar}\left[\frac{r^2}{a}+\frac{2r}{a^2}+\frac{2}{a^3}\right]_{\rho}^{R}$$

$$= 4c\pi\, e^{-ar}\left[\frac{r^2}{a}+\frac{2r}{a^2}+\frac{2}{a^3}\right]_{R}^{\rho}$$

in eine Reihe entwickelt:

$$S = 4c\pi\left[1 - ar + \frac{a^2r^2}{1.2} - \frac{a^3r^3}{1.2.3} + \frac{a^4r^4}{1.2.3.4} - \frac{a^5r^5}{5!}\right]\left[\frac{r^2}{a}+\frac{2r}{a^2}+\frac{2}{a^3}\right]$$

$$= 4c\pi\Big[\frac{r^2}{a} - r^3 + \frac{ar^4}{2} - \frac{a^2r^5}{3!} + \frac{a^3r^6}{4!} - \frac{a^4r^7}{5!} + \dots$$

$$+ \frac{2r}{a^2} - \frac{2r^2}{a} + \frac{2r^3}{2} - \frac{2ar^4}{3!} + \frac{2a^2r^5}{4!} - \frac{2a^3r^6}{5!} + \dots$$

$$+ \frac{2}{a^3} - \frac{2r}{a^2} + \frac{2r^2}{2a} - \frac{2r^3}{3!} + \frac{2ar^4}{4!} - \frac{2a^2r^5}{5!} + \dots\Big]$$

$$= 4c\pi\left[\frac{2}{a^3} - \frac{2r^3}{3!} + ar^4\left(\frac{1}{2} - \frac{2}{3!} + \frac{2}{4!}\right) - a^2r^5\left(\frac{1}{3!} - \frac{2}{4!} + \frac{2}{5!}\right) + \dots\right]$$

beim Substituiren der Grenzwerte fällt das erste Glied heraus, also auch:

$$S = 4c\pi\left[-\frac{r^3}{3} + \frac{ar^4}{4} - \frac{a^2r^5}{10} + \frac{a^3r^6}{36} - \dots\right]_{R}^{\rho}$$

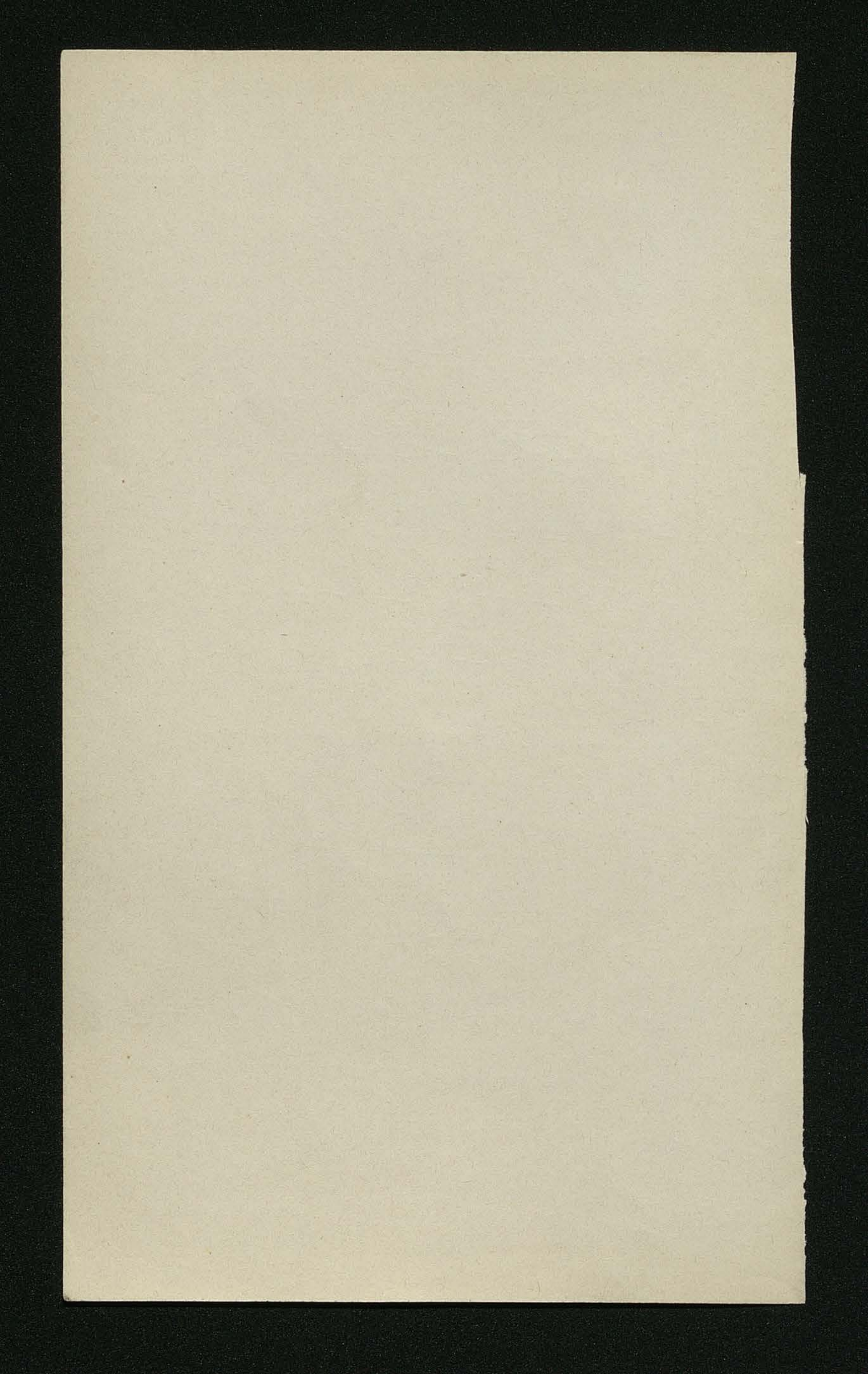
$$= 4c\pi\left[\frac{r^3}{3} - \frac{ar^4}{4} + \frac{a^2r^5}{10} - \frac{a^3r^6}{36} - \dots\right]_{\rho}^{R}$$

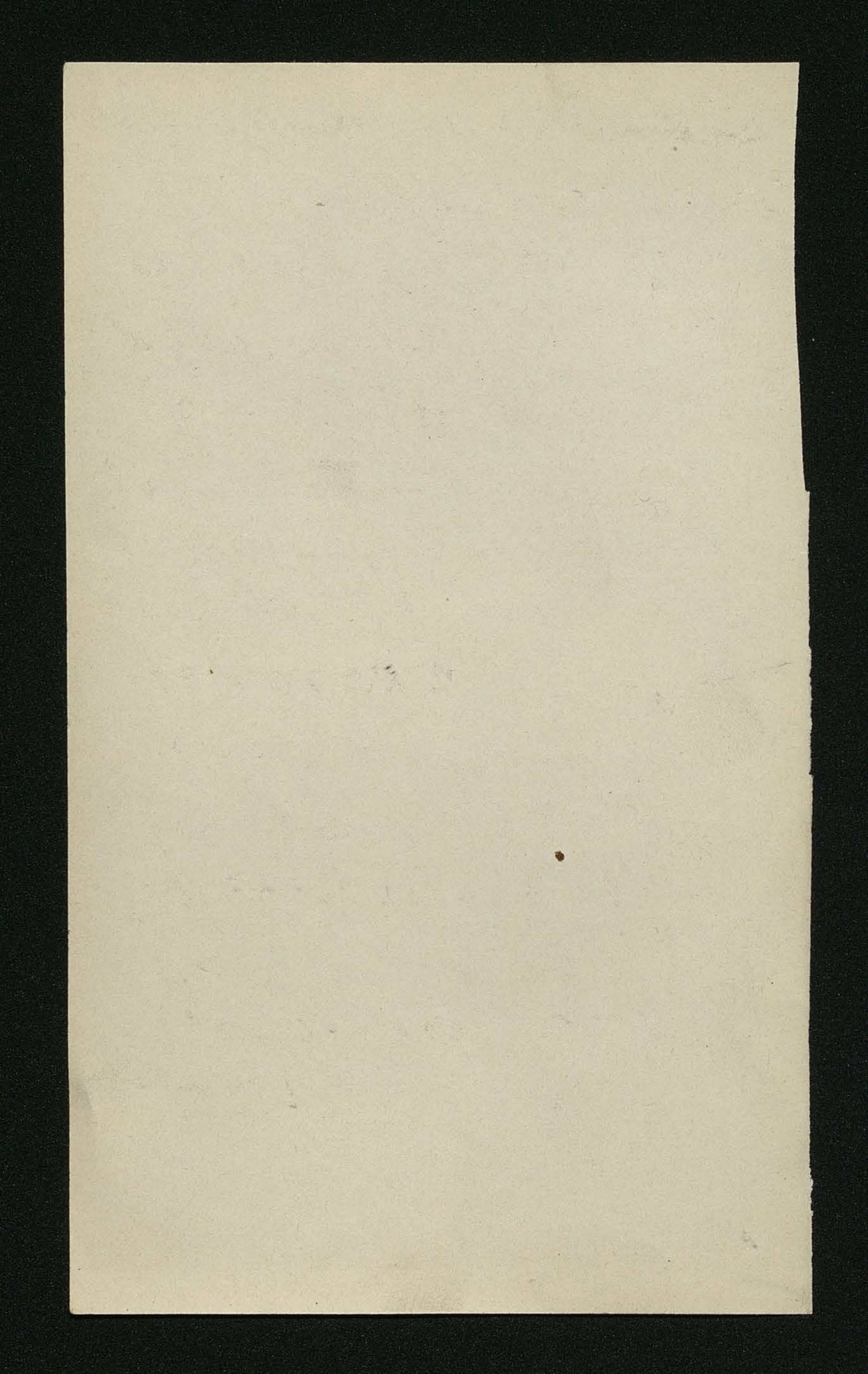
In der That gibt dies für $M = 0,\ a = 0$: $S = \frac{4c\pi}{3}\left[R^3 - \rho^3\right]$

~~Der Überschuss des Gesammtgewichtes über jenes,~~

wobei $M = 0,\ a = 0$:

$S - S_0 = C' = 4c\pi[$





Aberration [illegible] theorie [illegible]

[illegible]

L L' A A' B

[illegible] A [illegible]

[illegible] [[illegible] resp. [illegible]]

[illegible] A' [illegible] B

[illegible]

[illegible] A'B [illegible]

[illegible] q. e. d.

[illegible]

[illegible] AB [illegible] A'B' [illegible]

B' B C' C D A' A

[illegible] CC' [illegible]

CD [illegible]

[[illegible]]

Gleichgewichtsvertheilung von Elektricität auf einem System von 2 einander berührenden Kugeln

[illegible]

[illegible]

$$\frac{Q}{a} = \frac{q}{b} = \frac{Q+q}{a+b} \qquad Q + q = C$$

$$= \frac{C}{a+b}$$

[illegible] $Q = \frac{aC}{a+b}$

$$q = \frac{bC}{a+b} \qquad Q : q = a : b$$

$a < b$:

[illegible] a induc. [illegible] b [illegible]

[illegible] A [illegible]

$q + q = Q =$ [illegible]

[illegible] B ind. [illegible] $\frac{Q b}{a+b} + \frac{q a b}{(a+b)\left(a+b-\frac{a^2}{a+b}\right)}$?

[illegible] A ind. [illegible] $\frac{q a}{a+b}$

[illegible] induc. [illegible]:

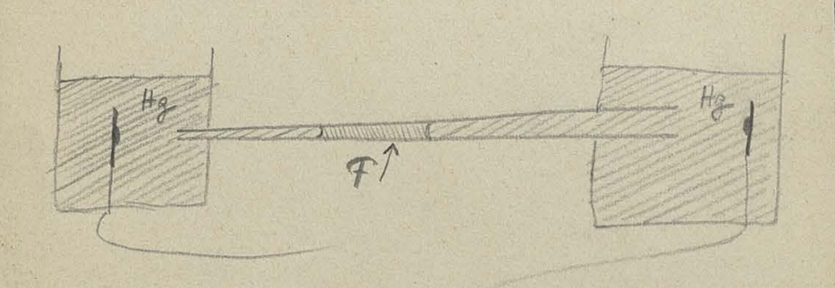
[illegible]

$$\frac{Q}{a} + \frac{q}{a+b} = \frac{q}{b} + \frac{Q}{a+b} \quad \Big| \quad Qab + Qb^2 + qab = qa^2 + qab$$

~~$\mathfrak{P} : q = a^2 : b^2$~~

$$C = \mathfrak{P} + q + \frac{\mathfrak{P} b + q a}{a+b} = \frac{\mathfrak{P}(a+2b) + q(2a+b)}{a+b}$$

Apparat zur Calibrirung von Röhren auf elektrischem Wege (Widerstandsbestimmung



In die Röhre wird ein Stück einer schlecht leitenden Flüssigkeit eingezogen F; sonst ist die Röhre ganz mit (z.B.) Quecksilber Hg angefüllt. Falls nun der Querschnitt ungleich ist, so ändert sich auch die Länge der schlecht leitenden Flüssigkeitssäule und aus beiden Gründen der Widerstand.

$w = \alpha \frac{l}{q}$

$\Sigma w = W = \alpha \int \frac{dl}{q}$

? ist dann das Ohm'sche Gesetz noch genau giltig? wenn der Querschnitt verschieden ist

x_1 λ_1

x_2 λ_2

v

l

$$\mathfrak{V}_1 = \alpha \int_0^{x_1} \frac{dx}{q} + \beta \int_{x_1}^{x_1+\lambda_1} \frac{dx}{q} + \alpha \int_{x_1+\lambda_1}^{l} \frac{dx}{q}.$$

$$\mathfrak{V}_2 = \alpha \int_0^{x_2} \frac{dx}{q} + \beta \int_{x_2}^{x_2+\lambda_2} \frac{dx}{q} + \alpha \int_{x_2+\lambda_2}^{l} \frac{dx}{q}.$$

$$x_2 = x_1 + \xi$$

$$\mathfrak{V}_1 - \mathfrak{V}_2 = -\alpha \int_{x_1}^{x_2} \frac{dx}{q} + \beta \left[\int_{x_1}^{x_2} \frac{dx}{q} - \int_{x_1+\lambda_1}^{x_2+\lambda_2} \frac{dx}{q} \right] + \alpha \int_{x_1+\lambda_1}^{x_2+\lambda_2} \frac{dx}{q}$$

$$= (\beta - \alpha) \left[\int_{x_1}^{x_2} \frac{dx}{q} - \int_{x_1+\lambda_1}^{x_2+\lambda_2} \frac{dx}{q} \right]$$

$$v = \text{const.} = \int_{x_1}^{x_1+\lambda_1} q\, dx = \int_{x_2}^{x_2+\lambda_2} q\, dx = \lambda_2 (q_2)_m$$

~~$W_1 = W_2 = (\beta - \alpha)$~~

Wenn der Widerstand α gegen den spec Widerstand β zu vernachlässigen ist, so kann man setzen:

$$W_1 = \beta \int_{x_1}^{x_1+\lambda_1} \frac{dx}{q} = \frac{\beta \lambda_1}{(q_1)}$$

$(q) =$ [illegible]

$$W_2 = \beta \int_{x_2}^{x_2+\lambda_2} \frac{dx}{q} = \frac{\beta \lambda_2}{(q_2)}$$

$$\frac{W_1}{W_2} = \frac{\lambda_1}{\lambda_2} \frac{(q_2)}{(q_1)}$$

$$v = \lambda_1 (q_1) = \lambda_2 (q_2)$$

$$\frac{\lambda_1}{\lambda_2} = \frac{(q_2)}{(q_1)}$$

$$= \left(\frac{\lambda_1}{\lambda_2}\right)^2 = \left[\frac{(q_2)}{(q_1)}\right]^2$$

$$\frac{(q_2)}{(q_1)} = \sqrt{\frac{W_1}{W_2}}$$

Eine genauere Formel erhält man wenn man auch den Widerstand W_0 = ohne ~~die~~ Tropfen

[de]r schlecht leitenden Flüssigkeit misst; dann

l – ~~mit Vernachlässigung von λ gegen l~~:

$$W_1 = \frac{\beta \lambda_1}{(q_1)} + W_0$$

$$\frac{(q_2)}{(q_1)} = \sqrt{\frac{W_1 - W_0}{W_2 - W_0}}$$

$$W_2 = \frac{\beta \lambda_2}{(q_2)} + W_0$$

~~Noch genauer:~~ Beweis: ~~wenn α und β bekannt~~:

$$W_1 = \alpha \int_0^l \frac{dx}{q} - \alpha \int_{x_1}^{x_1+\lambda_1} \frac{dx}{q} + \beta \int_{x_1}^{x_1+\lambda_1} \frac{dx}{q}$$

$$= W_0 + (\beta - \alpha) \int_{x_1}^{x_1+\lambda_1} \frac{dx}{q} = W_0 + (\beta - \alpha) \frac{\lambda_1}{(q_1)}$$

$$W_2 = W_0 + (\beta - \alpha) \frac{\lambda_2}{(q_2)}$$

$$\frac{W_1 - W_0}{W_2 - W_0} = \frac{\lambda_1}{\lambda_2} \frac{(q_2)}{(q_1)} = \left[\frac{(q_2)}{(q_1)}\right]^2$$

y | 0 | X | λ | $\varphi(x)$ | $f(x)$

$y = f(x)$ $\eta = \varphi(x)$

$$\eta = \frac{1}{\lambda}\int_{x-\frac{\lambda}{2}}^{x+\frac{\lambda}{2}} f(x)\, dx$$

Sehr oft bei physikal. Exper. wird eine fc $\varphi(x)$ bestimmt, während $f(x)$ gesucht wird;

jene stellt den arithm. Mittelwerth in einem festen Intervalle dar. z.B. Spectrallinien

[1). Übereinanderlagern der Spaltbilder

2). Bei Untersuchung mit Bolometer, Decke des ...

Calibriren von Röhren, Temperatur-Messung mittelst thermom. etc.]

Wie findet man $f(x)$?

$$\varphi(x) = \frac{1}{\lambda}\int_{x-\frac{\lambda}{2}}^{x+\frac{\lambda}{2}} f(x)\,dx = \frac{1}{\lambda}\int_{x-\frac{\lambda}{2}}^{x+\frac{\lambda}{2}} f(z)\,dz$$

$$\frac{d\varphi}{dx} = \frac{1}{\lambda}\left[f\left(x+\frac{\lambda}{2}\right) - f\left(x-\frac{\lambda}{2}\right)\right]$$

$$= \frac{1}{\lambda}\left[f(x) + \frac{\lambda}{2}f'(x) + \frac{\lambda^2}{4}\frac{1}{2}f''(x) + \frac{\lambda^3}{8}\frac{1}{2.3}f'''(x) + \cdots \right.$$
$$\left. - f(x) + \quad - \quad + \quad - \right]$$

$$= f'(x) + \frac{\lambda^2}{4}\frac{1}{2.3}f'''(x) + \frac{\lambda^4}{16}\frac{1}{5!}f^{V}(x) + \text{---}$$

~~$\lambda\frac{d\varphi}{dx} = \lambda f'(x) + \frac{\lambda^3}{4}\frac{1}{2.3}$~~

$$\varphi(x) = \int\frac{d\varphi}{dx}dx = f(x) + \frac{\lambda^2}{4}\frac{1}{2.3}f''(x) + \frac{\lambda^4}{16}\frac{1}{5!}f^{IV}(x) + \text{--}$$

I Näherungs-methode:

$$f_1(x) = \varphi(x) - \frac{\lambda^2}{4}\frac{1}{2.3}\cdot\varphi''(x) - \frac{\lambda^4}{16}\frac{1}{5!}\varphi^{IV}(x) - \frac{\lambda^6}{64}\frac{1}{7!}\varphi^{VI}(x)$$

$$f_2(x) = \varphi(x) - \frac{\lambda^2}{4}\frac{1}{2.3}f_1''(x) - \frac{\lambda^4}{16}\cdot\frac{1}{5!}f_1^{IV}(x) - \text{--}$$

$$f_3(x) = \varphi(x) - \frac{\lambda^2}{4}\frac{1}{2.3}f_2''(x) \text{ ———— }$$ etc.

oder:

$$f_2(x) = \varphi(x) - \frac{\lambda^2}{4}\frac{1}{2.3}\left[\varphi''(x) - \frac{\lambda^2}{4}\frac{1}{2.3}\varphi^{IV}(x) - \frac{\lambda^4}{16}\frac{1}{5!}\varphi^{VI}(x) - \text{--}\right]$$

$$-\frac{\lambda^4}{16}\cdot\frac{1}{5!}\left[\varphi^{IV}(x)-\frac{\lambda^2}{4}\cdot\frac{1}{2.3}\varphi^{VI}(x)-\frac{\lambda^4}{16}\frac{1}{5!}\varphi^{VIII}(x)-\right.$$

$$-\frac{\lambda^6}{64}\cdot\frac{1}{7!}\left[\varphi^{VI}(x)-\frac{\lambda^2}{4}\cdot\frac{1}{2.3}\varphi^{VIII}(x)-\right.$$

$$-\,-$$

$$=\varphi(x)-\frac{\lambda^2}{4}\cdot\frac{1}{2.3}\varphi''(x)+\left[\frac{\lambda^4}{16}\left(\frac{1}{2.3}\right)^2-\frac{\lambda^4}{16}\cdot\frac{1}{5!}\right]\varphi$$

$$+\frac{\lambda^6}{64}\left[\frac{1}{2.3}\cdot\frac{1}{5!}+\frac{1}{5!}\frac{1}{2.3}-\frac{1}{7!}\right]\varphi^{VI}(x)$$

etc.

Mit Beschränkung auf 4te Potenz

[Coeffic. e 6ten $=\frac{1}{322560}$!]

$$f_2(x)=\varphi(x)-\frac{\lambda^2}{24}\varphi''(x)+\frac{7\lambda^4}{5760}\varphi^{IV}(x)$$

$$f_3(x)=\varphi(x)-\frac{\lambda^2}{4}\cdot\frac{1}{2.3}\left[\varphi''(x)-\frac{\lambda^2}{24}\varphi^{IV}(x)+\frac{7}{5}\right.$$

$$+\frac{\lambda^4}{16}\cdot\frac{1}{5!}\varphi^{IV}(x)$$

$$=\varphi(x)-\frac{\lambda^2}{24}\varphi''(x)+\frac{7\lambda^4}{5760}\varphi^{IV}(x)=$$

$$f(\lambda) = \text{proxima} = \varphi(x) - \frac{\lambda^2}{24}\varphi''(x) + \frac{7\lambda^4}{5760}\varphi^{IV}(x)$$

$$\varphi\left(x+\frac{\lambda}{2}\right) + \varphi\left(x-\frac{\lambda}{2}\right) = \frac{1}{\lambda}\int_{x}^{x+\lambda} + \frac{1}{\lambda}\int_{x-\lambda}^{x} = \frac{1}{\lambda}\int_{x-\lambda}^{x+\lambda} f(x)\,dx$$

$$\frac{d}{dx}\left[\varphi\left(x+\frac{\lambda}{2}\right) + \varphi\left(x-\frac{\lambda}{2}\right)\right] = \frac{1}{\lambda}\left[f(x+\lambda) - f(x-\lambda)\right]$$

Coordinaten-Verschiebung

[φ Index φ_1 φ_2 usw. (x!]

$$\varphi(x) = \frac{1}{\lambda}\int_{x}^{x+\lambda} f_x(x)\,dx$$

$$\frac{d\varphi_1(x)}{dx} = \frac{1}{\lambda}\left[f_x(x+\lambda) - f_x(x)\right]$$

$$\varphi_{1x}(x+\lambda) = \frac{1}{\lambda}\left[f_x(x+2\lambda) - f_x(x+\lambda)\right]$$

$$\frac{d}{dx}\left[\varphi_1(x) + \varphi_1(x+\lambda)\right] = \frac{1}{\lambda}\left[f_x(x+2\lambda) - f_x(x)\right]$$

$$= 2\frac{d\varphi_2(x)}{dx}$$

$$\varphi_2(x) = \frac{1}{2\lambda}\int_{x}^{x+2\lambda} f(x)\,dx$$

$$\varphi_2(x) = \frac{1}{2}\left[\varphi_1(x) + \varphi_1(x+\lambda)\right] + C_1$$

$$\varphi_1(x) = \frac{1}{2}\left[\varphi_{\frac{1}{2}}(x) + \varphi_{\frac{1}{2}}(x+\lambda)\right] + C_2$$

$$\varphi_1(x+\lambda) = \frac{1}{2}\left[\varphi_{\frac{1}{2}}(x+\lambda) + \varphi_{\frac{1}{2}}(x+2\lambda)\right] + C_2$$

$$\varphi_2(x) = \frac{1}{2}\left[\frac{1}{2}\varphi_{\frac{1}{2}}(x) + \varphi_{\frac{1}{2}}(x+\lambda) + \frac{1}{2}\varphi_{\frac{1}{2}}(x+2\lambda)\right] + C_1 + C$$

$$= \frac{1}{4}\varphi_{\frac{1}{2}}(x) + \frac{1}{2}\varphi_{\frac{1}{2}}(x+\lambda) + \frac{1}{4}\varphi_{\frac{1}{2}}(x+2\lambda) + C_1 + C$$

etc. etc.

$$f(x+\lambda) = \lambda \frac{d\varphi(x)}{dx} + f(x)$$

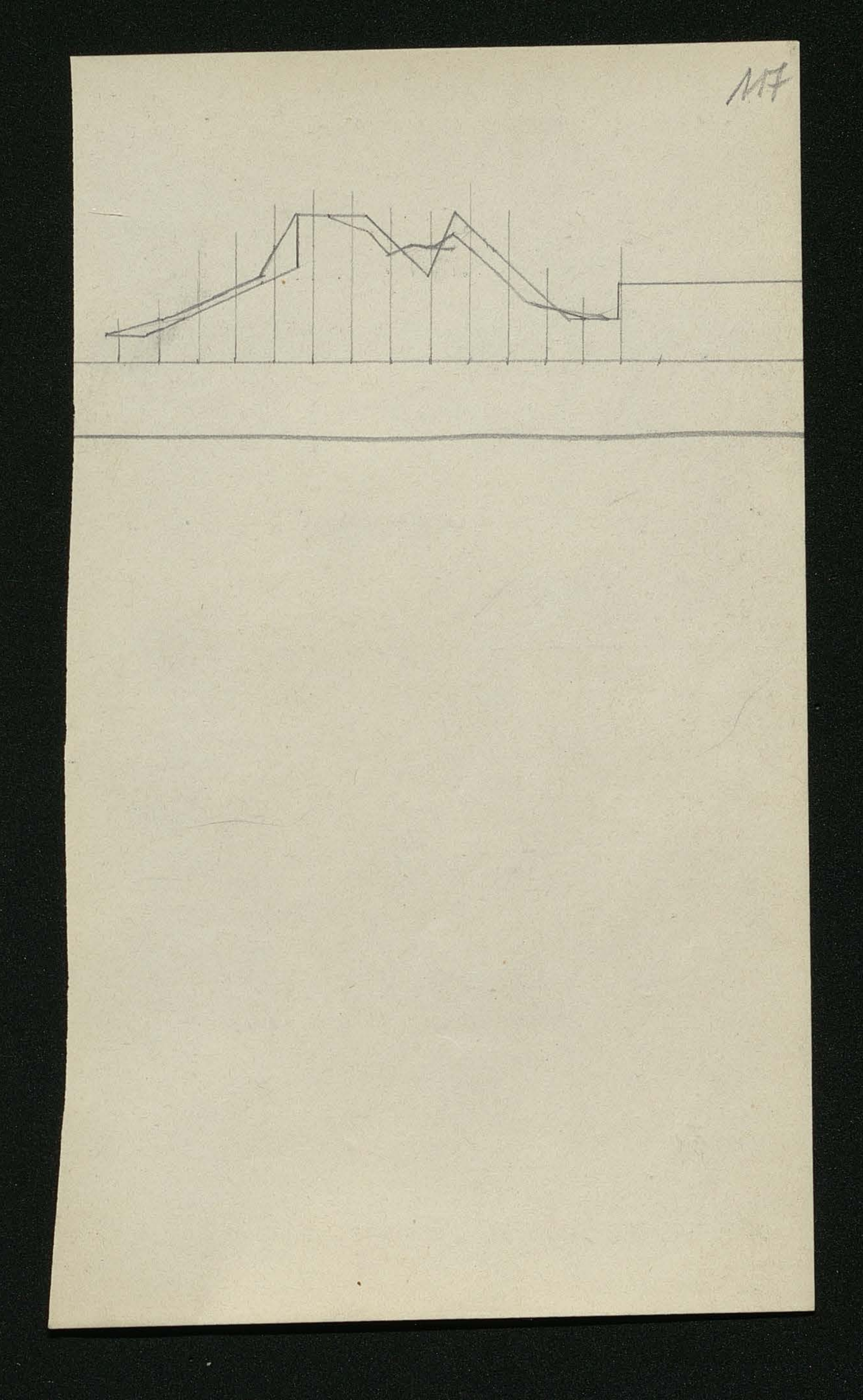
$$f(\lambda) = \lambda \frac{d\varphi(0)}{dx} + f(0)$$

$$f(2\lambda) = \lambda \frac{d\varphi(\lambda)}{dx} + f(\lambda)$$

$$= \lambda\left[\frac{d\varphi(\lambda)}{dx} + \frac{d\varphi(0)}{dx}\right] + f(0)$$

$$f(3\lambda) = \lambda\left[\frac{d\varphi(2\lambda)}{dx} + \frac{d\varphi(\lambda)}{dx} + \frac{d\varphi(0)}{dx}\right] + f(0)$$

etc. wenn also ein Werth $f(0)$ bekannt ist so können daraus alle anderen gefunden werden



Bewegung eines Punktes welcher von einem fixen Centrum umgek. prop. der 3 Pot. der Entfernung angezogen wird.

$$\frac{d^2x}{dt^2} = -k\frac{x}{r^4} \quad \Big| \quad \frac{dx}{dt}$$

$$\frac{d^2y}{dt^2} = -k\frac{y}{r^4} \quad \Big| \quad \frac{dy}{dt}$$

$$\frac{d}{dt}\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right] = -\frac{k}{r^3}\frac{dr}{dt}$$

$$= \frac{d}{dt}\left[\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2\right] = \frac{k}{2}\frac{d}{dt}\left(\frac{1}{r^2}\right)$$

$$\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2 = \frac{k}{2r^2} + A$$

$$\boxed{A = \left(\frac{dr}{dt}\right)_0^2 + r_0^2\left(\frac{d\varphi}{dt}\right)_0^2 - \frac{k}{2r_0^2}}$$

$$r^2\frac{d\varphi}{dt} = 2c$$

$$\left(\frac{dr}{dt}\right)^2 + \frac{4c^2}{r^2} = \frac{k}{2r^2} + A$$

$$\left(\frac{dr}{dt}\right)^2 = \frac{k-8c^2}{2r^2} + A$$

$$\frac{dr}{\sqrt{\frac{k-8c^2}{2r^2} + A}} = dt$$

$$\frac{r\,dr\,\sqrt{2}}{\sqrt{k-8c^2+2Ar^2}} = dt$$

$$t = \frac{1}{A\sqrt{2}}\sqrt{k-8c^2+2Ar^2} + B$$

$$4A^2(t-B)^2 = k-8c^2+2Ar^2$$

$$r = \sqrt{\frac{2A^2(t-B)^2+8c^2-k}{4A}}$$

$$= \frac{1}{2}\sqrt{2A\,(t-B)^2 + \frac{8c^2-k}{A}}$$

$$\frac{4c^2}{r^4}\left(\frac{dr}{d\varphi}\right)^2 = \frac{k-8c^2}{2r^2} + A$$

$$8c^2\left(\frac{dr}{d\varphi}\right)^2 = \frac{k-8c^2}{8c^2}\,r^2 + \frac{2Ar^4}{4c^2}$$

$$\frac{dr}{\sqrt{(k-8c^2)r^2+2Ar^4}} = \frac{d\varphi}{2c\sqrt{2}}$$

$$2c = r^2 \frac{d\varphi}{dt}$$

$$= r^2 \frac{d\varphi}{dr} \frac{dr}{dt}$$

$$= \frac{r^2 \, 2c\sqrt{2}}{\sqrt{(k-8c^2) r^2 + 2Ar^4}} \qquad \frac{\sqrt{k-8c^2 + 2Ar^2}}{r\sqrt{2}}$$

$$\frac{\varphi}{2c\sqrt{2}} = \int \frac{dr}{r\sqrt{(k-8c^2) + 2Ar^2}} + C$$

I. $k - 8c^2 > 0$

$$\frac{\varphi}{2c\sqrt{2}} = -\frac{1}{\sqrt{k-8c^2}} \log\left[\frac{\sqrt{k-8c^2} + \sqrt{k-8c^2+2Ar^2}}{r}\right]$$

$$e^{(C-\varphi)\frac{\sqrt{k-8c^2}}{2c\sqrt{2}}} = \frac{\sqrt{k-8c^2} + \sqrt{k-8c^2+2Ar^2}}{r}$$

$$\cancel{k-8c^2} + 2Ar^2 = \cancel{k-8c^2} + 2r\sqrt{k-8c^2}\, e^{(C-\varphi)\frac{\sqrt{k-8c^2}}{2c\sqrt{2}}} + r^2 e^{(C-\varphi)\frac{\sqrt{k-8c^2}}{c\sqrt{2}}}$$

$$r_1 = 0$$

$$r_2 = \frac{2\sqrt{k-8c^2}\; e^{(C-\varphi)\frac{\sqrt{k-8c^2}}{2c\sqrt{2}}}}{2A - e^{(C-\varphi)\frac{\sqrt{k-8c^2}}{c\sqrt{2}}}}$$

II. $k - 8c^2 < 0$

$$\frac{\varphi}{2c\sqrt{2}} = -\frac{1}{\sqrt{8c^2-k}} \arcsin \frac{1}{r}\sqrt{\frac{8c^2-k}{2A}} + \text{Const.}$$

$$\frac{1}{r}\sqrt{\frac{8c^2-k}{2A}} = \sin \frac{(C-\varphi)\sqrt{8c^2-k}}{2c\sqrt{2}}$$

$$r = \sqrt{\frac{8c^2-k}{2A}} \frac{1}{\sin \frac{(C-\varphi)\sqrt{8c^2-k}}{2c\sqrt{2}}}$$

$t = \infty \quad r = \infty$

II. $C - \varphi_1 = 0 \qquad \varphi_1 = C$

$C - \varphi_2 = \pi \qquad \varphi_2 = C - \pi$

I.

$$2A - e^{(C-\varphi)\frac{\sqrt{k-8c^2}}{c\sqrt{2}}} = 0$$

$$(C-\varphi)\frac{\sqrt{k-8c^2}}{c\sqrt{2}} = \log 2A$$

$$\varphi = C - \frac{c\sqrt{2}}{\sqrt{k-8c^2}} \log(2A)$$

$t=0 \qquad r = \sqrt{AB^2 + \frac{8c^2-k}{2A}}$

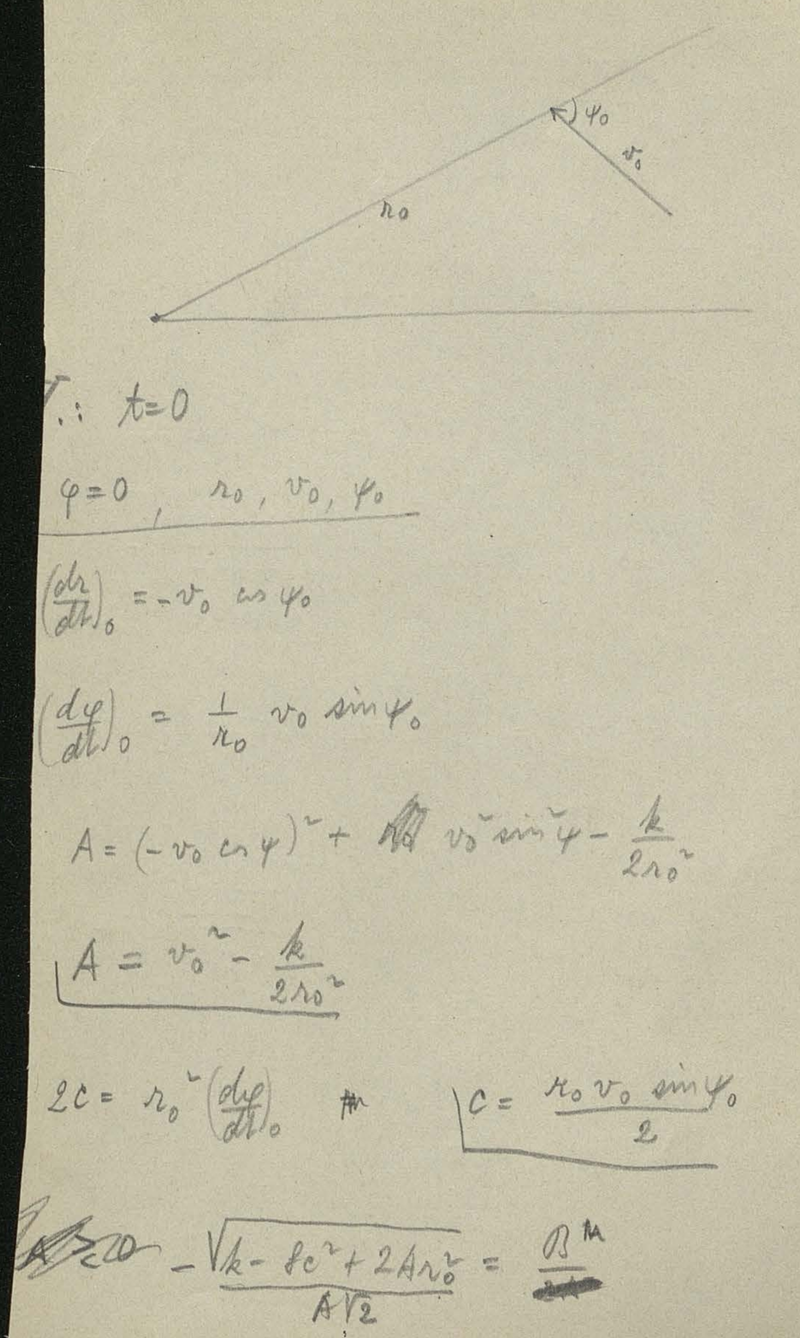
I. $\varphi = -\frac{2c\sqrt{2}}{\sqrt{k-8c^2}} \log\left[\frac{\sqrt{k-8c^2} + \sqrt{2\,AB}}{\sqrt{AB^2+\frac{8c^2-k}{2A}}}\right] + C$

II.

$$\varphi = -\frac{2c\sqrt{2}}{\sqrt{8c^2-k}} \arcsin\left(\sqrt{\frac{8c^2-k}{2A}}\,\frac{1}{\sqrt{AB^2+\frac{8c^2-k}{2A}}}\right) + C$$

$$= -\frac{2c\sqrt{2}}{\sqrt{8c^2-k}} \arcsin\left(\sqrt{\frac{8c^2-k}{2A^2B^2+8c^2-k}}\right) + C$$

$$= -\frac{2c\sqrt{2}}{\sqrt{8c^2-k}} \arcsin\left[\sqrt{\frac{1}{\frac{2A^2B^2}{8c^2-k}+1}}\right] + C$$



T.: $t=0$

$\varphi=0, \quad r_0, v_0, \psi_0$

$$\left(\frac{dr}{dt}\right)_0 = -v_0 \cos\psi_0$$

$$\left(\frac{d\varphi}{dt}\right)_0 = \frac{1}{r_0} v_0 \sin\psi_0$$

$$A = (-v_0 \cos\psi)^2 + v_0^2 \sin^2\psi - \frac{k}{2r_0^2}$$

$$A = v_0^2 - \frac{k}{2r_0^2}$$

$$2C = r_0^2 \left(\frac{d\varphi}{dt}\right)_0 \qquad C = \frac{r_0 v_0 \sin\psi_0}{2}$$

$$A \lessgtr 0 \qquad -\frac{\sqrt{k - 8C^2 + 2Ar_0^2}}{A\sqrt{2}} = B^m$$

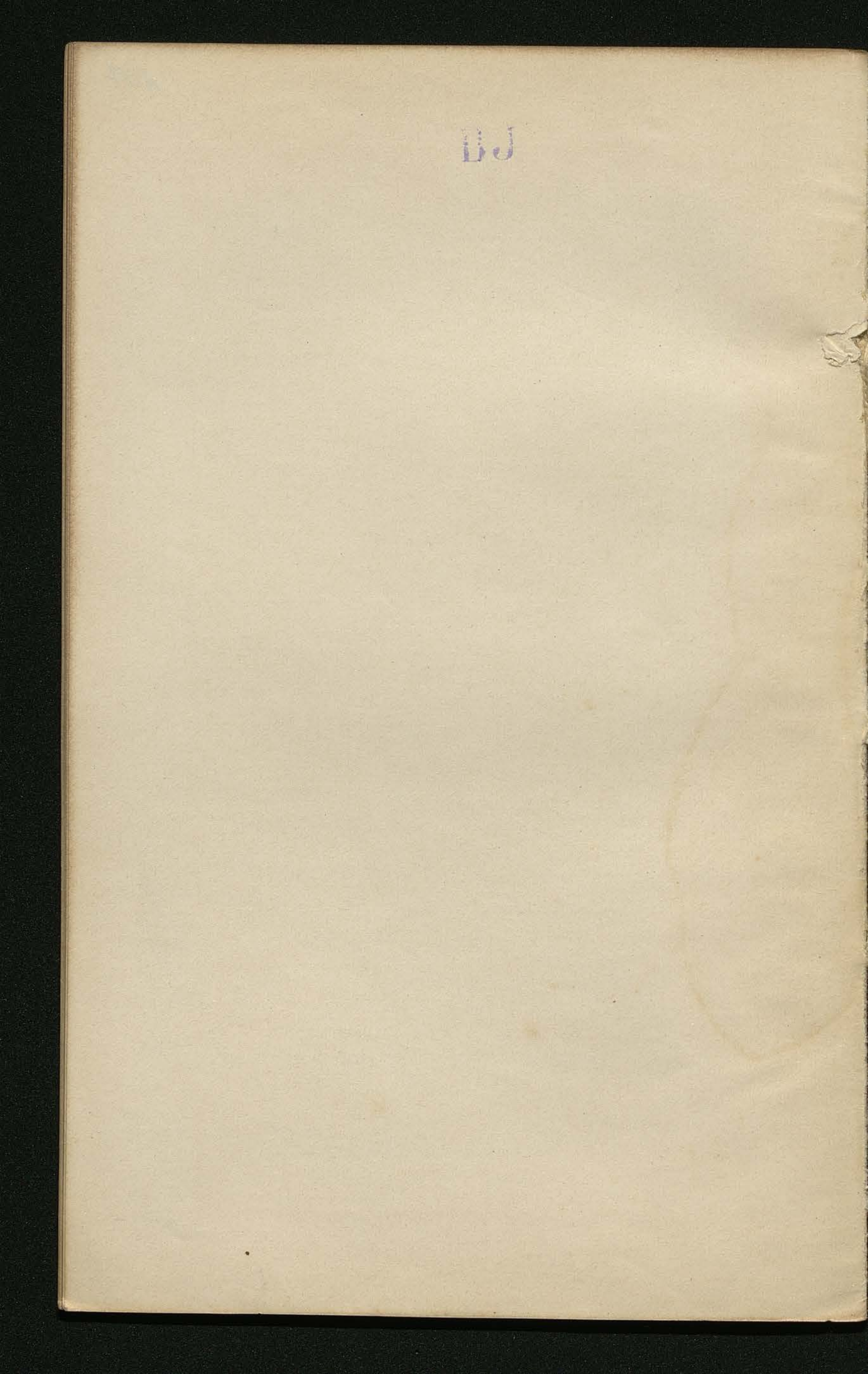
$$B = -\frac{\sqrt{\cancel{k} - 8c^2 + 2 v_0^2 r_0^2 - \cancel{k}}}{\left(v_0^2 - \frac{k}{2 r_0^2}\right)\sqrt{2}}$$

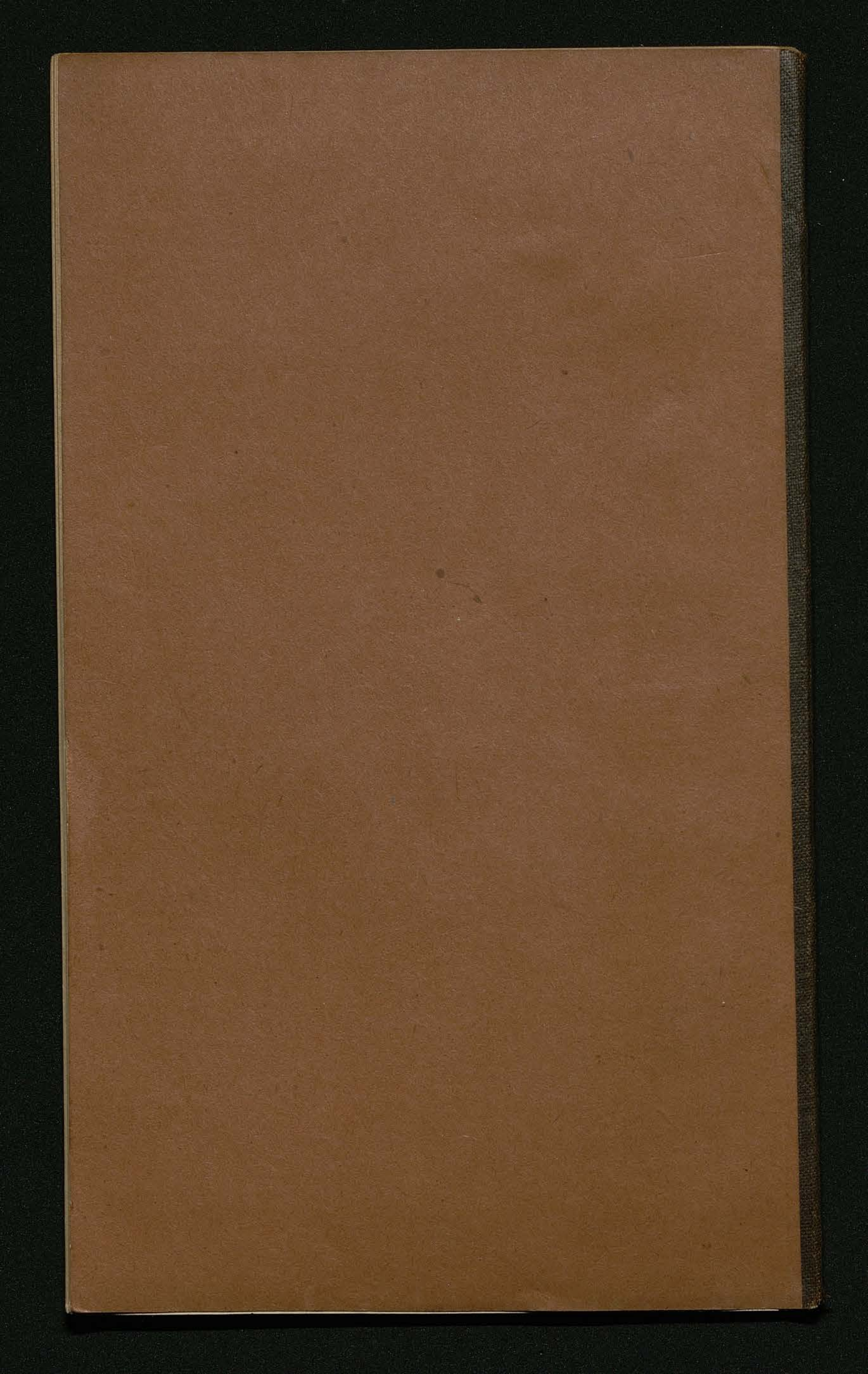
$$B = -\frac{\sqrt{v_0^2 r_0^2 - 4c^2}}{v_0^2 - \frac{k}{2 r_0^2}}$$

$$C = \frac{2c\sqrt{2}}{\sqrt{8c^2 - k}} \arcsin\left[\sqrt{\frac{1}{2\,\frac{\frac{4 r_0^4 (v_0^2 r_0^2 - 4c^2)}{(2 r_0^2 v_0^2 - k)^2}}{8c^2 - k} + 1}}\right]$$

$$= \frac{2c\sqrt{2}}{8c^2 - k} \arcsin\left[\sqrt{\frac{1}{2 v_0^2 r_0^2 \cos^2 \varphi_0}}\right]$$

$$= \frac{2c\sqrt{2}}{\sqrt{8c^2 - k}} \arcsin\left[\frac{\sqrt{8c^2 - k}}{\sqrt{2 v_0^2 r_0^2 - k}}\right] +$$



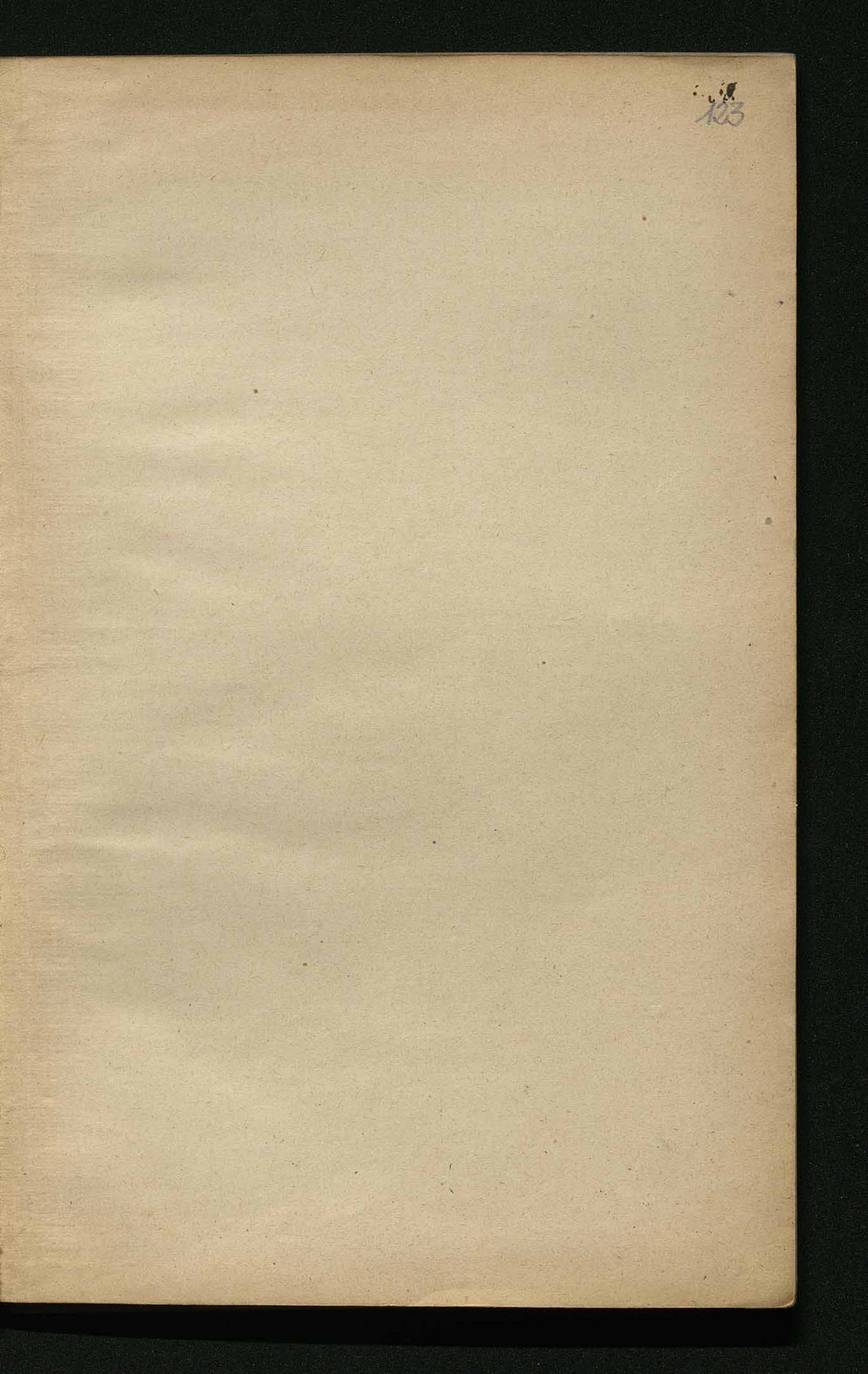


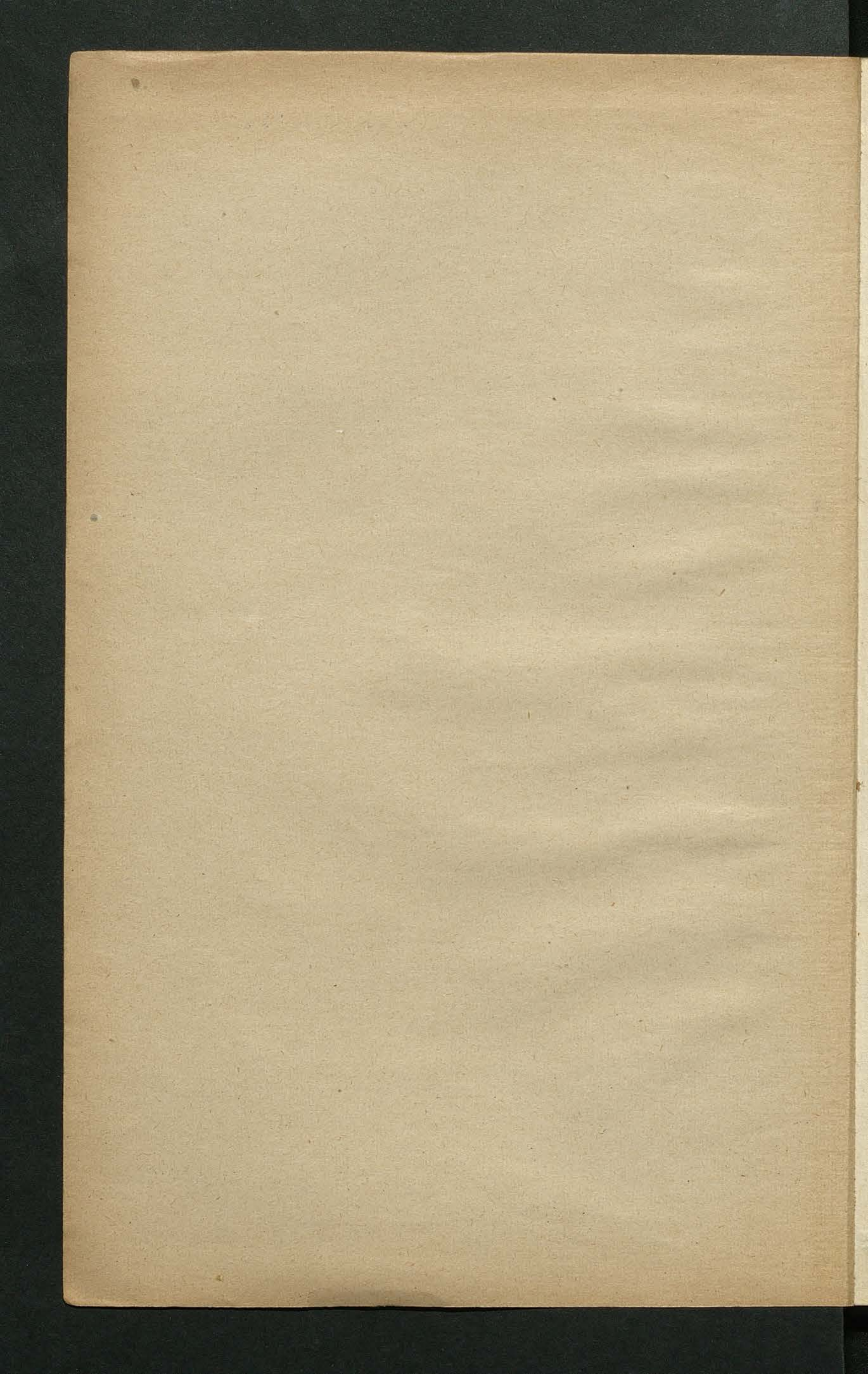
9451

1b2



10





$$c\, 2\pi r\, dr\, dx \frac{\partial u}{\partial t} = \sout{k\, 2\pi r\, dx \frac{\partial^2 u}{\partial r^2} dr}$$

$$= k \frac{\partial}{\partial r}\left[2\pi r \frac{\partial u}{\partial r}\right] dx\, dr$$

$$r \frac{\partial u}{\partial t} = m \frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r}\right) = m\left[\frac{\partial u}{\partial r} + r \frac{\partial^2 u}{\partial r^2}\right]$$

$$u = r^{\alpha} t^{\beta}$$

$$\sout{\beta\, r^{\alpha+1} t^{\beta-1} = m \alpha^2 r^{\alpha-1} t^{\beta}} \qquad \alpha r^{\alpha} t^{\beta}$$

$$u = r^{\alpha} e^{t\beta}$$

$$\sout{\beta\, r^{\alpha+1} e^{t\beta} = m \alpha^2 r^{\alpha-1} e^{t\beta}} \qquad \alpha r^{\alpha} e^{t\beta}$$

$$u = e^{\alpha r} t^{\beta}$$

$$\sout{\beta r\, e^{\alpha r} t^{\beta-1} = m\left[\alpha e^{\alpha r} t^{\beta} + r \alpha^2 e^{\alpha r} t^{\beta}\right]} \qquad r\alpha\, e^{\alpha r} t^{\beta}$$

ρ

θ

$$r = \frac{\frac{c^2}{\mu}}{1 + \sqrt{1 + \frac{2c^2C}{\mu^2}} \cos\varphi}$$

$\mu = k(m + m_1)$

$c = \sqrt{p k (m + m_1)}$

$C = \frac{k(m+m_1)(\varepsilon^2 - 1)}{p}$

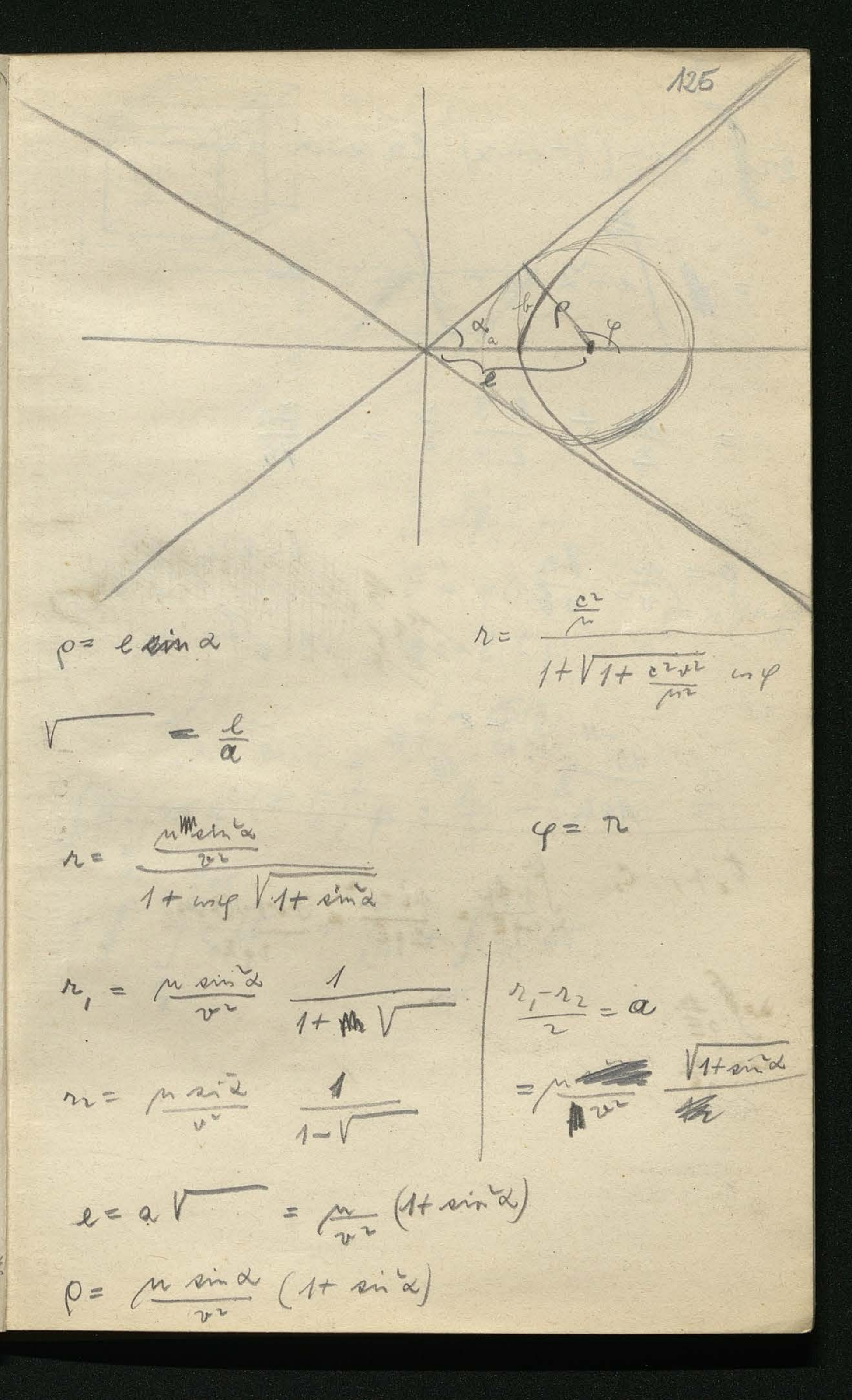
$$\frac{v^2}{2} = \frac{\mu}{r} + C$$

$$C = \frac{v^2}{2} \Big|^{r = \infty}$$

$$\cos\alpha \sqrt{1 + \frac{2c^2C}{\mu^2}} = -1$$

$$\cos^2\alpha \,(\mu^2 + 2c^2C) = \mu^2$$

$$c = \sqrt{\frac{\mu^2(1 - \cos^2\alpha)}{2C}} = \frac{\mu \sin\alpha}{\sqrt{2C}} = \frac{\mu \sin\alpha}{v}$$


α
a
b
ρ
φ
e
ρ = e sin α
√ = e/a
r = (c²/μ) / (1 + √(1 + c²v²/μ²) cos φ)
r = (μ sin²α / v²) / (1 + cos φ √(1 + sin²α))
φ = π
r₁ = (μ sin²α / v²) · 1/(1 + √)
r₂ = (μ sin²α / v²) · 1/(1 − √)
(r₁ − r₂)/2 = a
= (μ / v²) · √(1 + sin²α)
e = a √ = (μ/v²)(1 + sin²α)
ρ = (μ sin α / v²)(1 + sin²α)

$$\frac{1}{2\pi}\int_0^{\frac{\pi}{2}} \sin\alpha\,(1+\sin^2\alpha)\; 2\pi \sin\alpha \, d\alpha$$

$$= \int_0^{\frac{\pi}{2}} \sin^2\alpha \, d\alpha + \int \sin^4\alpha \, d\alpha$$

$$= \frac{\pi}{4} + \frac{1.3}{2.4}\,\frac{\pi}{2} = \frac{7\pi}{16}$$

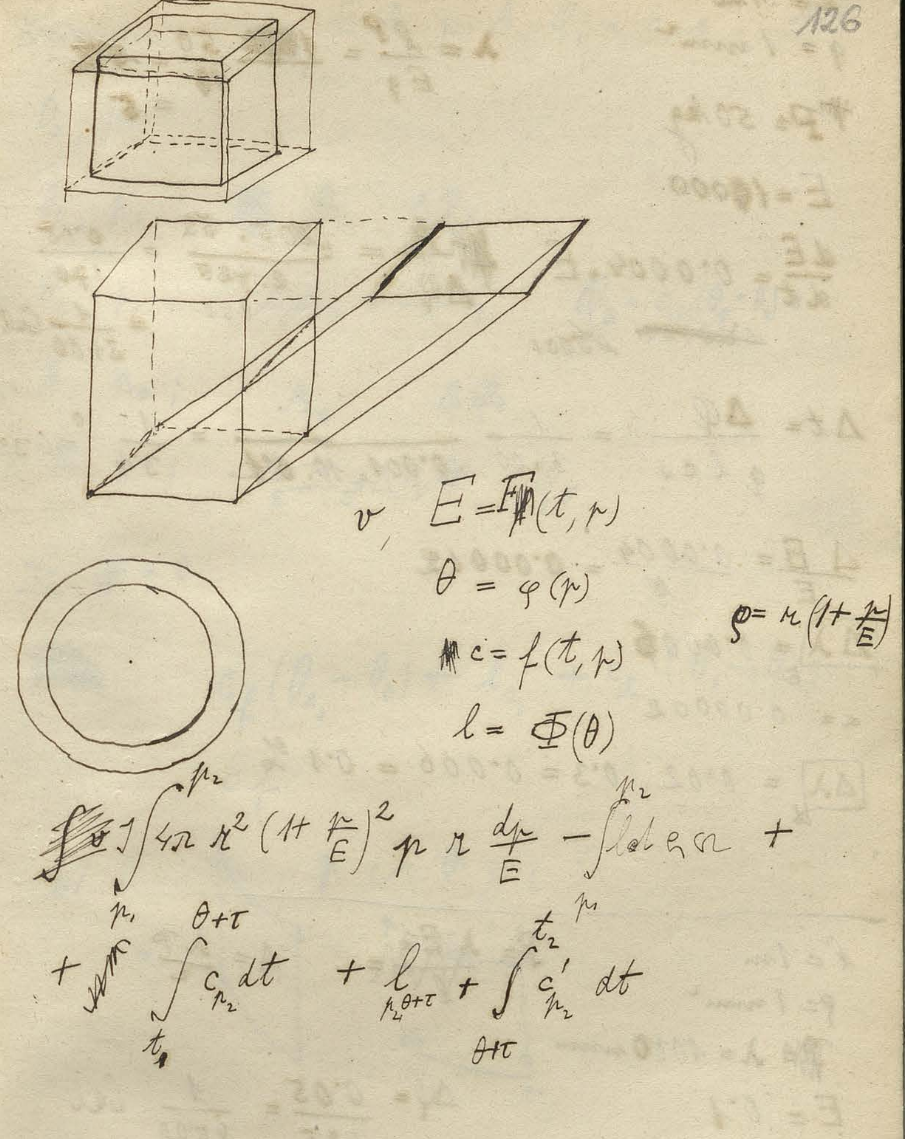
$$\rho = \frac{\mu}{v^2}\,\frac{7\pi}{16}$$

$$\mu = \frac{10^{-11}}{400^2} \cdot 6.10^{-8} = 10^{-15} = 6.10^{-23}$$

$t_1 \; p_1 \; C_1$ $$\int_{p_1}^{p_2} \frac{p\,dp}{\rho E} = \frac{p_2^2 - p_1^2}{2\rho E} = \frac{(p_2 - p_1)(p_2 + p_1)}{2\rho E}$$

$$\lambda = \int \frac{p}{\rho E}$$

$$\Delta\lambda = \frac{\Delta p}{\rho E}$$



$$v, \quad E = F(t, p)$$

$$\theta = \varphi(p)$$

$$c = f(t, p)$$

$$l = \Phi(\theta)$$

$$\varrho = r\left(1 + \frac{p}{E}\right)$$

$$J \int_{p_1}^{p_2} 4\pi r^2 \left(1 + \frac{p}{E}\right)^2 p \, r \frac{dp}{E} - \int_{p_1}^{p_2} l \, d\,[illegible] \; +$$

$$+ \int_{t_1}^{\theta+\tau} c_{p_2} \, dt + l_{p_2, \theta+\tau} + \int_{\theta+\tau}^{t_2} c'_{p_2} \, dt$$

$l = 1\,m$

$q = 1\,mm^2$

$P = 50\,kg$

$E = 10000$

$\lambda = \frac{lP}{Eq} = \frac{50}{10} = 5$

$\frac{dE}{dt} = 0.0004 \cdot E$

~~0.000001~~ ~~0~~.0001

$\Delta Q = \frac{0.005 \cdot 50}{425} = \frac{0.05}{170} = \frac{1}{3400}\,Cal$

850

$\Delta t = \frac{\Delta Q}{q\,l\,c\,s} = \frac{1}{3400 \cdot 0.001 \cdot 10 \cdot 0.1} = \frac{1}{3.4}^{0} = 0.3^{0}$

$\frac{\Delta E}{E} = \frac{0.0004}{2} = 0.00012$

$\boxed{\Delta\lambda}_E = 0.0006$

$\alpha = 0.00002$

$\boxed{\Delta\lambda}_\alpha = 0.02 \cdot 0.3 = 0.006 = 0.1\,\%$

$l = 1\,m$

$q = 1\,mm^2$

$\lambda = 1000\,mm$

$E = 0.1$

$P = 0.1\,kg$

$P = \frac{\lambda E q}{l} =$

$A = \frac{\lambda P}{2}$

$\Delta\varphi = \frac{0.05}{425} = \frac{1}{8500}\,Cal$

Kreisprozess: $E, \theta, p, A, Q, \alpha, c, s$

1). θ_0, p_0

2). θ_0, p_1; A_2 ΔQ_2

3). θ_1, p_1; $\Delta_E A_3 + \Delta_\alpha A_3$ $Q_3 = c_{(p)}(\theta_1 - \theta_0)s$

4). θ_1, p_0; A_4 ΔQ_4

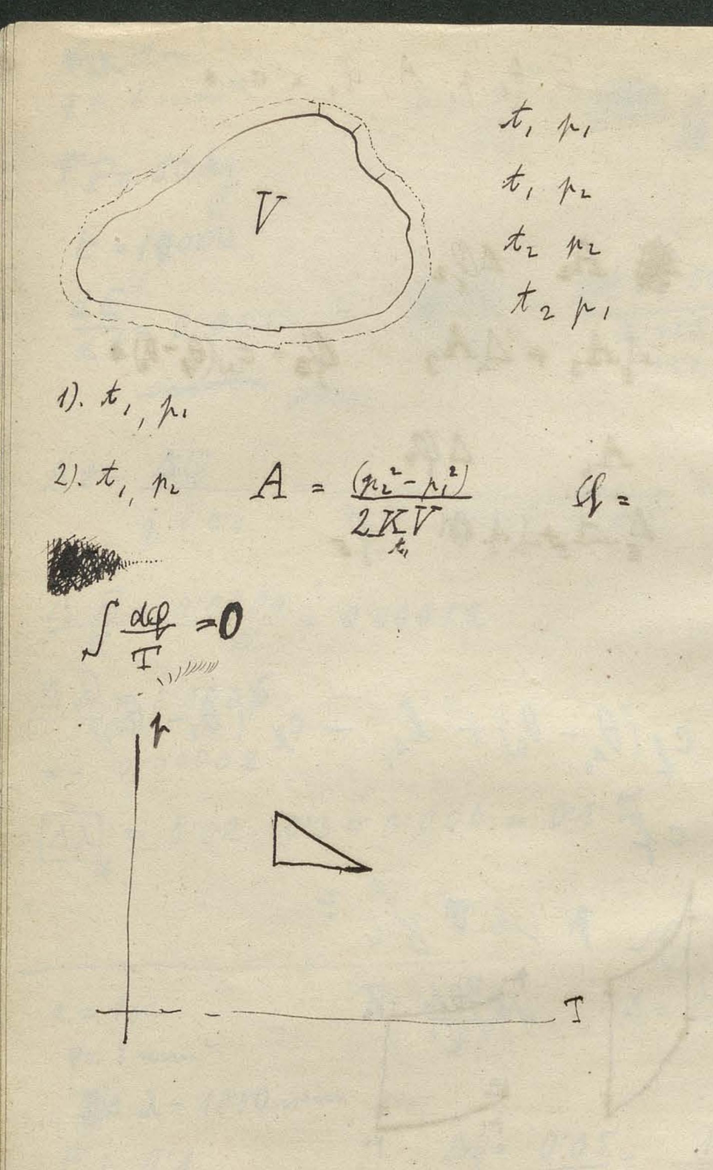
5). θ_0, p_0; $\Delta_E A_5 + \Delta_\alpha A_5$ Q_5

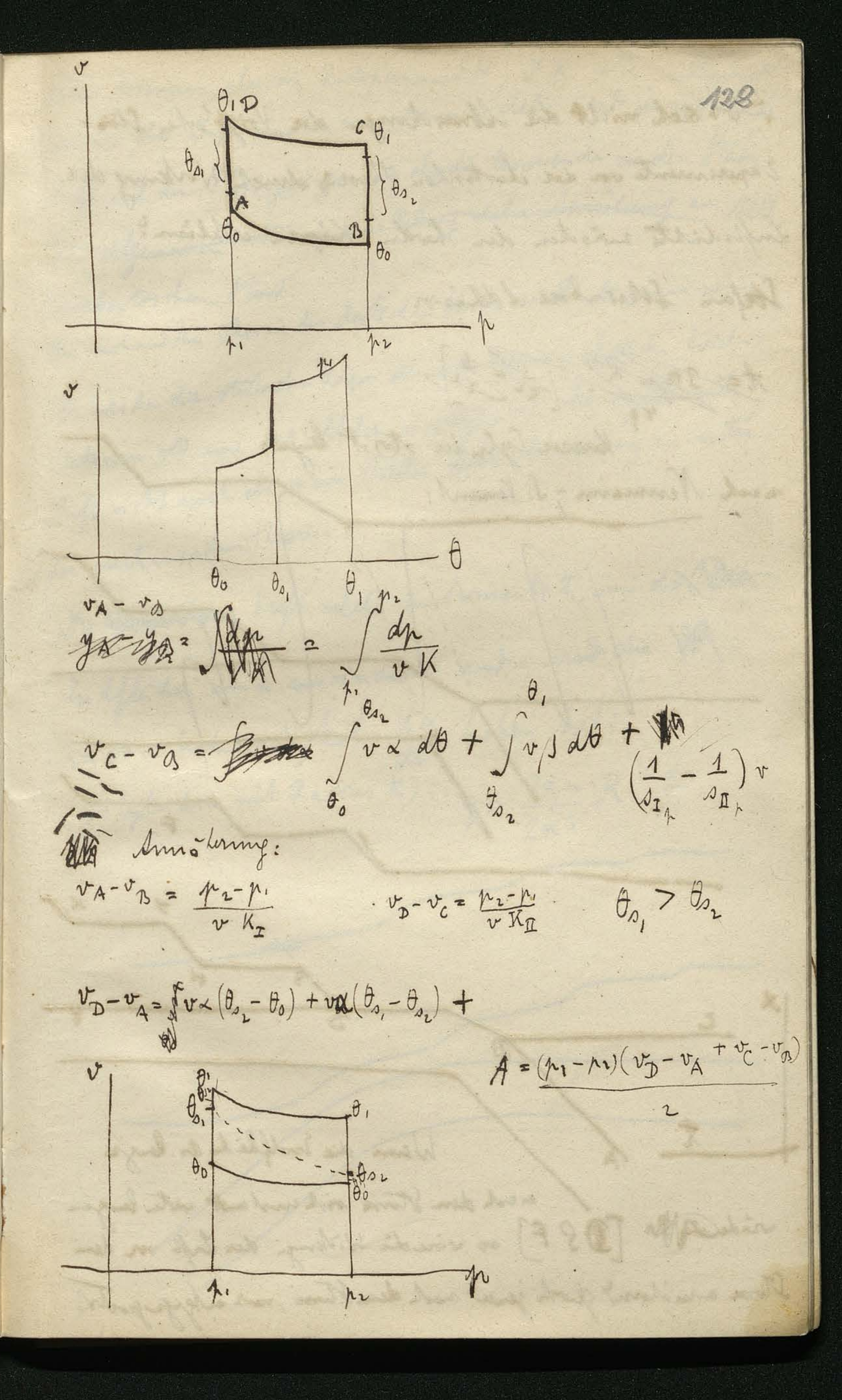
$$\sum \frac{Q}{T} = 0$$

$$c_f(\theta_{s_1} - \theta_0) + l_{s_1} + c_l(\theta_1 - \theta_{s_1})$$

c f

v p_1 θ_1 p_2 θ_0 θ_0 p_1 p_2 p


V
t₁ p₁
t₁ p₂
t₂ p₂
t₂ p₁
1). t₁, p₁
2). t₁, p₂
A = (p₂² − p₁²) / 2KV_{t₁}
Q =
∫ dQ/T = 0
p
T

128
v
θ₁ D
C θ₁
θ_{s1}
θ_{s2}
A
θ₀
B
θ₀
p
p₁
p₂
v
μ
θ
θ₀
θ_{s1}
θ₁
$v_A - v_B = \int_{p_1}^{p_2} \frac{dp}{vK}$
$v_C - v_B = \int_{\theta_0}^{\theta_{s2}} v\alpha\, d\theta + \int_{\theta_{s2}}^{\theta_1} v\beta\, d\theta + \left(\frac{1}{s_{I_p}} - \frac{1}{s_{II_p}}\right) v$
Annäherung:
$v_A - v_B = \frac{p_2 - p_1}{v K_I}$
$v_D - v_C = \frac{p_2 - p_1}{v K_{II}}$
$\theta_{s1} > \theta_{s2}$
$v_D - v_A = v\alpha(\theta_{s2} - \theta_0) + v\alpha(\theta_{s1} - \theta_{s2}) +$
$A = \frac{(p_1 - p_2)(v_D - v_A + v_C - v_B)}{2}$
v
θ₁
θ_{s1}
θ₁
θ₀
θ_{s2}
θ₀
p
p₁
p₂

? Ob sich nicht die Abweichungen der Voigt'schen Stoss-Experimente von der elastischen Theorie durch Wirkung der Luftschichte zwischen den beiden Körpern erklären?

Stefan Scheinbare Adhäsion

~~$t = \frac{3\pi\mu R^4}{4q}\left[\frac{1}{e^2} - \frac{1}{\epsilon^2}\right]$~~

Kurzer Cylinder stösst langen:

nach Neumann - S. Venant:

Wenn die Endfläche des langen nach dem Stosse sich konstant weiter bewegen würde ~~[illegible]~~ [B G F] so wäre die Wirkung der Luft vor dem Stosse annähernd gleich jener nach dem Stosse, nur entgegengesetzt

Nun kommen aber die Rutepausen SH, JK etc., während welcher die Luft einströmen kann; daher wird die vorwiegende Kraft nach dem Stoße geringer sein als die trennende vor dem Stoße.

~~Im Allgemeinen Wirkung einer Dehnung daher Annäherung an unelastischen Stoß.~~

Die mechanische Theorie des elastischen Stoßes würde stimmen wenn z.B. zwischen die stoßenden Körper ein sehr biegsames elastisches Zwischenmedium z.B. eine Spiralfeder eingelegt wäre. Die eingeklemmte Luft wirkt auch als so ein Polster daher Annäherung an Sector der mechanischen Theorie.

Die ~~[illegible]~~ Kraft welche im Momente t, wo die ~~[illegible]~~ Endflächen um α ~~auseinander~~ sind, und die ~~[illegible]~~ relative Geschwindigkeit $\frac{d\alpha}{dt}$ haben ist:

(bei Cylindern mit Radius R): $q = \frac{3\pi\mu}{2\alpha^3} R^4 \frac{d\alpha}{dt}$

$t = -\infty$:

$\frac{\partial u}{\partial t} = c_1$ $\quad -\xi < x < -(\xi + a_1)$

$\frac{\partial u}{\partial t} = c_2$ $\quad \alpha - \xi < x < \alpha - \xi + a_2$

$=$

$\xi : \alpha - \xi = c_1 : c_2$ $\quad \frac{c_2}{c_1} \xi < \alpha < \frac{c_2}{c_1} \xi + a_2$

$\alpha - \xi = \frac{c_2}{c_1} \xi$

$\alpha = \frac{c_1 + c_2}{c_1} \xi$

$\frac{\partial u}{\partial t} = \omega f'(x + \omega t) - \omega \varphi'(x - \omega t)$

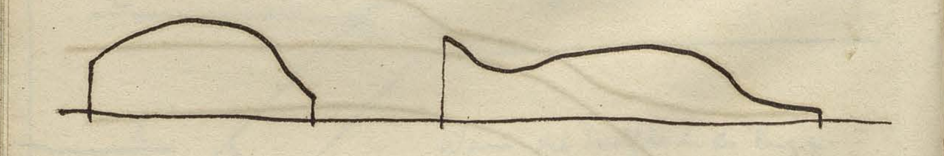
$\frac{c_1}{\omega} =$

$\xi = \infty$.

$f'(x) = \frac{c_1}{2\omega}$ $\quad \varphi'(x) = -\frac{c_1}{2\omega}$ $\quad -\xi < x < -(\xi + a_1)$

$f'(x) = \frac{c_2}{2\omega}$ $\quad \varphi'(x) = -\frac{c_2}{2\omega}$ $\quad \frac{c_2}{c_1}\xi < x < \frac{c_2}{c_1}\xi + a_2$

$\frac{\partial u}{\partial x} = 0$



$$X_x = \left(2K \frac{1+3L}{1+2L}\right) \frac{\partial u_0}{\partial x} = A \frac{\partial u_0}{\partial x} =$$

$$A\left(\frac{\partial u_0}{\partial x}\right)_1 = -A\left(\frac{\partial u_0}{\partial x}\right)_2 = \frac{3\pi\mu}{2} R^4 \frac{1}{\alpha^3} \frac{d\alpha}{dt}$$

$$= B \frac{1}{\alpha^3} \frac{d\alpha}{dt}$$

$$-\xi < x < -(\xi + a_1)$$

$$\frac{c}{c_1}\xi < x < \frac{c}{c_1}\xi + b$$

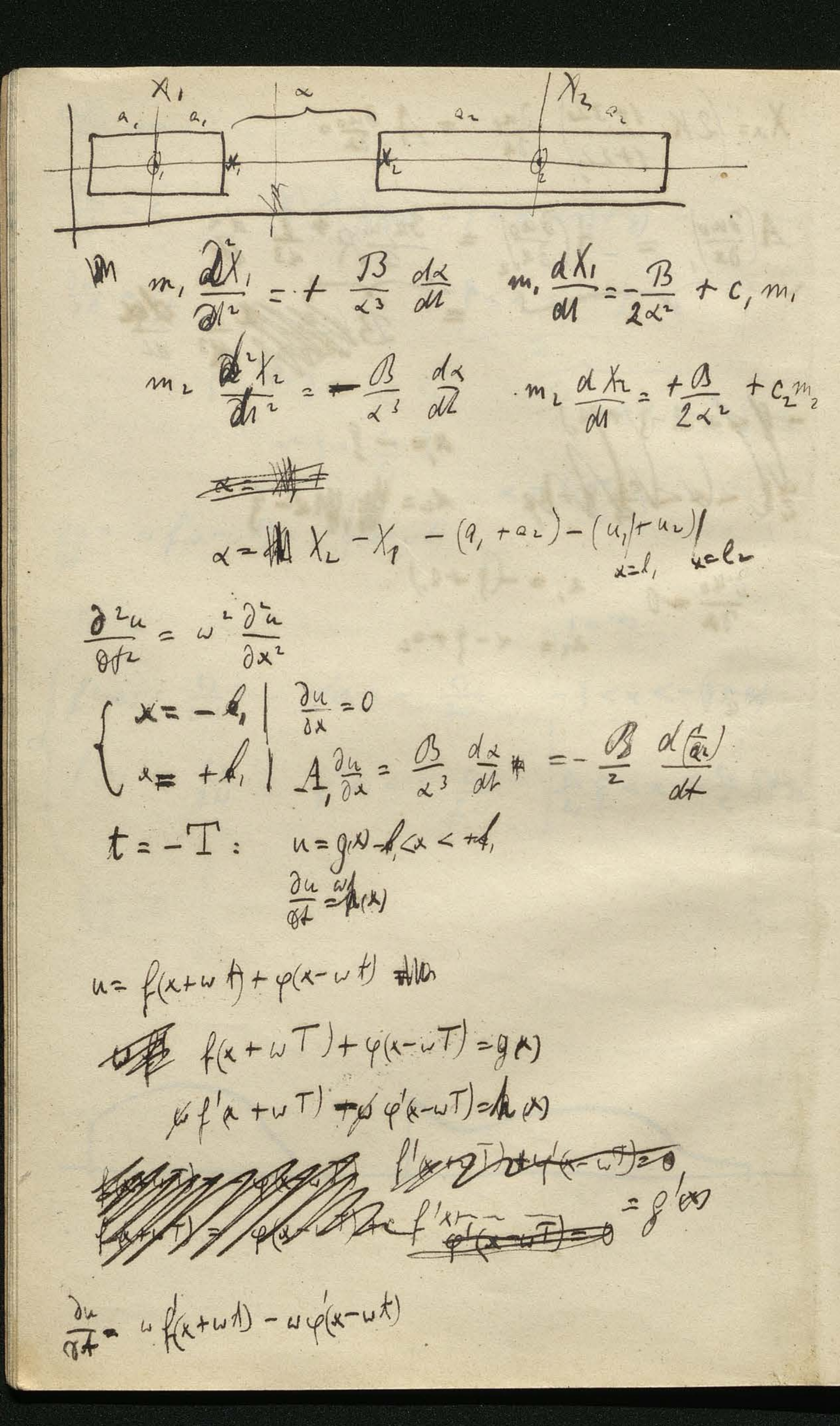
$$x_1 = -\xi$$

$$x_2 = \alpha - \xi$$

$$\frac{\partial u_0}{\partial x} = 0 \qquad x_1 = -(\xi + a_1)$$

$$x_2 = \alpha - \xi + a_2$$

u



$$m_1 \frac{\partial^2 X_1}{\partial t^2} = + \frac{B}{\alpha^3} \frac{d\alpha}{dt} \qquad m_1 \frac{dX_1}{dt} = -\frac{B}{2\alpha^2} + c_1 m_1$$

$$m_2 \frac{d^2 X_2}{dt^2} = - \frac{B}{\alpha^3} \frac{d\alpha}{dt} \qquad m_2 \frac{dX_2}{dt} = + \frac{B}{2\alpha^2} + c_2 m_2$$

$$\alpha = X_2 - X_1 - (a_1 + a_2) - (u_1|_{x=l_1} + u_2|_{x=l_2})$$

$$\frac{\partial^2 u}{\partial t^2} = \omega^2 \frac{\partial^2 u}{\partial x^2}$$

$$\begin{cases} x = -l_1 & \frac{\partial u}{\partial x} = 0 \\ x = +l_1 & A_1 \frac{\partial u}{\partial x} = \frac{B}{\alpha^3}\frac{d\alpha}{dt} = -\frac{B}{2}\frac{d\left(\frac{1}{\alpha^2}\right)}{dt} \end{cases}$$

$t = -T$: $u = g(x) \quad -l_1 < x < +l_1$

$$\frac{\partial u}{\partial t} = \omega h(x)$$

$$u = f(x + \omega t) + \varphi(x - \omega t)$$

$$f(x + \omega T) + \varphi(x - \omega T) = g(x)$$

$$f'(x + \omega T) - \varphi'(x - \omega T) = h(x)$$

$$= g'(x)$$

$$\frac{\partial u}{\partial t} = \omega f'(x + \omega t) - \omega \varphi'(x - \omega t)$$

~~$m_1 \frac{dX_1}{dt} + m_2 \frac{dX_2}{dt} = m_1 c_1 + m_2 c_2$~~

$$\frac{d\alpha}{dt} = \frac{dX_2}{dt} - \frac{dX_1}{dt} - \left(\frac{du_1}{dt} + \frac{du_2}{dt}\right)$$

$$= \frac{+B}{2\alpha^2 m_1} + \frac{B}{2\alpha^2 m_2} - c_1 + c_2 - \left(\frac{du_1}{dt} + \frac{du_2}{dt}\right)$$

~~$= - \frac{B}{2\alpha^2} \frac{1}{m_1}$~~

$$= + \frac{B\,(m_2 + m_1)}{2\, m_1 m_2} \frac{1}{\alpha^2} - - - - -$$

$$\frac{d(\alpha + u_1 + u_2)}{dt} = + \frac{B(m_2+m_1)}{2\, m_1 m_2} \frac{1}{\alpha^2} + c_2 - c_1$$

~~A_1~~

$$f'(x + \omega T) = \frac{1}{2}\left(h(x) + g'(x)\right)$$

$$\varphi'(x - \omega T) = \frac{1}{2}\left(-h(x) + g'(x)\right)$$

$$f(x + \omega T) = \frac{1}{2}\left[g(x) + \int h(x)\,dx\right]$$

$$\varphi(x - \omega T) = \frac{1}{2}\left[g(x) - \int h(x)\,dx\right]$$

$-l_1 < x < +l_1$

$$f(-l_1 + \omega t) + \varphi(-l_1 - \omega t) = 0$$

$$A_1 \left\{ f'(l_1 + \omega t) + \varphi'(l_1 - \omega t) \right\} = - \frac{B_1}{2} \frac{d\left(\frac{1}{\alpha^2}\right)}{dt}$$

$$u_1 = a_1 + \frac{b_1}{t} + \frac{c_1}{t^2} + \frac{d_1}{t^3} + \frac{e_1}{t^4} + \frac{f_1}{t^5} +$$

$$u_2 = a_2 + \frac{b_2}{t} + \frac{c_2}{t^2} + \frac{d_2}{t^3} + \text{———}$$

$$a_1\ b_1\ \cdots = f(x)$$

$$\frac{\partial u_1}{\partial t} = -\left[\frac{b_1}{t^2} + \frac{2 b_2}{t^3} + \frac{3 d_3}{t^4} + \text{—}\right.$$

$$\frac{\partial^2 u_1}{\partial t^2} = + 2\frac{b_1}{t^3} + 2.3\,\frac{c_2}{t^4} + 3.4\,\frac{d_3}{t^5} + \cdots$$

~~$2\frac{b_1}{t^3} + 2.3\,\frac{b_2}{t^4} + 3.4\,\frac{b_3}{t^5} + \cdots = \frac{\partial^2 a_1}{}$~~

$$2\frac{b_1}{t^3} + 2.3\,\frac{c_1}{t^4} + 3.4\,\frac{d_1}{t^5} + \cdots \quad \cdots =$$

$$\frac{\partial^2 a_1}{\partial x^2} + \frac{1}{t}\,\frac{\partial^2 b_1}{\partial x^2} + \frac{1}{t^2}\,\frac{\partial^2 c_1}{\partial x^2} + \cdots\cdots$$

$$\frac{\partial^2 a_1}{\partial x^2} = 0 \qquad 2 b_1 = \frac{\partial^2 d_1}{\partial x^2}$$

$$\frac{\partial^2 b_1}{\partial x^2} = 0 \qquad 2.3.c_1 = \frac{\partial^2 e_1}{\partial x^2}$$

$$\frac{\partial^2 c_1}{\partial x^2} = 0 \qquad 3.4.d_1 = \frac{\partial^2 f_1}{\partial x^2}$$

———.

$$a_1 = M_1 x + N_1$$

$$b_1 = M_2 x + N_2$$

$$c_1 = M_3 x + N_3$$

$$d_1 = M_2 \frac{x^3}{3} + N_2 x^2 + O_2 x + P_2$$

$$e_1 = M_3 x^3 + 3 N_3 x^2 + O_3 x + P_3$$

$$2 M_2 x + 2 N_2 = \frac{d^2 d_1}{dx^2}$$

$$M_2 x^2 + 2 N_2 x + O_2 = \frac{d d_1}{dx}$$

$$M_2 \frac{x^3}{3} + N_2 x^2 + O_2 x + P_2 = d_1$$

$$\frac{d^2 f_1}{dx^2} = 3.4 . \left[P_2 + O_2 x + N_2 x^2 + M_2 \frac{x^3}{3}\right]$$

$$\frac{d f_1}{dx} = 3.4. \left[P_2 x + O_2 \frac{x^2}{2} + N_2 \frac{x^3}{3}\right] + \frac{M_2 x^4}{} + \varphi_4$$

$$f_1 = 3.4 \left[P_2 \frac{x^2}{2} + O_2 \frac{x^3}{2.3} + N_2 \frac{x^4}{3.4}\right] + M_2 \frac{x^5}{5} + \varphi_4 x + R_4$$

$$f_1 = M_2 \frac{x^5}{5} + N_2 x^4 + 2 . O_2 x^3 + 6 P_2 x^2 + \varphi_4 x + R_4$$

$$\left\{ \begin{aligned} -\frac{B}{2} \frac{m_2 + m_1}{m_1 m_2} \frac{1}{\alpha^2} &= \frac{d(\alpha + u_1 + u_2)}{dt} + c_1 - c_2 \\ A_1 \frac{\partial u_1}{\partial x} &= -\frac{B}{2} \frac{d}{dt}\left(\frac{1}{\alpha^2}\right) \end{aligned} \right\} \Big\| x = a_1$$

$$A_1 \frac{m_2 + m_1}{m_1 m_2} \frac{\partial u_1}{\partial x} = \frac{d^2}{dt^2} (\alpha + u_1 + u_2) \quad \| x = l_1$$

$$=$$

$$2 A_1 \left\{ \frac{1}{t} \frac{\partial b_1}{\partial x} + \frac{1}{t^2} \frac{\partial c_1}{\partial x} + \frac{1}{t^3} \frac{\partial d_1}{\partial x} + \frac{1}{t^4} \frac{\partial e_1}{\partial x} + \frac{1}{t^5} \frac{\partial f_1}{\partial x} + \right.$$

$$= -B \frac{d\left(\frac{1}{\alpha^2}\right)}{dt}$$

damit einer der beiden Stäbe zurückspringt, müsste ein Moment sein, wo $\frac{dX_1}{dt}$ oder $\frac{dX_2}{dt} = 0$ also:

$$-\frac{B}{2\alpha^2} + c_1 m_1 = 0 \quad \text{oder} \quad +\frac{B}{2\alpha^2} + c_2 m_2 = 0$$

~~[illegible]~~ | verlangt ein $c_2 < 0$

$$\frac{B}{2\alpha^2} = m_1 c_1$$

$$\alpha = \sqrt{\frac{B}{2 m_1 c_1}}$$

dann ist:

$$m_2 \frac{dX_2}{dt} = m_2 c_2 + m_1 c_1$$

A

Wenn die Länge der störenden Cylinder im Verhältnis zu ihrem Querschnitt sehr kurz ist, so dass diese als Scheiben betrachtet werden können, wird die elektrische Verschiebung sehr klein werden; wenn man angenähert annimmt:

$\frac{dX}{dt} = \frac{d\alpha}{dt}$; so wird sein:

$$m_1 \frac{dx}{dt} = + \frac{B}{2x^2} - m_1 c_1$$

$$\frac{dx}{dt} - \frac{B}{2m_1} \frac{1}{x^2} + c_1 = 0$$

$$\frac{dx}{\frac{B}{2m_1}\frac{1}{x^2} - c_1} = dt$$

$$\int \frac{x^2 dx}{\frac{B}{2m_1} - c_1 x^2} = \int dt$$

$$\int \frac{\frac{2m_1}{B} x^2}{1 - \frac{2m_1 c_1}{B} x^2} dx = t + C$$

$$x \sqrt{\frac{2m_1 c_1}{B}} = y$$

$$x = \sqrt{\frac{B}{2m_1 c_1}}\, y$$

$$= \int \frac{2m_1}{B} \frac{B}{2m_1 c_1} \sqrt{\frac{B}{2m_1 c_1}}$$

$$\int \frac{x^2 dx}{1 - x^2} = \int \frac{1}{2} \left\{ \frac{-x}{1+x} + \frac{x}{1-x} \right\} dx$$

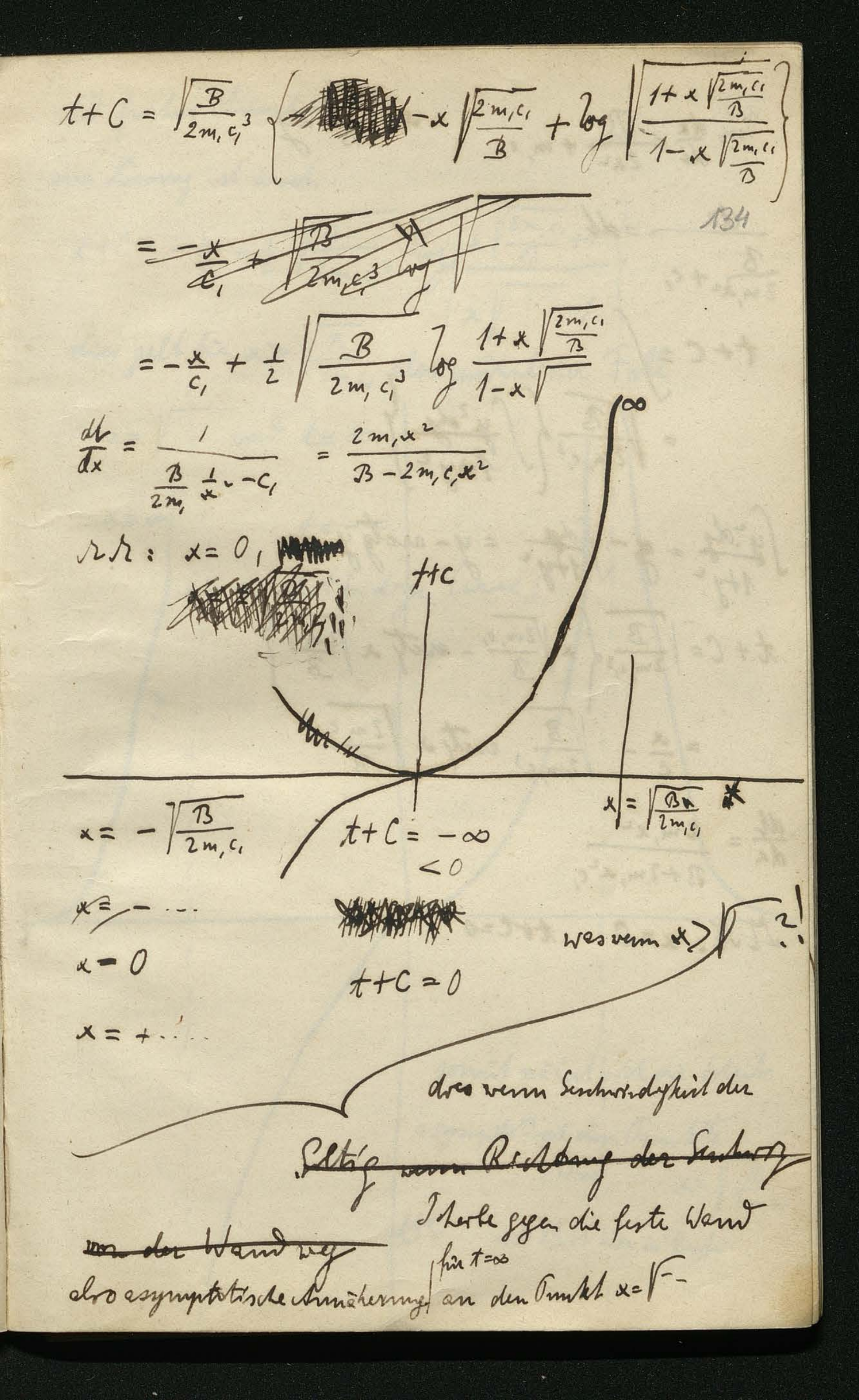
$$1+x = y$$

$$J_1 = \int \frac{y-1}{y} dy = y - \lg y$$
$$= (1+x) - \lg(1+x)$$

$$1-x = y$$

$$J_2 = -\int \frac{1-y}{y} dy = -\lg y + y$$
$$= (1-x) - \lg(1-x)$$

$$J = \frac{1}{2} \left\{ -2x + \lg \frac{1+x}{1-x} \right\}$$
$$= -x + \lg \sqrt{\frac{1+x}{1-x}}$$



$$t+C = \sqrt{\frac{B}{2m_1c_1^3}}\left\{ [illegible] - x\sqrt{\frac{2m_1c_1}{B}} + \log\sqrt{\frac{1+x\sqrt{\frac{2m_1c_1}{B}}}{1-x\sqrt{\frac{2m_1c_1}{B}}}}\right\}$$

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~~$= -\frac{x}{c_1} + \sqrt{\frac{B}{2m_1c_1^3}} \log \sqrt{\;}$~~

$$= -\frac{x}{c_1} + \frac{1}{2}\sqrt{\frac{B}{2m_1c_1^3}}\log\frac{1+x\sqrt{\frac{2m_1c_1}{B}}}{1-x\sqrt{\;}}$$

$$\frac{dt}{dx} = \frac{1}{\frac{B}{2m_1}\frac{1}{x^2}-c_1} = \frac{2m_1x^2}{B-2m_1c_1x^2}$$

rr: $x = 0$, [illegible]

was wenn $x > \sqrt{\;}$?!

das wenn Geschwindigkeit der

~~[illegible] Richtung der Geschw.~~

Stoße gegen die feste Wand

~~von der Wand weg~~

also asymptotische Annäherung für $t=\infty$ an den Punkt $x = \sqrt{\;}$ –

$$m_1 \frac{dx}{dt} = \frac{B}{2x^2} + m_1 c_1 \qquad x\sqrt{\frac{2m_1c_1}{B}} = y$$

$$\frac{dx}{\frac{B}{2m_1 x^2} + c_1} = dt$$

$$t + C = \int$$

$$= \sqrt{\frac{B}{2m_1 c_1^3}} \left\{ \int \frac{y^2 dy}{1+y^2} \right\}$$

$$\int \frac{y^2 dy}{1+y^2} = y - \int \frac{dy}{1+y^2} = y - \operatorname{arctg} y$$

$$t + C = \sqrt{\frac{B}{2m_1 c_1^3}} \left\{ x\sqrt{\frac{2m_1c_1}{B}} - \operatorname{arctg} x \sqrt{\frac{2m_1c_1}{B}} \right\}$$

$$= \frac{x}{c} - \sqrt{\frac{B}{2m_1 c_1^3}} \operatorname{arctg} x \sqrt{\frac{2m_1 c_1}{B}}$$

$$\frac{dt}{dx} = \frac{2m_1 x^2}{B + 2m_1 x^2 c_1}$$

M.A.: $x = 0 \qquad t + C = 0$

Ad frühere Annahme: c_1

eine Lösung ist auch:

$$t + C = -\frac{x}{c_1} + \frac{1}{2}\sqrt{\frac{B}{2m_1 c_1^3}} \log \left\{ \frac{x\sqrt{\frac{2m_1c_1}{B}}+1}{x\sqrt{}-1} \right\}$$

dies gilt für $x > \sqrt{\frac{B}{2m_1 c_1}}$ also gewöhnlicher Fall

für $x = \sqrt{}$ wird $t = \infty$

$x = \infty$ $t = -\infty$

also vollständige Curve:

?

x

t+C

somit nähert sich die Scheibe asymptotisch dem Punkte $x = \sqrt{\frac{B}{2m_1 c_1}}$ und bleibt hier in Ruhe

$t\infty$

Vorübung:

Eine ~~Platte~~ kreisförmige Platte bewegt sich mit der constanten Geschwindigkeit c senkrecht gegen eine Wand (in Luft); welches sind die durch Reibung und Compression der Luft entstehenden Kräfte?

Annahme: für $x = R$ sei Dichtigkeit der Luft $= \rho = \rho_0$

I). In jedem Zeitmomente muss: verdrängte Luftmasse = ausweichende + Verdichtung

$$\int_0^R \rho\, 2\pi x\,dx \cdot \Delta a = 2\pi R \cdot a \cdot v_R\, \Delta t + \int_0^R 2\pi x\,dx \cdot a \frac{\partial\rho}{\partial t}\Delta t$$

$$\frac{\Delta a}{\Delta t} = c \qquad \int_0^R \rho x\,dx = \frac{aR}{c} \cdot v_R + \frac{a}{c}\int_0^R \frac{\partial\rho}{\partial t}\, x\,dx$$

$$\int_0^R \left[\rho - \frac{a}{c}\frac{\partial\rho}{\partial t}\right] x\,dx = \frac{aR}{c} v_R$$

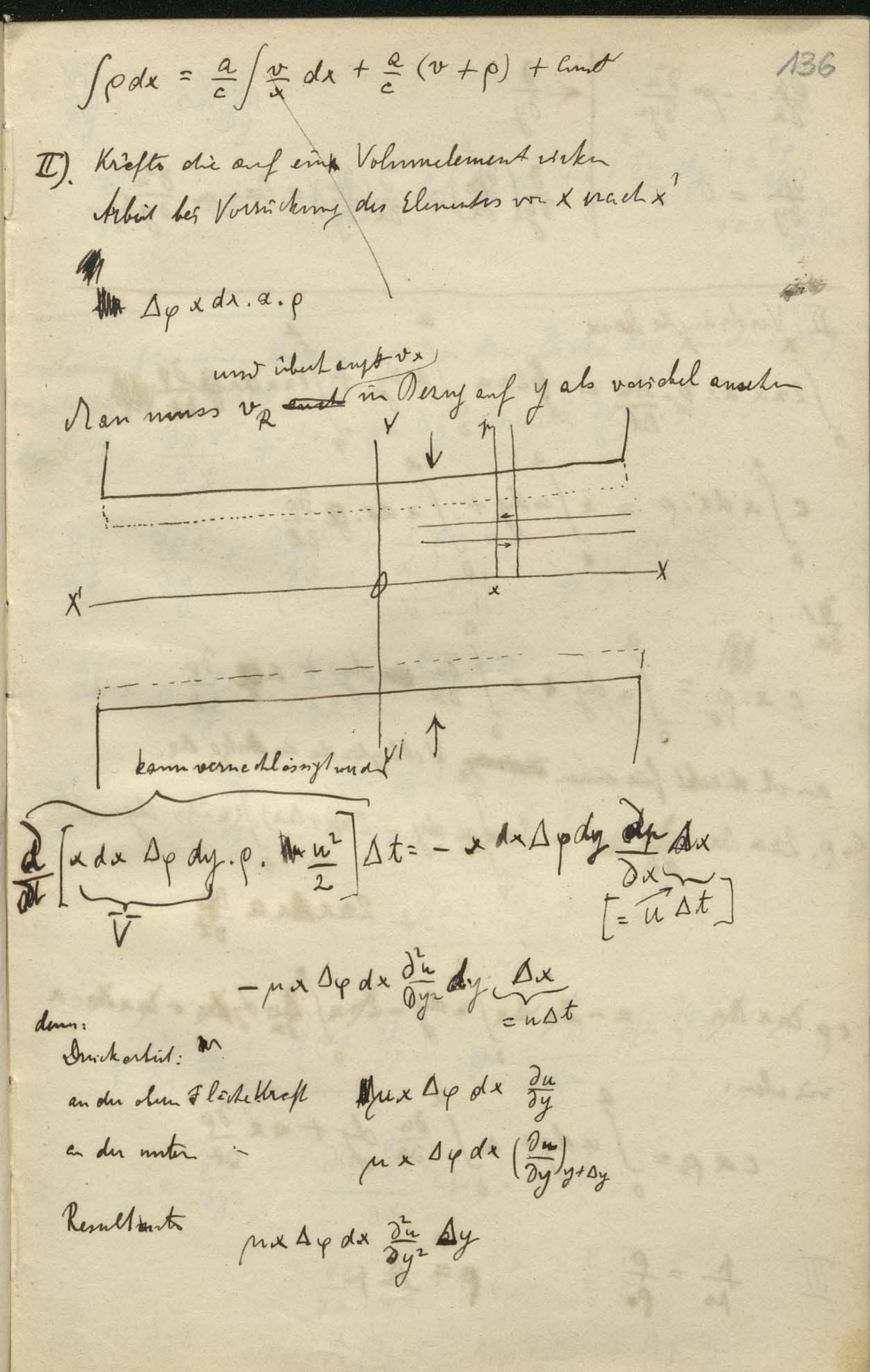
Ebenso für beliebigen Punkt x:

$$\int_0^x \rho x\,dx = \frac{ax}{c} v + \frac{a}{c}\int_0^x \frac{\partial\rho}{\partial x}\, x\,dx$$

$\frac{\partial}{\partial x}$:

$$\rho x = \frac{a}{c}\left(v + x\frac{\partial v}{\partial x}\right) + \frac{a}{c}\frac{\partial\rho}{\partial x}\cdot x$$

$$\rho = \frac{a}{c}\frac{v}{x} + \frac{a}{c}\frac{\partial v}{\partial x} + \frac{a}{c}\frac{\partial\rho}{\partial x}$$



$$\int \rho\, dx = \frac{a}{c}\int \frac{v}{x}\, dx + \frac{a}{c}(v+\rho) + \text{Const}$$

II). Kräfte die auf ein Volumelement wirken

Arbeit bei Vorrückung des Elementes von X nach X'

$\Delta\varphi\, x\, dx \cdot \alpha \cdot \rho$

Man muss v_R ~~einst~~ [und überhaupt v_x] in Bezug auf y als variabel ansehen

$$\frac{d}{dt}\Big[\underbrace{x\, dx\, \Delta\varphi\, dy}_{V} \cdot \rho \cdot \frac{u^2}{2}\Big]\Delta t = -\, x\, dx\, \Delta\varphi\, dy\, \frac{\partial p}{\partial x}\,\underbrace{\Delta x}_{[= u\,\Delta t]}$$

$$-\mu\, x\, \Delta\varphi\, dx\, \frac{\partial^2 u}{\partial y^2}\, dy\, \underbrace{\Delta x}_{= u\Delta t}$$

denn:

Druckarbeit:

an der oberen Fläche Kraft $\quad \mu\, x\, \Delta\varphi\, dx\, \frac{\partial u}{\partial y}$

an der unteren – $\quad \mu\, x\, \Delta\varphi\, dx \left(\frac{\partial u}{\partial y}\right)_{y+\Delta y}$

Resultante $\quad \mu\, x\, \Delta\varphi\, dx\, \frac{\partial^2 u}{\partial y^2}\, \Delta y$

$$\frac{\partial p}{\partial x} = -\mu \frac{\partial^2 u}{\partial y^2} \qquad \leftarrow \frac{\partial}{\partial y}$$

$$\frac{\partial p}{\partial y} = 0 \qquad \frac{\partial}{\partial y}\left(\frac{\partial p}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial p}{\partial y}\right) = 0 = -\mu \frac{\partial^3 u}{\partial y^3}$$

II Verdrängte Masse

$$\int_0^x 2\pi x \, dx \, \rho_0 \cdot \frac{\Delta a}{\Delta t} = 2\pi x \int_0^a u \, dy + \int_0^x 2\pi x \, dx \cdot a \frac{\partial \rho}{\partial t}$$

$$c \int_0^x x \, dx \cdot \rho = x \int_0^a u \, dy + \int_0^x x \, dx \cdot a \frac{\partial \rho}{\partial t}$$

$\frac{\partial}{\partial x}$:

$$c\, x \cdot \rho_0 = \int_0^a u \, dy + x \int_0^a \frac{\partial u}{\partial x} dy + x a \frac{\partial \rho}{\partial t}$$

auch direkt für einen Hohlcylinder der Dicke Δx:

$$c \cdot \rho \cdot 2\pi x \, \Delta x \cdot \frac{\Delta a}{\Delta t} = 2\pi x \int_0^a u \, dy - 2\pi(x + \Delta x) \int_0^a (u + \Delta u) \, dy + 2\pi x \Delta x \cdot a \frac{\partial \rho}{\partial t}$$

$$c \rho \, 2\pi x \, \Delta x = -2\pi \Delta x \int_0^a u \, dy - 2\pi x \int_0^a \frac{\partial u}{\partial x} dy \, \Delta x + 2\pi x \Delta x \, a \frac{\partial \rho}{\partial t}$$

wie oben:

$$c x \rho_0 = \int_0^a u \, dy + x \int_0^a \frac{\partial u}{\partial x} dy + a x \frac{\partial \rho}{\partial t}$$

III $\frac{p}{p_0} = \frac{\rho}{\rho_0} \qquad \rho = A p$

~~$\frac{\partial}{\partial y}$~~: $c\,x\,\frac{\partial}{\partial y}$ =

$$\frac{\partial \Pi}{\partial x}: \quad c\rho + c\,x\frac{\partial \rho}{\partial x} = 2\int_0^a \frac{\partial u}{\partial x}\,dy + x\int_0^a \frac{\partial^2 u}{\partial x^2}\,dy + a\frac{\partial \rho}{\partial t} + a\,x\,\frac{\partial^2(\rho\,x)}{\partial t\,\partial x}$$

---

ebenso für einen Hohlzylinder von der Dicke Δy in der Höhe y:

$$2\pi x\,\Delta x\,\Delta y)\;\; 2\pi x\,\Delta x\,\Delta y \quad \left(\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial y}\right) + -2\pi x\,\rho u\,\Delta y - 2\pi(x+\Delta x)\left[\rho u + \frac{\partial(\rho u)}{\partial x}\Delta x\right]\Delta y = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial y} + \underbrace{\frac{\partial(\rho u)}{\partial x} + \frac{1}{x}\rho u}_{=} = 0 \quad \rightarrow \frac{1}{x}\frac{\partial(\rho u x)}{\partial x}$$

$$\frac{\partial \rho}{\partial t} + \rho\frac{\partial v}{\partial y} + \frac{\partial(\rho u)}{\partial x} + \frac{\rho u}{x} = 0$$

$$\frac{\partial^2}{\partial y^2}: \quad \rho\frac{\partial^3 v}{\partial y^3} + \underbrace{\frac{\partial\left(\rho\frac{\partial^2 u}{\partial y^2}\right)}{\partial x} + \frac{\rho}{x}\frac{\partial^2 u}{\partial y^2}}_{\frac{1}{x}\frac{\partial\left[\rho x \frac{\partial^2 u}{\partial y^2}\right]}{\partial x}} = 0$$

$$\rho\frac{\partial^3 v}{\partial y^3} \pm \frac{1}{x\mu}\frac{\partial\left[\rho x\frac{\partial p}{\partial x}\right]}{\partial x} = \frac{A}{2x\mu}\frac{\partial\left[x\frac{\partial(p^2)}{\partial x}\right]}{\partial x}$$

$$\mu\rho\frac{\partial^3 v}{\partial y^3} = \frac{\partial\left(\rho\frac{\partial p}{\partial x}\right)}{\partial x} + \frac{1}{x}\rho\frac{\partial p}{\partial x}$$

$$= \frac{\partial \rho}{\partial x}\frac{\partial p}{\partial x} + \rho\frac{\partial^2 p}{\partial x^2} + \frac{\rho}{x}\frac{\partial p}{\partial x}$$

$$= A\left(\frac{\partial p}{\partial x}\right)^2 + A\,p\,\frac{\partial^2 p}{\partial x^2} + A\frac{p}{x}\frac{\partial p}{\partial x}$$

$$\frac{\partial^4 v}{\partial y^4} = 0 \qquad v = c_0 + c_1 y + c_2 y^2 + c_3 y^3$$

$$c = f(x, t)$$

$$\frac{\partial^3 v}{\partial y^3} = \text{const}_y = c_3$$

$$\frac{\partial^3 v}{\partial y^3} \rho = -\frac{1}{x} \cdot \frac{\partial \left(\rho x \frac{\partial^2 u}{\partial y^2}\right)}{\partial x}$$

$$\frac{\partial^3 v}{\partial y^3} \rho x = -\frac{\partial}{\partial x}\left[\rho x \frac{\partial^2 u}{\partial y^2}\right] = \frac{1}{\mu} \frac{\partial}{\partial x}\left[\rho x \frac{\partial p}{\partial x}\right]$$

~~$p A c_3 p x =$~~

$$\mu a_3 p x = \frac{\partial}{\partial x}\left[p x \frac{\partial p}{\partial x}\right] = \frac{\partial (p x)}{\partial x} \frac{\partial p}{\partial x} + p x \frac{\partial^2 p}{\partial x^2}$$

$$\frac{\partial^2 p}{\partial x^2} + \frac{1}{p x} \frac{\partial (p x)}{\partial x} \frac{\partial p}{\partial x} = \mu a_3$$

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial [\log (p x)]}{\partial x} \frac{\partial p}{\partial x} = \mu a_3$$

$$\frac{\frac{\partial^2 p}{\partial x^2}}{\frac{\partial p}{\partial x}} \partial x + \frac{\partial \log (p x)}{\partial x} \partial x = \frac{\mu a_3}{\frac{\partial p}{\partial x}} \partial x$$

$$\frac{\partial \left(\log \frac{\partial p}{\partial x}\right)}{\partial x}$$

$$\frac{\partial \log \left(p x \frac{\partial p}{\partial x}\right)}{\partial x} \cdot \frac{\partial p}{\partial x} = \mu \frac{\partial^3 v}{\partial y^3}$$

$u = a_0 + a_1 y + a_2 y^2 + \ldots$ $\qquad a = fc(x\,t)$

$p = f(x, t)$

$\frac{\partial p}{\partial x} = -2\mu a_2$ falsch! $\quad p = b_0 - 2\mu \int a_2 dx$

$b = fc(t)$

~~$p = b_0 - 2a_2\mu x + \ldots$~~

für $x = R$ bleibt p = constant $= p_0$

$p_0 = (b_0) - 2(a_2)\mu R + \ldots$

$(b_0) = p_0 + 2(a_2)\mu R$

~~auch in der Form:~~

~~$\frac{\partial p}{\partial t} + p\frac{\partial v}{\partial y} + \frac{1}{x}\frac{\partial(\mu u x)}{\partial x} = 0$~~

~~$\frac{\partial^2 p}{\partial x^2} = -2\mu \frac{\partial a_2}{\partial x} = -\mu$~~

für $x = R$

$p = p_0 + 2(a_2)\mu(R - x)$

$= fc(t)$

~~in der Form:~~

~~$\frac{\partial p}{\partial t} + p\frac{\partial v}{\partial y} + \frac{1}{x}\frac{\partial(\mu u x)}{\partial x} = 0$~~

$u = 0$ für alle t und $x = 0$

$y = a = E - ct$

$$0 = a_0 + a_1(E - ct) + a_2(E - ct)^2$$

$$= a_0 + a_1 E + a_2 E^2 - [a_1 + 2a_2 E]ct + a_2 c^2 t^2$$

---

für $y = 0$ wird u ein relat. Maximum haben also $\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0$ und für $\begin{matrix} y=0 \\ x=R \end{matrix}$ ein absolutes; dann muss also auch $\left.\frac{\partial u}{\partial x}\right|_{\substack{x=R \\ y=0}} = 0$ sein also: $\left.\frac{\partial a_0}{\partial x}\right|_{x=R} = 0$

aus Verdrängter Masse:

$$A c x p = \int_0^a u\,dy + x \int_0^a \frac{\partial u}{\partial x} dy + A a x \frac{\partial p}{\partial t}$$

~~$A c x [p_0 + 2 a_2 \mu (R - x)]$~~ $= a_0 a + a_1 \frac{a^2}{2} + a_2 \frac{a^3}{3}$

$$A c x \left[b_0 - 2 \mu \int a_2 dx \right] \quad + x \left(\frac{\partial a_0}{\partial x} a + \frac{\partial a_1}{\partial x} \frac{a^2}{2} + \frac{\partial a_2}{\partial x} \frac{a^3}{3} \right)$$

~~$+ A a x 2 \mu (R - x) \frac{\partial a_2}{\partial t}$~~

$$+ A a x \left[\frac{\partial b_0}{\partial t} - 2 \mu \int \frac{\partial a_2}{\partial t} dx \right]$$

$$\frac{\partial p}{\partial t} + p \frac{\partial v}{\partial y} + \frac{1}{x} \frac{\partial (p u x)}{\partial x} = 0 \qquad \text{(IV)}$$

$$\frac{\partial}{\partial y} =$$

$$\frac{\partial}{\partial t}\left(\frac{\partial p}{\partial y}\right) + \frac{\partial p}{\partial y} \frac{\partial v}{\partial y} + p \frac{\partial^2 v}{\partial y^2} + \frac{1}{x} \frac{\partial}{\partial x} \left\{ x \frac{\partial (p u)}{\partial y} \right\} = 0$$

$$\frac{\partial^2 (p u)}{\partial x \partial y} + \frac{1}{x} \frac{\partial (p u)}{\partial y} = 0$$

$$\frac{\partial}{\partial x} \left\{ u \frac{\partial p}{\partial y} + p \frac{\partial u}{\partial y} \right\} + \frac{1}{x} \left\{ u \frac{\partial p}{\partial y} + p \frac{\partial u}{\partial y} \right\} = 0$$

$$p \frac{\partial^2 v}{\partial y^2} + \frac{p}{x} \frac{\partial u}{\partial y} + \frac{\partial}{\partial x} \left(p \frac{\partial u}{\partial y} \right) = 0$$

$$\frac{\partial p}{\partial x} \frac{\partial u}{\partial y} + p \frac{\partial^2 u}{\partial x \partial y}$$

$$\frac{1}{x} \frac{\partial}{\partial x} \left(x p \frac{\partial u}{\partial y} \right)$$

$$\mu \frac{\partial^2 v}{\partial y^2} + \frac{1}{x} \frac{\partial}{\partial x}\left\{\mu x \frac{\partial u}{\partial y}\right\} = 0$$

$$\mu x \frac{\partial^2 v}{\partial y^2} + \frac{\partial}{\partial x}\left\{\mu x \frac{\partial u}{\partial y}\right\} = 0$$

$$\mu x \frac{\partial^2 v}{\partial y^2} + \mu \frac{\partial}{\partial x}\left\{x \frac{\partial u}{\partial y}\right\} - \mu x \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} = 0$$

$$" + \mu x \frac{\partial^2 u}{\partial y \partial x} + \mu \frac{\partial u}{\partial y} + x \frac{\partial \mu}{\partial x} \frac{\partial u}{\partial y} = 0$$

$$\mu x \frac{\partial}{\partial y}\left\{\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x}\right\} + \frac{\partial u}{\partial y} \frac{\partial}{\partial x}(\mu x) = 0$$

$$\frac{\partial}{\partial y}\left\{\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x}\right\} + \frac{\partial u}{\partial y} \frac{\partial \log(\mu x)}{\partial x} = 0$$

$$\frac{\partial}{\partial y}\left\{\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x}\right\} + \frac{\partial}{\partial y}\left\{u \frac{\partial \log(\mu x)}{\partial x}\right\} = 0$$

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} + u \frac{\partial \log \mu x}{\partial x} = \text{const}_y = f_c(x, t)$$

selbstverständlich! nach (III) $= \frac{1}{\mu} \frac{\partial \mu}{\partial t}$

$$\frac{\partial^2 v}{\partial y^2} = 2 c_2 + 2 \cdot 3 \cdot y\, c_3$$

~~$$\mu x (2 c_2 + 6 c_3 y) = -\frac{\partial}{\partial x}\left(\mu x \frac{\partial u}{\partial y}\right)$$~~

$$\mu x \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial y^2} + \frac{\partial u}{\partial y} \frac{\partial}{\partial x}\left\{x \mu \frac{\partial u}{\partial y}\right) = 0$$

$$\frac{\partial^2 v}{\partial y^2} + \frac{\partial u}{\partial y} \frac{\partial}{\partial x}\left\{\log\left(\mu x \frac{\partial u}{\partial y}\right)\right\} = 0$$

$$a\int_{-\infty}^{+\infty} e^{-\mu x^2}dx = 1 = \frac{a}{\sqrt{\mu}}\int_{-\infty}^{+\infty} e^{-\mu x^2}dx\sqrt{\mu} = a\sqrt{\frac{\pi}{\mu}} = 1$$

$$a = \sqrt{\frac{\mu}{\pi}}$$

$$\sqrt{\frac{\mu}{\pi}}\int_0^x e^{-\mu x^2}dx = \sqrt{\mu} \ldots 0.5 x$$

$$= \frac{1}{\sqrt{\pi}}\int_0^{\frac{y}{\sqrt{\mu}}} e^{-y^2}dy \qquad x = \frac{y}{\sqrt{\mu}}$$

$$\frac{x}{l_{50}} = y$$

$$l_{50}\sqrt{\frac{\mu}{\pi}}\int_0^{l_{50}\cdot y} e^{-\mu y^2 l_{50}^2} \qquad dy = \ldots \qquad \mu = f(l_{50})$$

$$2\int y = 1$$

$$= 0.5 \qquad l_{50} = 2.0845 \int_0$$

$$l_{50}\sqrt{\frac{\mu}{\pi}}\int_0^{l_{50}} e^{-\mu l_{50}^2 y^2}dy = \frac{1}{2}$$

$$y\, l_{50} = x$$

$$\int_0^1 e^{-\mu x^2}$$

$$\sqrt{\frac{\mu}{\pi}}\int_0^{\frac{l_{50}}{2}} e^{-\mu x^2}dx = \frac{1}{4} = \frac{1}{\sqrt{\pi}}\int_0^{\sqrt{\mu}\cdot\frac{l_{50}}{2}} e^{-y^2}dy$$

$$\mu = f(l_{50})^2$$

$$l_{50} = \frac{0.6\ldots}{\sqrt{\mu}} \qquad \mu = \frac{0.4546}{l_{50}^2}$$

$$\frac{\Sigma x N}{\Sigma N} = \xi$$

$$\frac{1}{\sqrt{\mu}}$$

$$\int_0^a e^{-x^2} dx =$$

$$\frac{x}{a-x} = y$$

$$x = \frac{ay}{1+ay}$$

$$a\int_0^\infty e^{-\left(\frac{ay}{1+y}\right)^2} d\left(\frac{y}{1+y}\right)$$

$$\xi = \frac{\Sigma nx}{\Sigma n} = \frac{\int_0^\infty x e^{-\mu x^2} dx}{\int_0^\infty e^{-\mu x^2} dx}$$

$$l_{50} = 2 . \xi . 0{\cdot}845$$

$$x\sqrt{\mu} = y$$

$$= \frac{\int_0^\infty y e^{-y^2} dy}{\sqrt{\mu}\int_0^\infty e^{-y^2} dy}$$

$$= \frac{-1}{2\sqrt{\mu}} \frac{e^{-y^2}\Big|_0^\infty}{\frac{1}{2}\sqrt{\pi}} = \frac{1}{\sqrt{\pi\mu}}$$

$$l_{50} = \frac{1{\cdot}69}{\sqrt{\pi\mu}}$$

$$\frac{1}{\sqrt{\pi}} = 0{\cdot}56419$$

0·82880 -1
1·65760

$$\frac{0{\cdot}6742}{\sqrt{\mu}} = l_{50}$$

~~0.455~~ 0.674

Prob: $\frac{1}{2} = \frac{1}{\sqrt{\pi}} \int_0 e^{-y^2} dy$

~~$\int 1 - \frac{y^2}{1} + \frac{y^4}{1.2} - \frac{y^6}{1.2.3} +$~~

~~$y - \frac{y^3}{3} + \frac{y^5}{1.2.5} - \frac{y^7}{1.2.3.7} + \frac{y^9}{1.2.3.4.9}$~~

~~0.65760 − 1 log 0.455~~

$\frac{93}{16.6.11}$

$\frac{3}{110}$ 40

0.97280 − 2   0.09393 : 3

0.28800 − 2   0.01941 : 10

0.60320 − 3   0.00401 : 6.7

0.91840 − 4   0.00083 : 6.4.9

$\frac{24.9}{216}$

0.45457	0.03131
0.00194	0.00009
0.45651	0.03140
− 0.03140	
0.42511	

$\sqrt{\pi} = 1.77245$

$\frac{\sqrt{\pi}}{2} = 0.88622$

0.166666
0.041666
0.0046295

1
0.1
0.0046
0.0001

1.1047

0.3333
0.0238

35
0.85

0.670

$\int_0^x e^x = 1 + x + \frac{x^2}{2} + \dots$

$= e^x - 1$

0.3371

Rest: $\frac{\sqrt{\pi}}{4} = \int_0$

0.22789
−0.24857
0.07932
05864

09016

0.05350

$l_{50} = \frac{0.05350}{\sqrt{\gamma}}$

$\gamma = \frac{0.09016}{l_{50}^2}$

2 %	1	1'75	4
7 %	1'5		10'5
16 %	1		16
25 %	0'5		12'5

1	. $\frac{15}{8}$	15
1	. $\frac{13}{8}$	13
$\frac{5}{2}$	. $\frac{11}{8}$	27'5
$\frac{9}{2}$	. $\frac{9}{8}$	40'5
7	. $\frac{7}{8}$	49
9	. $\frac{5}{8}$	45
12	. $\frac{3}{8}$	36
13	. $\frac{1}{8}$	13
50 (l_{50})		239'0 : 8 = 30

7 9

$\frac{3}{5} l_{50} = \frac{6}{5} e_{50}$

$e_{50} = \frac{5}{6} m = 0'83$

Probe: 0·47675

$$\frac{\sqrt{\pi}}{4} = \int_0^{0\cdot1084} e^{-y^2} dy$$

0·67830 - 1
0·03490 - 1
0·39150 - 2
0·74810 - 7

0·002463
0·000559
:42

0·47675
0·00246
—
0·47921
-0·0362
—
0·4430

- 0·0361 -
0·0001
—
0·0362

= 0·4431 stimmt!

1 kg A | 1 kg B = Molecularge[w]

n_1 | n_2 = Z. d. Atome

N_1 | N_2 = Z. d. Molecule

$\nu_1 = \frac{n_1}{N_1}$ | $\nu_2 = \frac{n_2}{N_2}$ = Z. d. At. i. 1 Molec.

a | b = Atomgewicht

$a n_1 = 1 = b n_2$ = 1 [illegible]

c_1 | c_2 = sp. W. = W. z. [illegible] [illegible] 1° [illegible]

$a c_1 = b c_2$ (Dulong Petit)

$\frac{c_1}{n_1} = \frac{c_2}{n_2}$ (u = [illegible] Atome)

$$c_1 = J \frac{n_1\, a\Delta(u_1^2)}{2} = J\frac{\Delta(u_1^2)}{2} \qquad c_2 = J\frac{n_2\, b\Delta(u_2^2)}{2} = J\frac{\Delta(u_2^2)}{2}$$

$$\frac{c_1}{n_1} = J\frac{a\Delta(u_1^2)}{2} \qquad \frac{c_2}{n_2} = J\frac{b\Delta(u_2^2)}{2}$$

wenn gleiche Temp. so $\frac{a\Delta(u_1^2)}{2} = \frac{b\Delta(u_2^2)}{2}$

Besteht Gleichheit der Temp. darin, dass die mittl. kinetische Energie der Atome oder der Molecüle gleich ist?

Glaube das letztere.

Dann ist Beweis in Müller-P. und Winkelmann falsch!

$v =$ Geschw. d. ~~Atome~~ Molecüle

Aus Temp. Gleichheit:

$$\frac{A\Delta(v_1^2)}{2} = \frac{B\Delta(v_2^2)}{2}$$

$$A = \frac{n_1 a}{N_1} \qquad B = \frac{n_2 b}{N_2}$$

$$\frac{\overbrace{n_1 a}^{=1}\Delta(v_1^2)}{2N_1} = \frac{\overbrace{n_2 b}^{=1}\Delta(v_2^2)}{2N_2} \qquad \frac{\Delta(v_1^2)}{N_1} = \frac{\Delta(v_2^2)}{N_2}$$

$$c_1 = J\,\frac{N_1 \cdot A\Delta(v_1^2)}{2} \qquad c_2 = J\,\frac{N_2\, B\Delta(v_2^2)}{2}$$

Dulong Petit:

$$\frac{c_1}{n_1} = \frac{c_2}{n_2}$$

$$J\,\frac{\overbrace{N_1 A}^{=1}\Delta(v_1^2)}{2\,n_1} = J\,\frac{\overbrace{N_2 B}^{=1}\Delta(v_2^2)}{2\,n_2} \qquad \frac{\Delta(v_1^2)}{n_1} = \frac{\Delta(v_2^2)}{n_2}$$

~~$A = n_1 a$~~

~~$\frac{n_1 a v_1^2}{n_1} = \frac{n_2 b v_2^2}{n_2}$~~

$$\downarrow$$

$$\frac{n_1}{N_1} = \frac{n_2}{N_2}$$

also würde bei genauer Giltigkeit des Dulong-Petit'schen Gesetzes folgen, dass die Molecüle (aller Körper bei derselben Temperatur) aus gleichviel Atomen zusammengesetzt sind! wenn man dabei die innere Arbeit $= 0$ voraussetzt!

$$a c_1 : b c_2 = a\,\Delta(v_1^2) : b\,\Delta(v_2^2)$$

$$= a N_1 \underbrace{\frac{\Delta(v_1^2)}{N_1}} : b N_2 \underbrace{\frac{\Delta(v_2^2)}{N_2}}$$

$$= a N_1 : b N_2$$

$$= \frac{N_1}{n_1} : \frac{N_2}{n_2} = \nu_1 : \nu_2$$

Also geben die Verhältnisse der Atomwärmen bei Voraussetzung einer inneren Arbeit = 0 zugleich die Verhältnisse der Zusammensetzung der Moleküle aus Atomen!

Dies alles bei Voraussetzung von gleichartigen Atomen

wenn wir jetzt voraussetzen

A	besteht aus	ν_1	Atomen mit Atomgew.	a_1
	"	ν_2	"	a_2
B		μ_1	"	b_1
		μ_2	"	b_2

so folgt ~~Temperaturgleichheit für~~

$$A = \nu_1 a_1 + \nu_2 a_2 \qquad B = \mu_1 b_1 + \mu_2 b_2$$

Temp. Gleichheit:

$$A\,\Delta(v_1^2) = B\,\Delta(v_2^2)$$

$$c_1 = J\,\frac{A N_1 \Delta(v_1^2)}{2} \qquad c_2 = J\,\frac{B N_2 \Delta(v_2^2)}{2}$$

$$= \frac{J}{2}\,\Delta(v_1^2) \qquad = \frac{J}{2}\,\frac{\Delta(v_2^2)}{2}$$

so folgt: $A c_1 = B c_2$ } = überhaupt allgemeinster Ausdruck des Dulong'-Petit'-und Neumann' Ges.

~~$(a_1 v_1 + a_2 v_2)\,c_1 = (b_1 \mu_1 + b_2 \mu_2)\,c_2$~~

~~da bekannt ist: $(a_1 v_1 + a_2 v_2) c_1 = a_1 c_2 (\ldots)$~~

~~2 B.: $(Q_1 V_1 + Q_2 V_2)\,C_3 = Q_1 \mu_1 c_1$~~

~~$= Q_2 \mu_2 c_2$~~

~~$$C_3 = \frac{Q_1 \mu_1 c_1}{Q_1 V_1 + Q_2 V_2} = \frac{Q_2 \mu_2 c_2}{Q_1 V_1 + Q_2 V_2}$$~~

es würde also aus der Annahme

1). Gleichheit der Temp. zweier Körper besteht darin, dass die kinetische Energie der Molecüle gleich ist

2). Innere Arbeit = 0 also spec. Wärme = Arbeit zur Vermehrung der kinetischen Energie der Molecüle

folgen, dass die Molecular-Wärme aller Körper gleich ist; dies nicht der Fall, also falsch!

Annahme 1 ist richtig denn vd. Avogadro'schen Gesetz etc.

also Annahme 2 zu corrigiren.

$$c_1 = J\,\frac{N_1 A \Delta(v_1^2)}{2} + J N_1 \frac{A \Delta(v_1)^2}{2} P_1$$

$$c_2 = J\,\frac{N_2 B \Delta(v_2^2)}{2} + J N_2 A(U_2)$$

P_1 = [illegible] pro 1 Molecül

Temperatur Gleichheit erfordert:

$$A\,\Delta(v_1^2) = B\,\Delta(v_2^2)$$

~~Dulong Petit: $a c_1 = b c_2$~~

$$A\,\Delta(v_1^2)\left(J N_1 + J N_1 P_1\right) = c_1$$

$$\frac{c_1}{N_1(1+P_1)} = \frac{c_2}{N_2(1+P_2)}$$

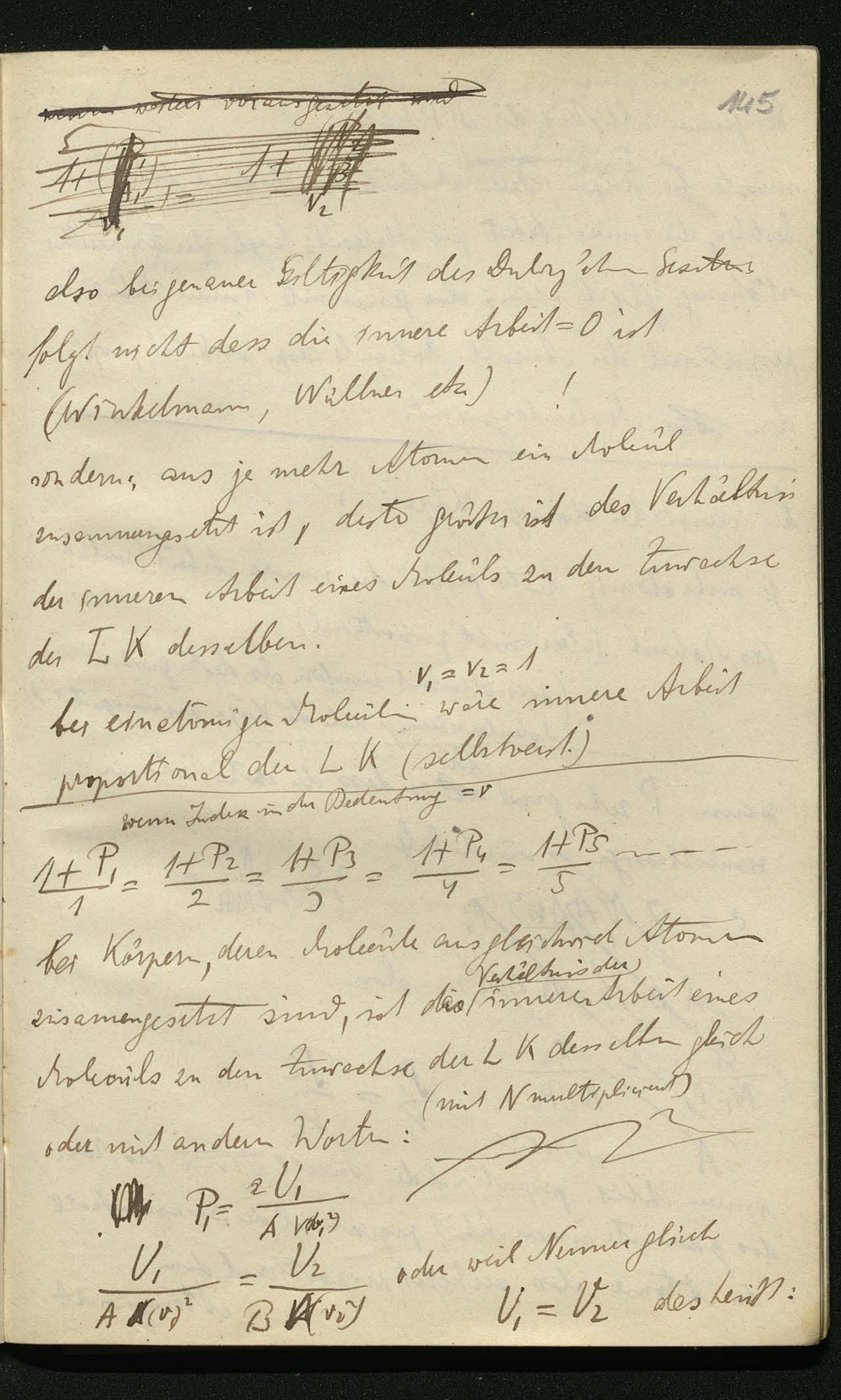
$$\frac{c_1}{N_1(1+P_1)} = \frac{c_2}{N_2(1+P_2)}$$

Dulong Petit: $a c_1 = b c_2$

~~$a : N_1(1+N_1 P_1) = b :$~~

$$a N_1(1+P_1) = b N_2(1+P_2)$$

$$\frac{1+P_1}{v_1} = \frac{1+P_2}{v_2}$$



~~wenn weiter vorausgesetzt wird~~

~~[illegible]~~

also bei genauer Gültigkeit des Dulong'schen Gesetzes folgt nicht dass die innere Arbeit = 0 ist (Winkelmann, Wüllner etc.) !

sondern, aus je mehr Atomen ein Molekül zusammengesetzt ist, desto grösser ist das Verhältnis der inneren Arbeit eines Moleküls zu den Zuwachse der L K desselben.

$v_1 = v_2 = 1$

bei einatomigen Molekülen wäre innere Arbeit proportional der L K (selbstverst.)

wenn Index in der Bedeutung $= v$

$$\frac{1+P_1}{1} = \frac{1+P_2}{2} = \frac{1+P_3}{3} = \frac{1+P_4}{4} = \frac{1+P_5}{5} \ldots$$

bei Körpern, deren Moleküle aus gleich viel Atomen zusammengesetzt sind, ist das Verhältnis der inneren Arbeit eines Moleküls zu den Zuwachse der L K desselben gleich (mit N multipliziert)

oder mit anderen Worten:

$$P_1 = \frac{2 V_1}{A\, (v_1)^2}$$

$$\frac{V_1}{A\,(v_1)^2} = \frac{V_2}{B\,(v_2)^2}$$

~~[illegible]~~ oder weil Nenner gleich $V_1 = V_2$ das heisst:

bei genauer Giltigkeit des D P Gesetzes

müsste bei Körpern, deren Molecüle aus gleichviel Atomen bestehen, die innere Arbeit pro Molecül bei gleicher Temperaturerhöhung gleich sein, also gesammte innere Arbeit = proportional der Anzahl Molecüle resp. verkehrt prop. dem ~~Ato~~ Moleculargewicht.

---

bei verschiedenatomigen Körpern:

je mehratomig desto größer die innere Arbeit (weil $k=\frac{1}{2}$) pro Molecül (aber nicht proportional)

(dies stimmt insofern als desto größer Molecularwärme je mehr der Körper zusammengesetzt ist)

---

wenn P sehr groß ~~ist~~ wäre, so dass der erste Posten zu vernachlässigen so würde folgen:

$$\frac{a\, c_1}{b\, c_2} = \frac{a N_1 P_1}{b N_2 P_2} =$$

$$c_1 = J N_1 A \frac{\Delta (v_1^2)}{2} P_1$$

$$A\, \Delta(v_1^2) = B\, \Delta(v_2^2) \quad \text{also}$$

$$\frac{c_1}{N_1 P_1} = \frac{c_2}{N_2 P_2} \qquad \frac{P_1}{V_1} = \frac{P_2}{V_2}$$

A pro Molecül

innere Arbeit proportional der Anzahl Atome pro Molec.

also gesammte innere Arbeit proportional der Gesammtzahl der Atome also verkehrt proportional dem Atomgewicht

weil darum aber spec. Wärme = Äquiv. nur für die innere

wie ist dies bei zusammengesetzten Körpern

Joulesches Gesetz ?

a

b R

$$V = \int_a \int_{r=0} \mu \frac{2 r \pi \, dr \, dx}{(r^2 + x^2)^{\frac{n}{2}}}$$

~~$V = \mu\pi \sqrt{r^2+x^2}$~~

~~$V = \mu\pi \frac{}{(r^2+x^2)^{\frac{n}{2}-1}}$~~

b

$$V = \int_a^b \mu\pi \frac{(r^2+x^2)^{1-\frac{n}{2}}}{1-\frac{n}{2}} \Bigg|_0^R dx =$$

$$= \frac{2\mu\pi}{2-n} \int_a^b \left\{ \frac{R^2+x^2}{(R^2+x^2)^{\frac{n}{2}}} - x^{2-n} \right\} dx$$

$$\int \frac{R^2+x^2}{(R^2+x^2)^{\frac{n}{2}}} dx = (R^2+x^2)^{1-\frac{n}{2}} x - \int \frac{x \left(1-\frac{n}{2}\right) 2x \, dx}{(R^2+x^2)^{\frac{n}{2}}}$$

$$R^2 \underbrace{\int \frac{dx}{(R^2+x^2)^{\frac{n}{2}}}}_{J_1} + \underbrace{\int \frac{x^2 dx}{--}}_{J_2} = (R^2+x^2)^{\frac{2-n}{2}} x - (2-n) \int \frac{x^2 dx}{---}$$

$$J_1 = \frac{(R^2+x^2)^{\frac{2-n}{2}}}{R^2} x - \frac{(3-n)}{R^2} J_2$$

$$\int \frac{dx}{(R^2+x^2)^{\frac{n}{2}}} = \frac{x}{(R^2+x^2)^{\frac{n}{2}}} + n\int \frac{x^2\,dx}{(R^2+x^2)^{\frac{n}{2}+1}}$$

$$-n\int \frac{dx}{\dots} = -\int \frac{n\,dx}{(R^2+x^2)^{\frac{n}{2}}}$$

$$(1-n)\,J_1 = \frac{x}{(R^2+x^2)^{\frac{n}{2}}} + n\int \frac{dx}{(R^2+x^2)^{\frac{n}{2}}}\,\frac{x^2 - R^2}{R^2+x^2}$$

$$= \frac{x}{(R^2+x^2)^{\frac{n}{2}}} - nR^2\int \frac{dx}{(R^2+x^2)^{\frac{n}{2}+1}}$$

$n = m-2$

$$(1-n+2)\underbrace{\int \frac{dx}{(R^2+x^2)^{\frac{n-2}{2}}}}_{J} = \frac{x}{(R^2+x^2)^{\frac{n-2}{2}}} - (n-2)R^2\underbrace{\int\frac{dx}{(R^2+x^2)^{\frac{n}{2}}}}_{J_1}$$

$$J = R^2 J_1 + J_2 = (R^2+x^2)^{\frac{2-n}{2}}x - (3-n)J_2$$

$$(3-n)J = \frac{x}{(R^2+x^2)^{\frac{n-2}{2}}} - (n-2)R^2 J_1$$

$$J = \frac{x}{(R^2+x^2)^{\frac{n-2}{2}}} - (n-2)(R^2+x^2)^{\frac{2-n}{2}}x - (n-2)(3-n)J_2$$

$$(3-n)^2 J_2 = (R^2+x^2)^{\frac{2-n}{2}}x\,(n-4) + \dots$$

$$J_2 = \quad x(R^2+x^2)^{\frac{2-n}{2}}$$

$$\frac{dJ_2}{dx} = (R^2+x^2)^{\frac{2-n}{2}} + (2-n)x^2(R^2+x^2)^{-\frac{n}{2}}$$

$$= (R^2+x^2)^{-\frac{n}{2}}\left[R^2 + (3-n)x^2\right]$$

Voraussetzung $n > 2$

V_1 V_2

$a V_1 v_1^2 = b V_2 v_2^2$

$a u_1^2 = b u_2^2$

~~$c = J \Delta (a V_1 v_1^2 + a u_1^2)$~~

~~$= J a V_1 v_1^2 (1 + \lambda)$~~

$$\frac{u_1^2}{V_1 v_1^2} = \frac{u_2^2}{V_2 v_2^2} = \lambda = 1$$

$u_1^2 = \lambda V_1 v_1^2$

$$c_1 = J N_1 \Delta (a V_1 v_1^2) + J N_1 V_1 \Delta (a u_1^2)$$

$$= J N_1 \Delta (a V_1 v_1^2) [1 + \lambda V_1] = J N_1 \Delta (a V_1 v_1^2)(1 + V_1)$$

$$\frac{c_1}{N_1 (1 + V_1)} = \frac{c_2}{N_2 (1 + V_2)}$$

$P_1 = V_1$

$$\frac{1 + V_1}{V_1} = \frac{1 + V_2}{V_2}$$

spec. Vol.

I). $\xrightarrow{\quad V \quad}$ $\overset{M}{\bigcirc}$

II). $\xrightarrow{\quad V \quad}$ $\circ \cdot \circ$ $m = \frac{M}{2}$, v

$$E_I = M\frac{V^2}{2}$$

$$E_{II} = \frac{m}{2}\left[(V+v)^2 + (V-v)^2\right]$$

$$= \frac{M}{2}\left[V^2 + v^2\right] = E_I + 2 \cdot \frac{m v^2}{2}$$

$$E_n = \frac{m}{2}\sum\left[(V \pm v_n)^2\right]$$

$$\sum v_n = 0$$

$$= \frac{m}{2}\left(\sum V^2 + \sum v_n^2\right)$$

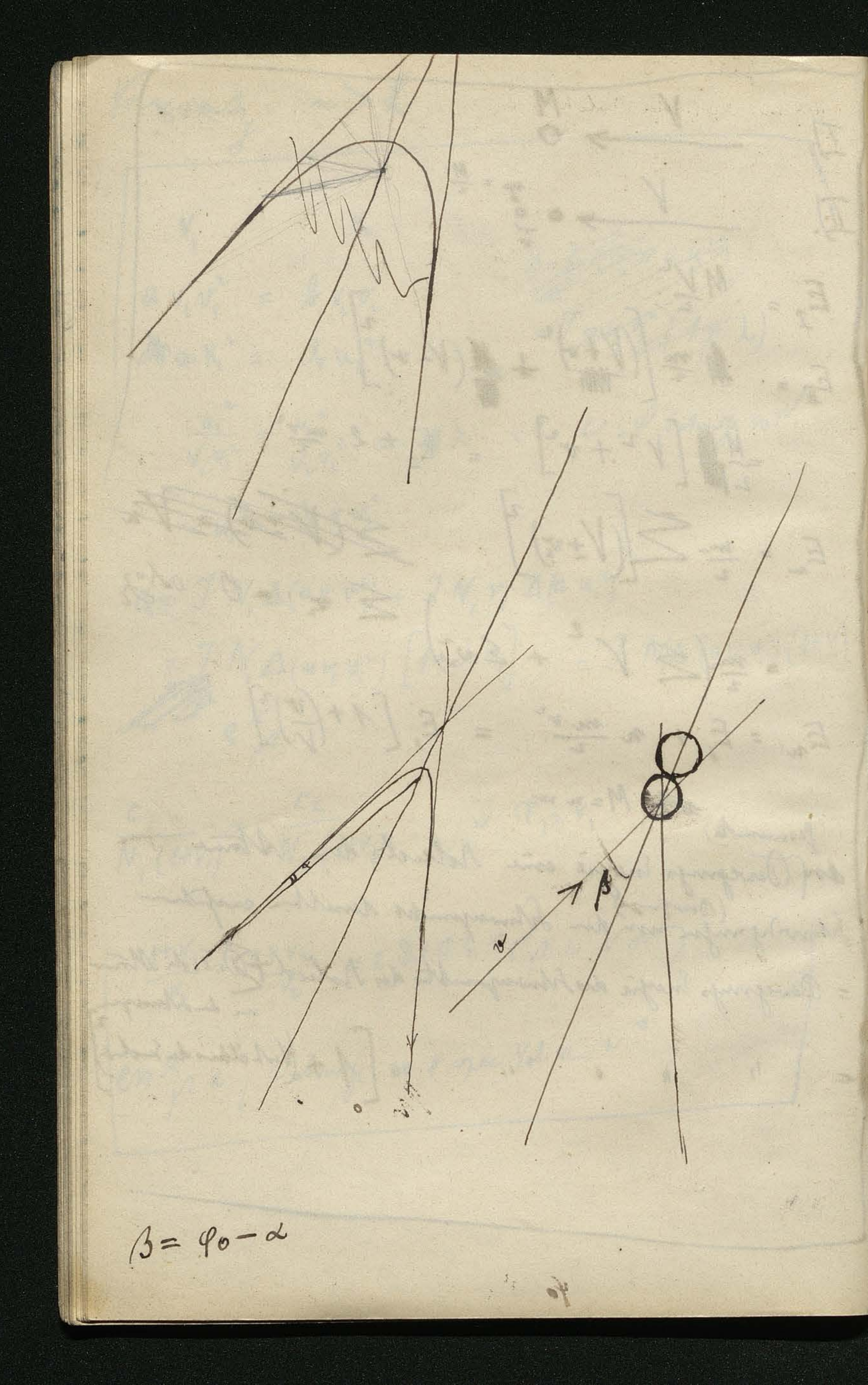
$$E_n = E_1 + n\frac{m v^2}{2} = E_1\left[1 + \left(\frac{v}{V}\right)^2\right]$$

$$M = n\,m$$

Die (gesammte) Bewegungs Energie eines Molecul, dessen Atome (Bewegungs) Schwingungen um den Schwerpunkt desselben ausführen

= Bewegungs Energie des Schwerpunktes des Molecul $+ \sum$ B. E. der Atome um den Schwerpunkt

= " " " " " $\left[1 + (\text{Verhältnis der Geschwind.})^2\right]$



$\beta = \varphi_0 - \alpha$

das, was Red. ds der Molekel berechnet wird, ist theilweise die Verkürzung des Weges in Folge der grössten Geschwindigkeit im Perihel, theils Wirkung der Deviation.

VI.

bei quadrat. Gesetz!

$$r = \frac{\frac{c^2}{\mu}}{1+\sqrt{1+\frac{2c^2C}{\mu^2}}\cos\varphi}$$

$\parallel C: v_0, r_0, \alpha$

$$C = \frac{v_0^2}{2} - \frac{\mu}{r_0}$$

$\mu = k(m+m_1)$

$$c = \frac{1}{2} r_0 v_0 \sin\alpha$$

$$\cos\varphi\sqrt{1+\frac{2c^2C}{\mu^2}} = -1$$

$$\varphi_0 = \arccos\left(\frac{-1}{\sqrt{1+\frac{2c^2C}{\mu^2}}}\right)$$

$r = \infty$

$$\varphi_0 = \frac{\pi}{2} + \arcsin\frac{1}{\sqrt{1+\frac{2c^2C}{\mu^2}}}$$

$$r = \frac{p}{1+\varepsilon\cos\varphi}$$

$$\frac{\frac{c^4}{\mu^2}}{\left(1+\sqrt{1+\frac{2c^2C}{\mu}}\cos\varphi\right)^2}\,d\varphi = c\,dt$$

$$\int\frac{A}{(1+B\cos\varphi)^2}\,d\varphi = A\left\{\frac{1}{-\left(1-\sqrt{1+\frac{2c^2C}{\mu}}\right)\frac{2c^2C}{\mu}}\right\}$$

$$\left\{\frac{\left(1+\frac{2c^2C}{\mu}-\sqrt{1+\frac{2c^2C}{\mu}}\right)\sin\varphi}{1+\sqrt{1+\frac{2c^2C}{\mu}}\cos\varphi} + (1-\sqrt{\ })\underbrace{\int\frac{d\varphi}{1+\sqrt{\ }\cos\varphi}}\right\}$$

$$= -\frac{1}{(1-\sqrt{\ })\,\mathrm{tg}\frac{\varphi}{2}}$$

$$= A\left\{\frac{1}{\frac{2c^2C}{\mu}\left(\sqrt{\ }-1\right)}\right\}\left\{\left(1+\frac{2c^2C}{\mu}-\sqrt{\ }\right)\sin\varphi\right.$$

Periheldistanz $r(\mathrm{Min}) =$ $\varphi = 0$

$$r = \frac{\frac{c^2}{\mu}}{1+\sqrt{1+\frac{2c^2C}{\mu}}}$$

$$2\left(\frac{r_0 \sin\varphi_0 - r\sin\varphi}{\sin\beta \quad v_0} - \Big|_{r_0}^{r}\right) = \ldots$$

$\lim_{r_0=\infty}$

$$\sin\beta = \cos \arcsin \frac{1}{\sqrt{}} = \sqrt{1-\sin^2 \arcsin}$$

$$= \sqrt{1 - \frac{1}{1+\frac{2c^2C}{\mu^2}}} = \frac{\frac{2c^2C}{\mu^2}}{\sqrt{1+\frac{2c^2C}{\mu^2}}}$$

$$r\left(1+\sqrt{}\cos\varphi\right) = \frac{c^2}{\mu}$$

$$\cos\varphi = \frac{\frac{c^2}{\mu}-r}{r\sqrt{}}$$

$$\sin\varphi = \sqrt{1 - \frac{\left(\frac{c^2}{\mu}-r\right)^2}{r^2\left(1-\frac{2c^2C}{\mu^2}\right)}} = \frac{\sqrt{2\frac{c^2r}{\mu} - \frac{c^4}{\mu^2} - \frac{2c^2Cr^2}{\mu^2}}}{r\sqrt{}}$$

$$= \frac{\sqrt{\frac{c^2}{\mu}\left(2r - \frac{c^2}{\mu} - 2Cr^2\right)}}{r\sqrt{}}$$

$$\operatorname{tg}\frac{\varphi}{2} = \sqrt{\frac{1-\cos\varphi}{1+\cos\varphi}} = \sqrt{\frac{r\sqrt{} - \frac{c^2}{\mu}+r}{r\sqrt{} + \frac{c^2}{\mu} - r}}$$

$$= \frac{\sqrt{1-\cos^2\varphi}}{1+\cos\varphi} = \frac{\sin\varphi}{1+\cos\varphi} \qquad \frac{1}{\operatorname{tg}\frac{\varphi}{2}} = \sqrt{\frac{1+\cos\varphi}{1-\cos\varphi}} = \frac{\sin\varphi}{1-\cos\varphi}$$

$$\left\{ \frac{\left(1+\frac{2c^2C}{\mu}+\sqrt{}\right)\sin\varphi}{1+\sqrt{}\cos\varphi} - \frac{\sin\varphi}{1-\cos\varphi} \right\}$$

$$= \frac{2c^2C}{\mu})\sin\varphi + \sqrt{}\sin\varphi - (\quad)\sin\varphi\cos\varphi - 2\sqrt{}\sin\varphi\cos\varphi$$

$$\overline{(1+\sqrt{}\cos\varphi)(1-\cos\varphi)}$$

$$= \frac{\frac{2c^2C}{\mu} + \sqrt{1+\frac{2c^2C}{\mu}} - \left(1+\frac{2c^2C}{\mu}+2\sqrt{1+\frac{2c^2C}{\mu}}\right)\cos\varphi}{(1+\sqrt{}\cos\varphi)(1-\cos\varphi)}$$

$$\sin\varphi_0 = \sin(\beta+\alpha)$$

$$= \sin\beta\cos\alpha + \cos\beta\sin\alpha$$

$$= \cos\alpha\,\frac{\frac{2c^2C}{\mu^2}}{\sqrt{}} - \sin\alpha\,\frac{1}{\sqrt{}}$$

$$\sin\varphi_0 = \frac{\frac{2c^2C}{\mu^2}\cos\alpha - \sin\alpha}{\sqrt{1+\frac{2c^2C}{\mu^2}}}$$

$$= \frac{\frac{C}{\mu^2}\,\frac{1}{2}r_0^2 v_0^2 \sin^2\alpha\cos\alpha - \sin\alpha}{\sqrt{}}$$

$$= \frac{\sin\alpha}{\sqrt{}}\left(\frac{C}{4\mu^2}r_0^2 v_0^2 \sin 2\alpha - 1\right)$$

$$\cos\varphi_0 = \sqrt{\frac{1 + \frac{2c^2C}{\mu^2} - \frac{4c^4C^2}{\mu^4}\cos^2\alpha + \frac{4c^2C}{\mu^2}\cos\alpha\sin\alpha - \sin^2\alpha}{1 + \frac{2c^2C}{\mu^2}}}$$

$$= 1 + \frac{2c^2C}{\mu^2} - \frac{4c^4C^2}{\mu^4} + \frac{4c^4C^2}{\mu^4}\sin^2\alpha - \sin^2\alpha + \frac{4c^2C}{\mu^2}\cos\alpha\sin\alpha$$

$$= \sqrt{\left(1 - \frac{4c^4C^2}{\mu^4}\right)\left(1 - \sin^2\alpha\right) + \frac{2c^2C}{\mu^2}\left(1 + 2\sin\alpha\cos\alpha\right)}$$

$$\sin\rho)\cos\varphi_0 = \sqrt{\frac{\left(1 - \frac{4c^4C^2}{\mu^4}\right)\cos^2\alpha + \frac{2c^2C}{\mu^2}\left(1 + \sin 2\alpha\right)}{1 + \frac{2c^2C}{\mu^2}}}$$

$$= \sqrt{\left(1 - \frac{2c^2C}{\mu^2}\right)\cos^2\alpha + \frac{\frac{2c^2C}{\mu^2}\left(1 + \sin 2\alpha\right)}{1 + \frac{2c^2C}{\mu^2}}}$$

Die Bentlenungung durch die $\frac{1}{r^2}$ Kraft gegenüber einer Stoß-Reflexion lässt sich folgendermaßen berechnen:

wenn (Stoß) ohne Kraft so bewegt sich Körper mit Geschw. v in Linie ABC

dabei Flächengeschw. constant

$c = \frac{p v}{2}$. [p = Perpendikel von AB auf O]

$c t = \frac{p}{2} v t$

$F = \frac{p}{2} s$

also Zeit = $\frac{\text{Sector Fläche}}{\text{Flächengeschw.}}$

dagegen bei Anziehungen zwar dieselbe Flächengeschwindigkeit aber es entfällt die Fläche zwischen Asympt. und Hyperbel.

also Zeitgewinn = $\frac{\text{Fläche } ABCD}{\text{Flächengeschw.}}$

$$\rho = \frac{p}{1 + \varepsilon \cos\varphi}$$

$$\frac{F}{2} = \int \frac{\rho^2 d\varphi}{2} = \frac{p}{2} \int_0^{\varphi_0} \frac{d\varphi}{(1 + \varepsilon \cos\varphi)^2}$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$y = \frac{b}{a}\sqrt{x^2 - a^2}$$

$$\int y\,dx = \frac{b}{a}\int\sqrt{x^2-a^2}\,dx$$

$$J = x\sqrt{x^2-a^2} - \int\frac{x^2}{\sqrt{x^2-a^2}}\,dx$$

$$= J + \int\frac{a^2\,dx}{\sqrt{x^2-a^2}}$$

$$2J = x\sqrt{x^2-a^2} - a^2\int\frac{dx}{\sqrt{x^2-a^2}}$$

$$J = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left(x+\sqrt{x^2-a^2}\right)$$

$$\int y\,dx = \frac{bx}{2}\sqrt{\left(\frac{x}{a}\right)^2-1} - \frac{ab}{2}\log\left(\frac{x}{a}+\sqrt{\left(\frac{x}{a}\right)^2-1}\right)$$

$$= \frac{xy}{2} - \frac{ab}{2}\log\left(\frac{x}{a}+\frac{y}{b}\right)$$

die Fläche ABCD $= \dfrac{ab}{2}\log\left(\dfrac{x}{a}+\dfrac{y}{b}\right)\Bigg|_{x=0}^{x=\infty}$

$$= \frac{ab}{2} + \frac{ab}{2}\log\left(\frac{x}{a}+\frac{y}{b}\right)\Bigg|_{x=a}^{x=\infty} = \infty\,!!!$$

Zeitgemin wird also ∞; daher muss man als obere Grenze die ~~mittlere Weglänge~~ [Entfernung] λ einführen

$$\frac{F(ABCD)}{2} = \frac{ab}{2}\left\{1+\log\left(\frac{\lambda}{a}+\frac{y_\lambda}{b}\right)\right\} = \frac{ab}{2}\left\{1+\log\frac{\lambda}{a}+\log\left(1+\frac{a}{b}\frac{y_\lambda}{x_\lambda}\right)\right\}$$

$$=(\text{angenähert})\ \frac{ab}{2}\left\{1+\log\frac{2\lambda}{a}\right\}$$

$\mathfrak{R}$ = Entfernung $\cdot \frac{a}{\sqrt{a^2+b^2}}$

$$r = \frac{\frac{c^2}{\mu}}{1+\cos\varphi\sqrt{1+\frac{2c^2C}{\mu^2}}}$$

$$c = \frac{r_0 v_0 \sin\alpha}{2}$$

$$C = \frac{v_0^2}{2} - \frac{\mu}{r_0}$$

$$\mu = k(m+m_1)$$

φ β α r_0

$$\beta = 180^0 - \varphi_{As}$$

$$\varphi_{As} = 90^0 + \arcsin\frac{1}{\sqrt{1+\frac{2c^2C}{\mu^2}}}$$

daraus: $\beta = \arccos\frac{1}{\sqrt{1+\frac{2c^2C}{\mu^2}}}$ $\cos\beta = \frac{1}{\varepsilon}$

~~$\sqrt{a^2+b^2} = e^2 =$~~

~~$(r)_{\varphi=0} = \sqrt{a^2+b^2} - a = \frac{\frac{c^2}{\mu}}{1+\sqrt{1+\frac{2c^2C}{\mu}}}$~~ $= a\left[\sqrt{1+\left(\frac{b}{a}\right)^2} - 1\right]$

$$\frac{b}{a} = \operatorname{tg}\beta = \operatorname{tg}\arccos\frac{1}{\sqrt{}} = \frac{\sqrt{1-\frac{1}{1+\frac{2c^2C}{\mu^2}}}}{\frac{1}{\sqrt{1+\frac{2c^2C}{\mu^2}}}} = \sqrt{\frac{2c^2C}{\mu^2}}$$

$$= \sqrt{\frac{1+\frac{2c^2C}{\mu^2}-1}{1+\cdots}}$$

~~$= \sqrt{\frac{\frac{2c^2C}{\mu^2}}{1+\frac{2c^2C}{\mu^2}}} = \frac{1}{\sqrt{1+\frac{\mu^2}{2c^2C}}}$~~

$$\frac{\frac{c^2}{\mu}}{1+\sqrt{1+\frac{2c^2C}{\mu^2}}} = a\left[\sqrt{1+\frac{2c^2C}{\mu^2}} - 1\right]$$

$\frac{c^2}{\mu} = \frac{2c^2C}{\mu^2}a$

$a = \frac{\mu}{2C}$

$$\frac{c^2}{\mu} = a\left\{\sqrt{\ldots} - 1 + \ldots\sqrt{1+\frac{2c^2C}{\mu^2}}\right\}$$

oder auch auf folgende Weise:

$a : r_0 = \sin\alpha : \sin\beta$

$a = \lim_{r_0=\infty} \frac{r_0 \sin\alpha}{\sin\beta}$

$$\sin\beta = \sqrt{\frac{2c^2C}{1+\frac{2c^2C}{\mu^2}}}$$

$$\frac{1}{\sin\beta} = \sqrt{\frac{1+\frac{2c^2C}{\mu^2}}{2c^2C}} = \frac{1}{\mu}\sqrt{1+\frac{1}{2c^2C}} = \frac{1}{\mu}\left[1+\frac{1}{c^2C}+\ldots\right]$$

$a = \lim \frac{r_0 \sin\alpha}{\mu}\left(1+\frac{1}{c^2C}\right)$?

$\lim = 0$

$= \frac{2c}{v_0\mu}$

$$\sqrt{a^2+b^2} = \sqrt{\frac{\mu^2}{4C^2}+\frac{c^2}{2C}} = \frac{\sqrt{\mu^2+2c^2C}}{2C}$$

$$= \frac{\mu}{2C}\sqrt{1+\frac{2c^2C}{\mu^2}}$$

$a = \frac{\mu}{2C}$

$b = \sqrt{\frac{c^2}{2C}}$

$= \frac{c}{\sqrt{2C}}$

$$\frac{\sqrt{a^2+b^2}}{a} = \varepsilon = \sqrt{1+\frac{2c^2C}{\mu^2}}$$ stimmt!

$$\Delta T = \frac{F}{c} = \frac{ab}{c}\left\{1 + \log \frac{2l}{a}\right\}$$

$$= \frac{\mu}{\sqrt{(2C)^3}}\left\{1 + \log 2\right.$$

l = Entfernung des Punktes vom ~~Focus = Entfernung vom gemeinsamen Schwerpunkt = doppelte Entfg der beiden M.~~ Mittelpunkt $\cdot \frac{a}{\sqrt{a^2+b^2}}$

(angenähert) ~~von einander~~

$$l = \left\{\sqrt{a^2+b^2} + \text{Entfernung vom Focus} \right\} \frac{a}{\sqrt{a^2+b^2}}$$

" Schwerpunkt

halbe ~~doppelte~~ Entfg der beiden P. von einander " λ

$$= a + \frac{a\lambda}{2\sqrt{a^2+b^2}} = a\left\{1 + \frac{\lambda}{2\sqrt{a^2+b^2}}\right\}$$

$$= a\left\{1 + \frac{\lambda}{2\frac{\mu}{2C}\sqrt{1+\frac{2c^2C}{\mu^2}}}\right\}$$

$$\Delta T = \frac{\mu}{\sqrt{(2C)^3}}\left\{1 + \log 2\left\{1 + \frac{\lambda}{\frac{\mu}{C}\sqrt{1+\frac{2c^2C}{\mu^2}}}\right\}\right\}$$

$$\int_0^R 2\pi r\, dr\, V \frac{2 v_0^2 r \mu}{\mu^2 + v_0^4 r^2} = 4\pi v_0^2 \mu V \int_0^R \frac{r^2\, dr}{\mu^2 + v_0^4 r^2}$$

$$= 4\pi \frac{\mu V}{v_0^2}\left(1 - \frac{\mu^2}{\mu^2 + v_0^4 r^2}\right)$$

$$= 4\pi\mu \frac{V}{v_0^2}\left[R - \frac{\mu}{v_0^2} \operatorname{arctg} \frac{v_0^2 R}{\mu}\right]$$

Drucke $\frac{\lambda}{\frac{\mu}{c}\sqrt{}}$ ist = $\frac{\text{Entfernung der beiden } \sigma}{\text{Excentr. der Bahn}}$ also sehr gross

daher transform

$$\frac{\Delta T}{r} = \frac{\mu}{(2C)^{\frac{3}{2}}}\left\{1 + \log 2 + \log\frac{\lambda}{\frac{\mu}{C}\sqrt{1+\frac{2c^2C}{\mu^2}}} + \log\left\{1+\frac{\frac{\mu}{c}\sqrt{}}{\lambda}\right\}\right\}$$

$$= \frac{\mu}{(2C)^{\frac{3}{2}}}\left\{1+\log\frac{2\lambda}{\frac{\mu}{C}\sqrt{1+\frac{2c^2C}{\mu^2}}} + \underbrace{\log(1+\delta)}_{\neq 0}\right\}$$

$$\longrightarrow \quad \lambda = 2r_0$$

$$\Delta T = \frac{\mu}{(2C)^{\frac{3}{2}}}\left\{1+\log\frac{4r_0C}{\frac{\mu}{}\varepsilon}\right\}$$

$$\varepsilon = \sqrt{1+\frac{2c^2C}{\mu^2}} = \sqrt{1+\frac{2r_0^2v_0^2\sin^2\alpha}{2\cdot 4\mu^2}\left(\frac{v_0^2}{2}-\frac{\mu}{r_0}\right)}$$

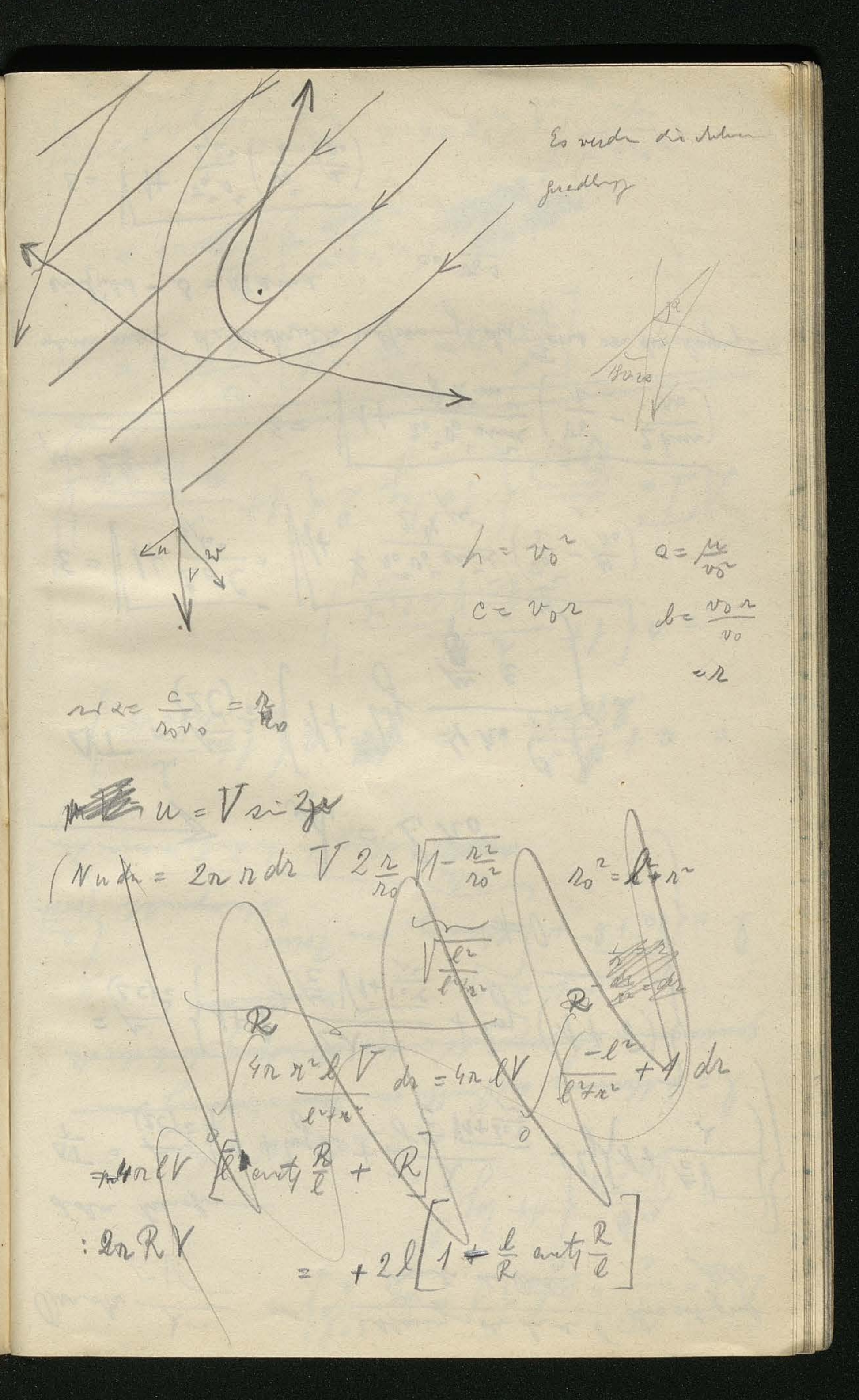
$\mu = 2km$

$$\varepsilon = \sqrt{1+\frac{r_0^2v_0^2\sin^2\alpha}{8k^2m^2}\left(\frac{v_0^2}{2}-\frac{2km}{r_0}\right)}$$

wenn man die senkrechte Entfernung des Focus von der Asymptote einführt $- s = r_0\sin\alpha$

$$c = \frac{v_0 s}{2}$$

$$\varepsilon = \sqrt{1+\frac{v_0^2s^2}{2\mu^2}\left(\frac{v_0^2}{2}-\frac{\mu}{r_0}\right)}$$



Es werden die Bahnen
gradlinig
$h = v_0^2$
$a = \frac{\mu}{v_0^2}$
$c = v_0 r$
$b = \frac{v_0 r}{v_0}$
$u = V \sin 2\varphi$
$N u\,dr = 2\pi r\,dr\, V\, 2\frac{r}{r_0}\sqrt{1-\frac{r^2}{r_0^2}}$
$r_0^2 = l^2 + r^2$
$\int_0^R 4\pi r^2 l \frac{V}{l^2+r^2}\,dr = 4\pi l V \int_0^R \left(\frac{-l^2}{l^2+r^2} + 1\right) dr$
$4\pi l V \left[l \arctan\frac{R}{l} + R \right]$
$: 2\pi R V$
$= +2l\left[1 + \frac{l}{R}\arctan\frac{R}{l}\right]$

$r = r_0 \frac{\pi}{4} e^{-\frac{2a\mu}{r_0}}$

α, α, r_0, β, μ, F

$a = \frac{\mu}{h}$

$b = \frac{c}{\sqrt{h}}$

$h = v_0^2 - \frac{2\mu}{r_0}$

$c = r_0 v_0 \sin\alpha$

$r_0 \sin\alpha = r$

$c = v_0 r$

$r = \frac{c}{v_0}$

$c = b\sqrt{h}$

$= b\sqrt{\frac{\mu}{a}}$

$=$

$u = \sqrt{2 - 2y} = \sqrt{\sin^2(\beta - \alpha)}$

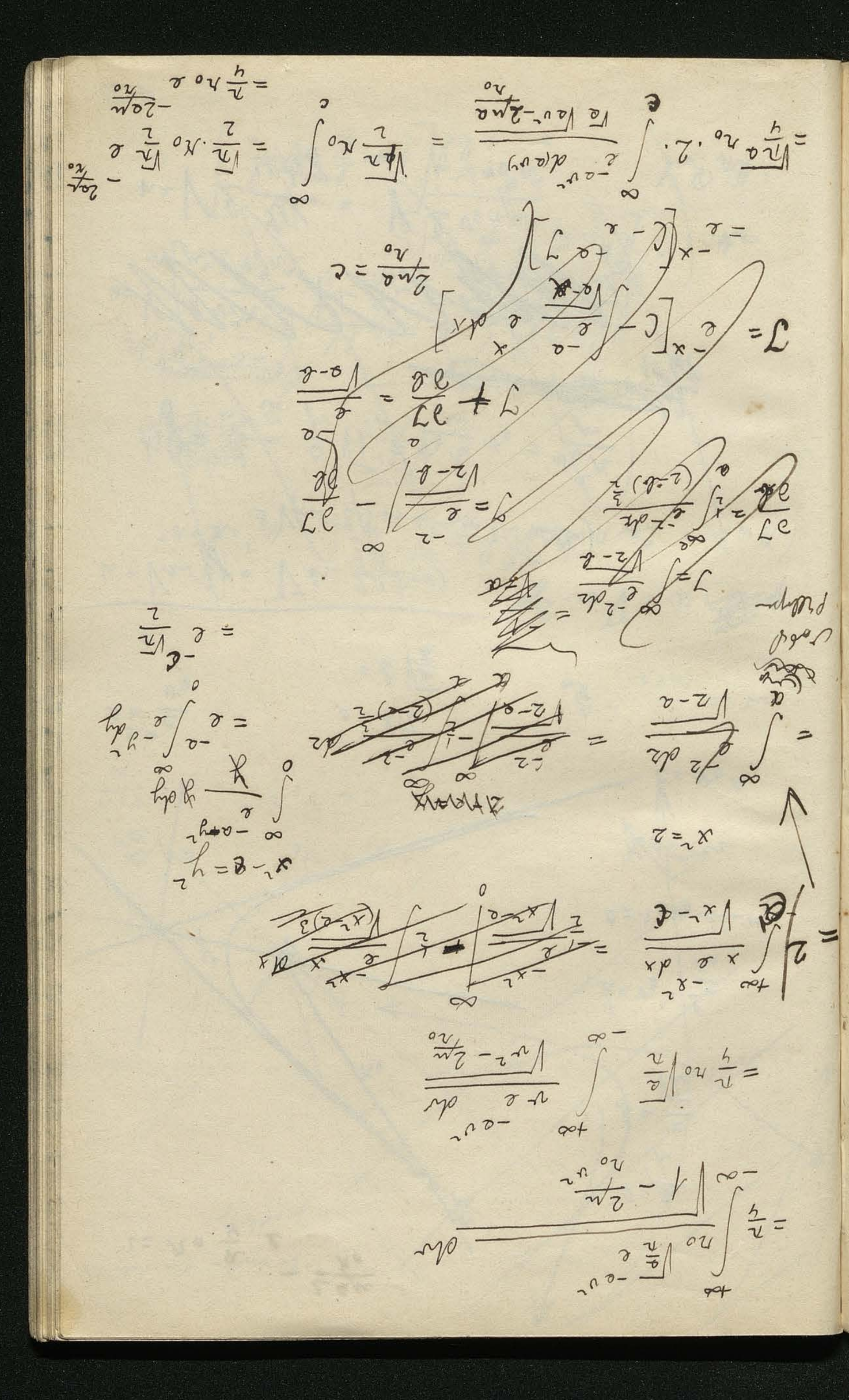
$= \sqrt{(\sin 2\beta \sin\alpha - \sin 2\beta \sin\alpha)}$

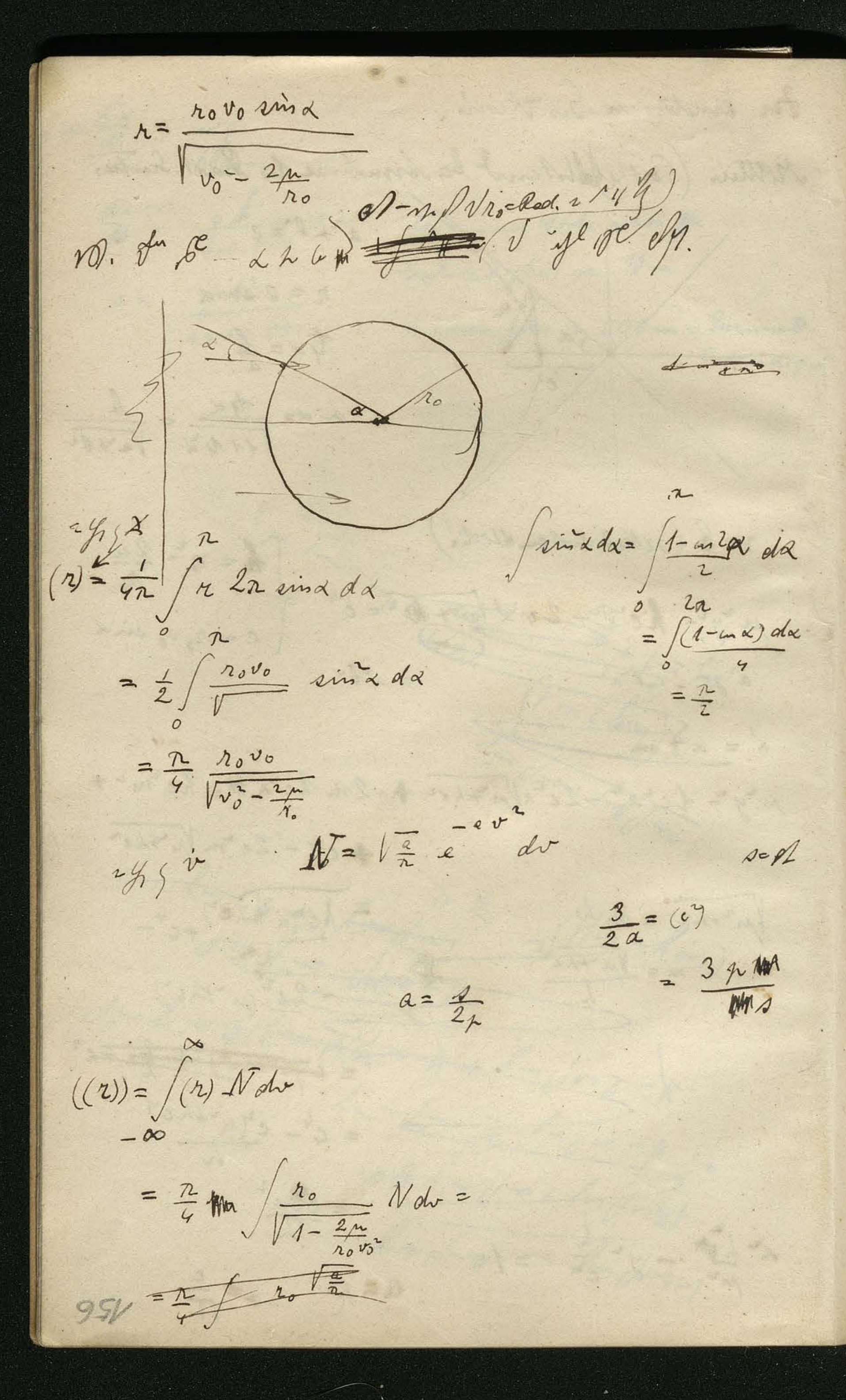
$\operatorname{tg}\beta = \frac{b}{a} = \frac{c}{\mu}\sqrt{h} = \frac{v_0 r}{\mu} v_0 = \frac{v_0^2 r}{\mu}$

~~$\beta = v_0^2 r$~~ ~~$\sin\beta = \frac{\operatorname{tg}\beta}{\sqrt{1+\operatorname{tg}^2\beta}}$~~

~~$u = \sqrt{\left[\frac{2 v_0^2 r \mu}{\mu^2 + v_0^4 r^2}\sqrt{1 - \frac{r^2}{r_0^2}} - \dots\right]}$~~

$u = \sqrt{2 \frac{\operatorname{tg}\beta}{1 + \operatorname{tg}^2\beta}} = \sqrt{\frac{2 v_0^2 r \mu}{\mu^2 + v_0^4 r^2}}$





$$r = \frac{r_0 v_0 \sin\alpha}{\sqrt{v_0^2 - \frac{2\mu}{r_0}}}$$

10.

$$(r) = \frac{1}{4\pi}\int_0^{\pi} r \, 2\pi \sin\alpha \, d\alpha$$

$$= \frac{1}{2}\int_0^{\pi} \frac{r_0 v_0}{\sqrt{}} \sin^2\alpha \, d\alpha$$

$$= \frac{\pi}{4} \frac{r_0 v_0}{\sqrt{v_0^2 - \frac{2\mu}{r_0}}}$$

$$\int_0^{\pi} \sin^2\alpha \, d\alpha = \int_0^{\pi} \frac{1-\cos 2\alpha}{2} \, d\alpha$$

$$= \int_0^{2\pi} \frac{(1-\cos\alpha)\, d\alpha}{4}$$

$$= \frac{\pi}{2}$$

$$N = \sqrt{\frac{a}{\pi}}\, e^{-a v^2}\, dv$$

$$\frac{3}{2a} = (c^2)$$

$$= \frac{3\, p}{s}$$

$$a = \frac{s}{2p}$$

$$((r)) = \int_{-\infty}^{\infty} (r)\, N \, dv$$

$$= \frac{\pi}{4} \int \frac{r_0}{\sqrt{1 - \frac{2\mu}{r_0 v_0^2}}} \, N \, dv =$$

$$\frac{\pi}{4}\int r_0 \sqrt{\frac{a}{\pi}}$$

Zur kinetischen Gastheorie.

Mittlerer (Perihelabstand bei Annahme des Quadrat. Gesetzes

$$a^2 + b^2 = e^2$$

$$r = e \sin\alpha$$

$$\operatorname{tg}\alpha = \frac{b}{a}$$

$$\sin\alpha = \frac{\operatorname{tg}\alpha}{\sqrt{1+\operatorname{tg}^2\alpha}} = \frac{b}{\sqrt{a^2+b^2}}$$

$r = b$ (selbstverständlich!)

$$\mu^2 y^2 - hc^2 x'^2 + 2c^2 x' \sqrt{\mu^2 + hc^2} = c^4$$

$$\left\{ \begin{aligned} h &= v_0^2 - \frac{2\mu}{r_0} \\ c &= r_0 v_0 \sin\alpha \end{aligned} \right.$$

$$a^2 y^2 - b^2 x^2 \qquad = a^2 b^2$$

$$x' = x + m$$

$$\mu^2 y^2 - hc^2 x^2 + 2c^2 x \sqrt{\mu^2 + hc^2} - 2m\, hc^2 x = hc^2 m^2 + c^4 - 2c^2 m \sqrt{\mu^2 + hc^2}$$

$$\sqrt{\mu^2 + hc^2} = mh$$

$$m = \frac{\sqrt{\mu^2 + hc^2}}{h}$$

$$= \frac{hc^2(\mu^2 + hc^2)}{h^2} + c^4 - \frac{2c^2}{h}(\mu^2 + hc^2)$$

$$= \sout{c^2(\mu^2 + hc^2)\left(1 - \frac{2}{h}\right) + c^4}$$

$$= c^4 - \frac{c^2(\mu^2 + hc^2)}{h}$$

$$= -\frac{c^2 \mu^2}{h}$$

$$x^2 \frac{h^2}{\mu^2} - y^2 \frac{h}{c^2} = 1$$

$$a = \frac{\mu}{h} \qquad b = \frac{c}{\sqrt{h}}$$

$1 - \cos\beta\cos\gamma = \sin\beta\sin\gamma$

~~$1 \geqq \cos\beta\cos\gamma + \sin\beta\sin\gamma$~~

~~$1 - \cos\beta\cos\gamma \geqq \sin\beta\sin\gamma$~~

$1 - 2\cos\beta\cos\gamma + \cos^2\beta\cos^2\gamma$ ~~$(1-\sin^2\beta)(1-\sin^2\gamma)$~~ $\geqq \sin^2\gamma\sin^2\beta$

~~$1 - \cos^2\beta - \cos^2\gamma \geqq 0$~~

$(\cos\gamma - \cos\beta)^2 \geqq 0$ ~~$\cos^2\beta\cos^2\gamma$~~ $\geqq 1 - \cos^2\gamma - \cos^2\beta$

~~$1 - \cos\beta\cos\gamma + \sin\beta\sin\gamma$~~

~~$2(\log 2 - 1)$~~

$\log 2 - 1 + \frac{1}{2}$ ~~$(2+1)$~~ ~~$\log$~~ $+ \log \ldots - 1$

$m\frac{d^2x}{dt^2} = \frac{mg}{600.000}$

$\frac{dx}{dt} = \frac{gt}{600.000} + c$ also in einer Woche schon [illegible] Geschwindigkeit von 10 m

$x = \frac{gt^2}{1.200.000}$

in einem Jahre = 500 m = Gesammtgeschwindigkeit

Einfluss der Rotation

$$\frac{k}{p} = \frac{~~12.1710~~ ~~19~~}{8.2.40.000.~~24.26~~} = \frac{1}{600.000}$$

$$\frac{d\varphi}{dt} = -\frac{\frac{d\varphi}{dt}}{600.000}$$

$$\frac{\frac{d^2\varphi}{dt^2}}{\frac{d\varphi}{dt}} = -\frac{1}{600000}$$

$$\lg \frac{d\varphi}{dt} = -\frac{1}{60000} t$$

$$\frac{d\varphi}{dt} = e^{-\frac{t}{600000}}$$

24. 60. 60

36.4
72
144
86400

$$\frac{4.75.1000}{60^2}$$

$\varphi = 45^0 \quad \sin\varphi = \cos\varphi = \frac{1}{\sqrt{2}}$

$$\Delta T_0^{6h} = -\frac{6.60.60}{10.000}\frac{1}{2\sqrt{2}}\left[\sin\delta + \frac{2}{\pi}\cos\delta\right]$$

$$= -\frac{2{\cdot}16}{2\sqrt{2}}\left[\sin\delta + \frac{2}{\pi}\cos\delta\right]$$

Maximum im Sommer = $\frac{1}{\sqrt{2}}$

Mittelwerth : $\Delta T_0^{6h} = -\frac{2}{3}\frac{1}{\sqrt{2}}$ sec $\# \frac{1}{2}$ sec $\quad \frac{2}{\sqrt{2}} = 1{\cdot}4$ sec!

also im halben Tag $\neq 1$ sec!

(Normaler Werth am Morgen und Abend)

$\varphi = 90^0$ NordPol:

$$\Delta T_0^{6h} = -6^h \frac{f}{2p}\sin\delta$$

$$= -\frac{6.60.60}{10.000}\cdot\frac{1}{2}\sin\delta = \frac{2{\cdot}16}{2}\sin\delta$$

$\neq \sin\delta$

$$\Delta T_0^{24h} = 4\sin\delta$$

$$\Delta T_0^{91t} = \underbrace{4.91}_{365} \; M(\sin\delta)_0^{\varepsilon} \text{ sec}$$

$$\frac{1-\cos\varepsilon}{\varepsilon} = \frac{1-\cos 23^0 30'}{23^0 30'} \Big/ \frac{90^0.2}{\pi}$$

$$= \frac{083 \,.\, 18 \,.\, 2}{47.3}$$

36 :
249
498
2988 : 141 = 0·21
168

$$\Delta T_0^{91t} = 365.0{\cdot}21$$

$\frac{730}{365}$ also in einem Halbjahr

$= 76$ sec $\quad 153$ sec $= 2\frac{1}{2}$ Minuten!

(Normaler Werth im Sommer und Winter)

der Wendekreise täglich zweimal Maximum = 20·6″ in der Richtung, wo Sonne vor einem ¼ Jahr auf- und untergegangen war und zwar nach O um 12h Nachts ± ~

W Mittag ± ~

Componente in der Vertikalen

$$I = f \sin h$$

$$= [\sin\delta \sin\varphi + \cos\delta \cos\varphi \cos t] f \qquad \Big\|\quad \tau = \pi\sqrt{\frac{l}{g}} \qquad \Delta\tau = \tau\left(\frac{I}{2\mu}\right)$$

$$\Delta T_{\odot}^{\frac{\pi}{2}} = \int_0^{\frac{\pi}{2}} dt \left[\frac{f}{2\mu}(\sin\delta \sin\varphi + \cos\delta \cos\varphi \cos t)\right]$$

$$= -\frac{f}{2\mu}\left[\sin\delta \sin\varphi \cdot \frac{\pi}{2} + \cos\delta \cos\varphi\right]$$

$$\Delta T_0^{6h} = -\frac{6h}{\frac{\pi}{2}} \frac{f}{2\mu}\left[\frac{\pi}{2}\sin\delta \sin\varphi + \cos\delta \cos\varphi\right]$$

$$= -6^h \frac{f}{2\mu}\left[\sin\delta \sin\varphi + \frac{2}{\pi}\cos\delta \cos\varphi\right]$$

z.B. $\varphi = 0$ Äquator

$$\Delta T_0^{6h} = -\frac{6^h}{10.000}\frac{1}{2}\frac{2}{\pi}\cos\delta$$

Maximum Äquinoctien

$$= -\frac{6.60.60}{10.000}\frac{1}{\pi}\cos\delta = -\frac{2\cdot16}{3\cdot14}\cos\delta = -\frac{2}{3}\cos\delta^{sec}$$

$$H^2\{\underbrace{\cos^2 A \cos^2\varphi + \sin^2\varphi}\} + 2f\cos A\cos\varphi\sin\delta \cdot H$$

$$= f^2(\sin^2\varphi - \sin^2\delta)$$

$\cos^2 A\cos^2\varphi + 1 - \cos^2\varphi$

$= 1 - \cos^2\varphi\sin^2 A$

$$H^2 + 2H\frac{f\cos A\cos\varphi\sin\delta}{1-\cos^2\varphi\sin^2 A} = f^2 \ldots$$

$$H = -\frac{f\cos A\cos\varphi\sin\delta}{\cos^2 A\cos^2\varphi + \sin^2\varphi} \pm \sqrt{\frac{f^2(\sin^2\varphi - \sin^2\delta)}{\cos^2 A\cos^2\varphi + \sin^2\varphi} + \frac{f^2\cos^2 A\cos^2\varphi\sin^2\delta}{(\quad)^2}}$$

$$= \frac{-f\cos A\cos\varphi\sin\delta \pm \sqrt{f^2(\sin^2\varphi - \sin^2\delta)(\cos^2 A\cos^2\varphi + \sin^2\varphi) + f^2\cos^2 A\cos^2\varphi\sin^2\delta}}{\cos^2 A\cos^2\varphi + \sin^2\varphi}$$

Bewegungs Richtung dort, wo Sonne vor einem Vierteljahr, um 6 Stunden später war. Somit für alle Punkte innerhalb

Componente welche in die Horizontale fällt:

$$H = f \cos h$$

$$\left[\sin h = \sin\delta \sin\varphi + \cos\delta \cos\varphi \cos t\right.$$

$$\cos A = \frac{-\sin\delta + \sin h \sin\varphi}{\cos h \cos\varphi}$$

$$H = fc.(A)$$

$$\cos h = \frac{H}{f}$$

~~$A^2 \cos^2\varphi \cos^2\varphi = \sin^2\delta$~~

$$\cos A = \frac{-\sin\delta + \sin\varphi\sqrt{1 - \frac{H^2}{f^2}}}{\frac{H}{f}\cos\varphi}$$

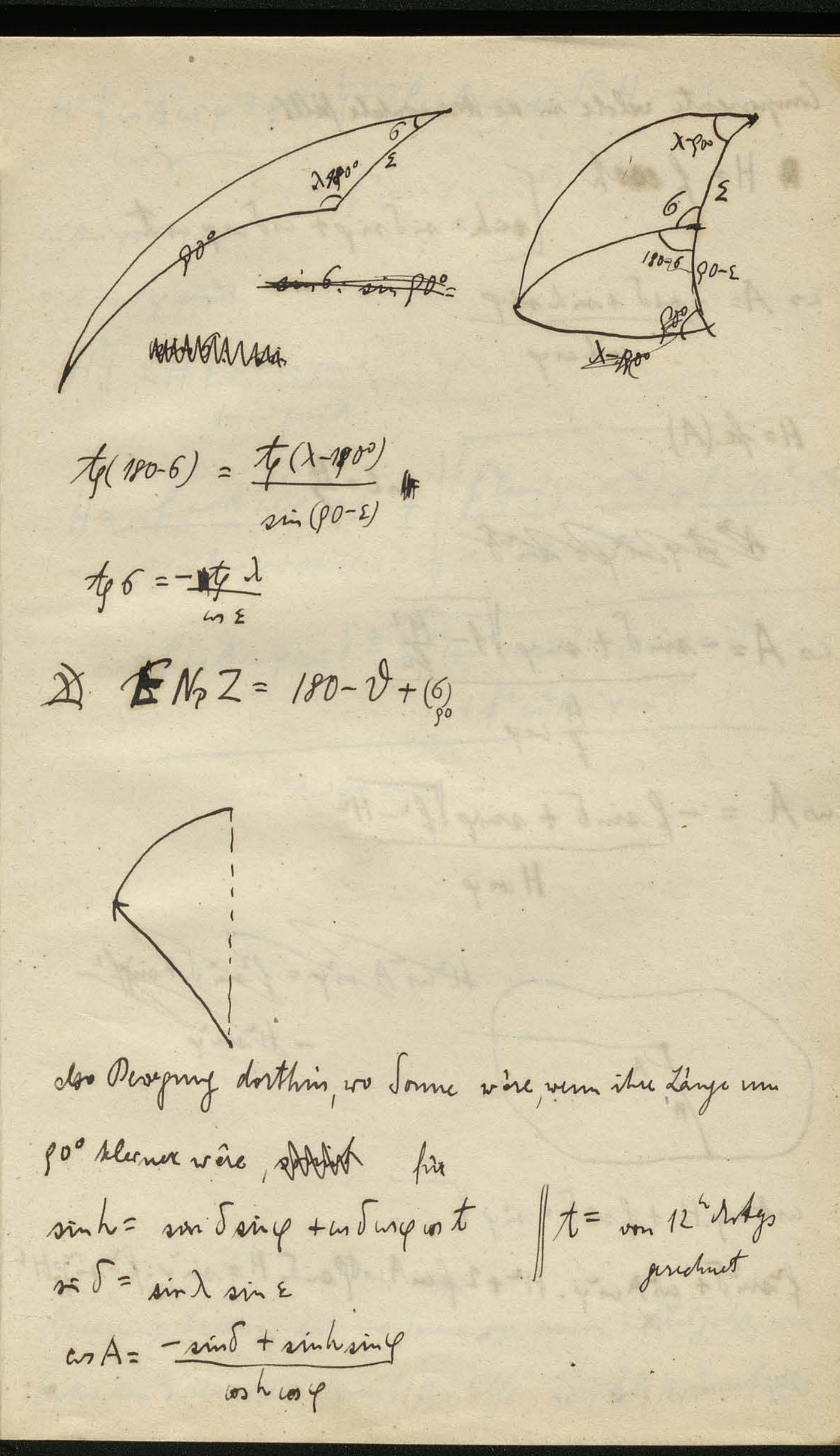
$$\cos A = \frac{-f\sin\delta + \sin\varphi\sqrt{f^2 - H^2}}{H\cos\varphi}$$

~~$H^2\cos^2 A \cos^2\varphi = f^2\sin^2\delta + \sin^2\varphi f^2$~~

~~$- H^2\sin^2\varphi$~~

$$\cos A \cos\varphi . H + f\sin\delta = \sin\varphi\sqrt{}$$

$$f^2\sin^2\delta + \cos^2 A\cos^2\varphi . H^2 + 2f\cos A\cos\varphi\sin\delta . H = \sin^2\varphi . f^2 - \sin^2\varphi . H^2$$



$$tg(180-\sigma) = \frac{tg(\lambda-180^0)}{\sin(90-\varepsilon)}$$

$$tg\,\sigma = -\frac{tg\,\lambda}{\cos\varepsilon}$$

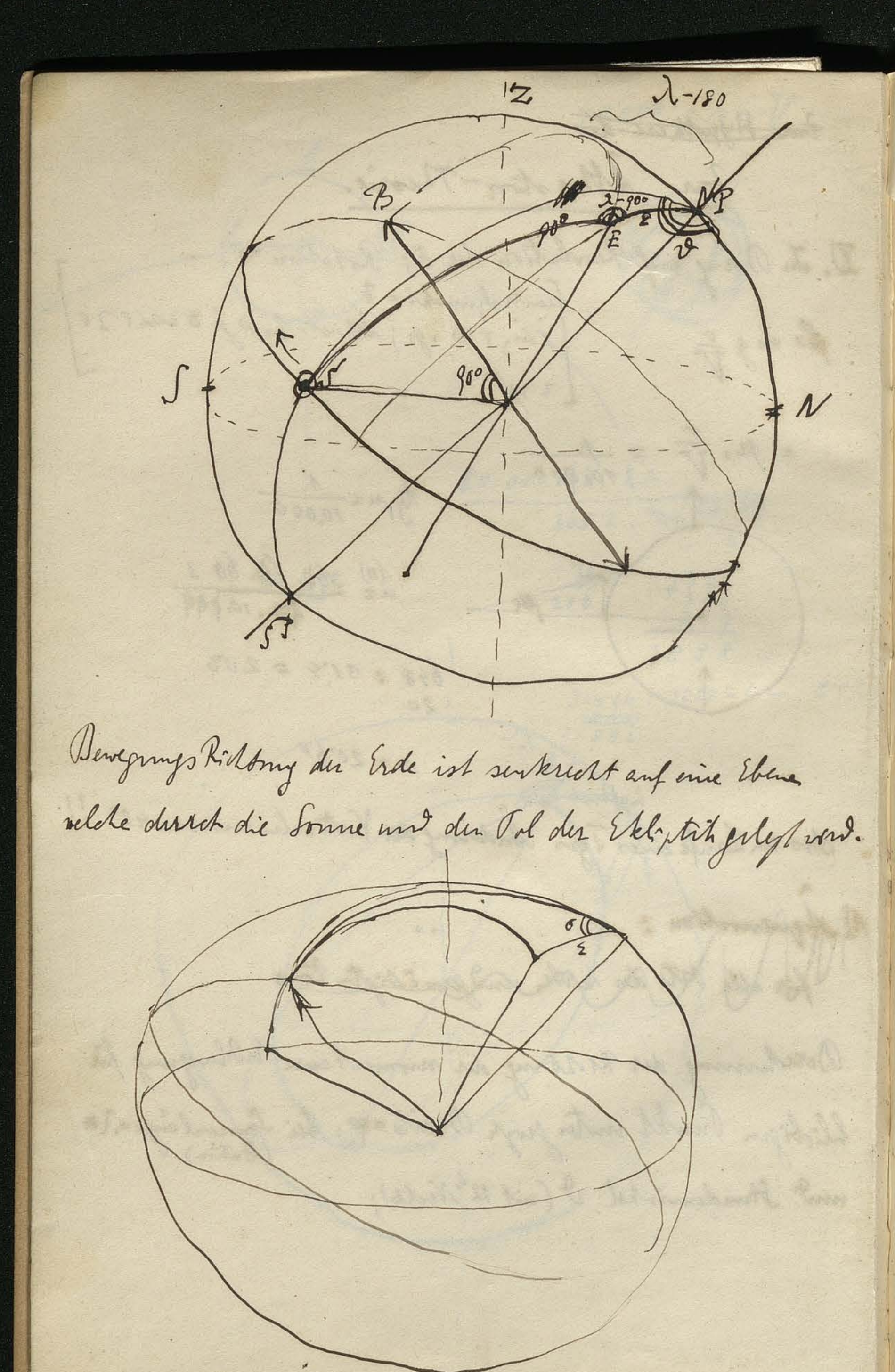
$\angle\,N_p Z = 180 - \vartheta + (\sigma)_{90}$

also Bewegung dorthin, wo Sonne wäre, wenn ihre Länge um 90° kleiner wäre, für

$\sin h = \sin\delta\sin\varphi + \cos\delta\cos\varphi\cos t$ ‖ t = von 12h Mittags gerechnet

$\sin\delta = \sin\lambda\sin\varepsilon$

$$\cos A = \frac{-\sin\delta + \sin h\sin\varphi}{\cos h\cos\varphi}$$



Bewegungs Richtung der Erde ist senkrecht auf eine Ebene welche durch die Sonne und den Pol der Ekliptik gelegt wird.

~~Zur Hypothese des~~

Zur Ätherstoss-Theorie.

I. In Bezug auf Revolution der ♁; Rotation = 0.

$f = m g \frac{v}{V}$

[Centralpunkt = ♁
[illegible] ⊙ [illegible]; Ätherst. [illegible] ♁ [illegible]
[illegible]]

$= p \cdot \frac{v}{V} = \frac{p}{10.000}$

$tg\,\mu = \frac{1}{10.000}$

$\mu = \frac{360 \cdot 60 \cdot 60 \cdot 3}{2\pi \cdot 10.000}$ (11)

$648 : 31{,}4 = 20{,}6$
20

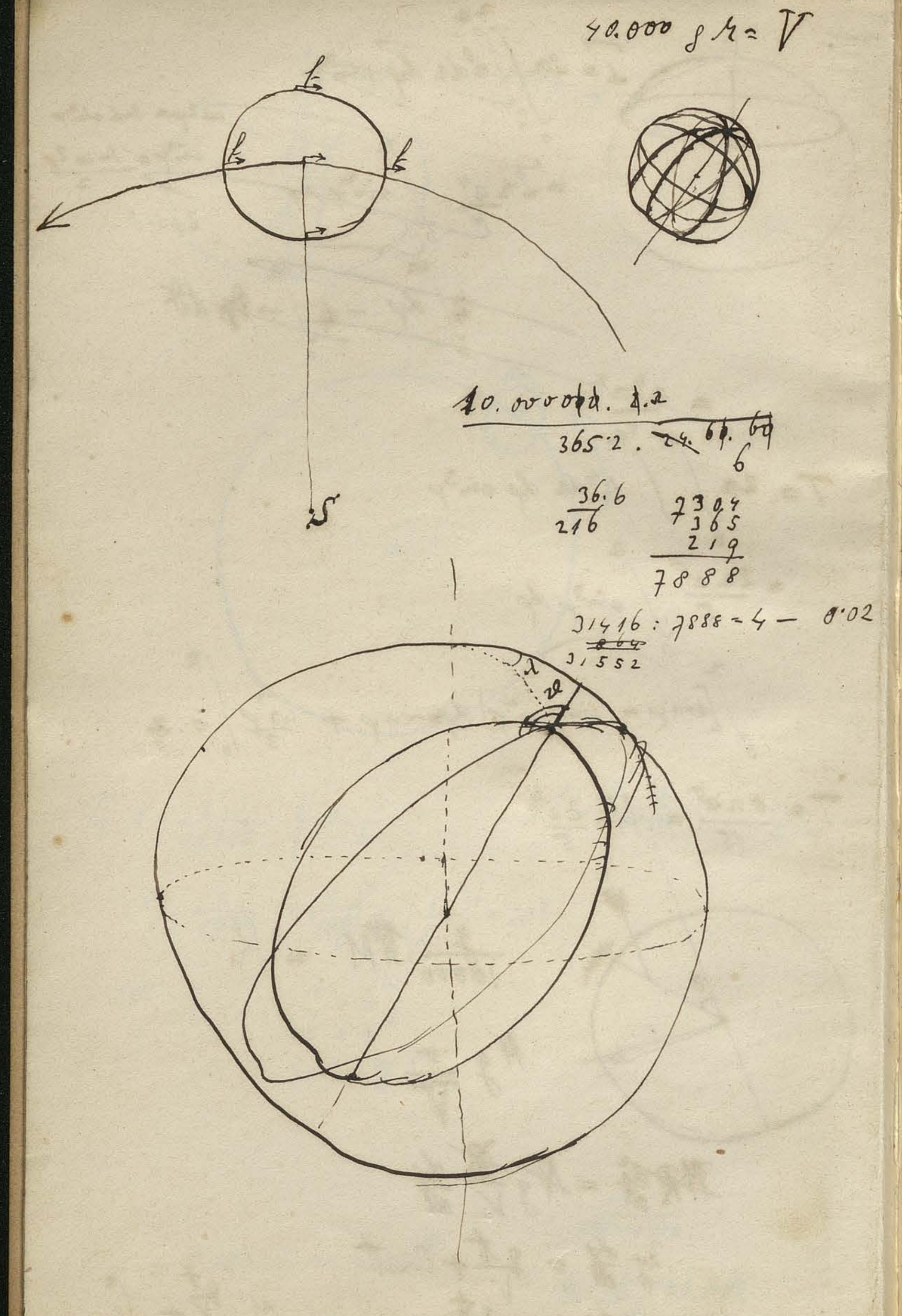
$\mu = 20{,}6''$

also im Laufe eines (halben) Tages Änderung der Verticalen um ca 40" !!

A) Äquinoctien :

~~für alle Orte der heissen und gemässigten Zone~~

Berechnung der Richtung der momentanen Erdbewegung für beliebigen Punkt unter geogr. Breite = φ; bei Sonnenlänge = λ* (Datum); und Stundenwinkel ϑ (seit 12^h Nachts);


365·2 . 24. 60. 60
6
36.6
216
7304
365
219
7888
31416 : 7888 = 4 — 0·02
31552

$$T = 2\pi \int_0^a \int_0^\pi r^3 dr\, d\varphi \sin^2\varphi$$

$$\cos 2\varphi = 1 - 2\sin^2\varphi \qquad \sin^2\varphi = \frac{1-\cos 2\varphi}{2} \qquad 2\varphi = -\vartheta$$

$$= 2\pi \frac{a^4}{4} \int_0^\pi \sin^2\varphi\, d\varphi$$

$$\frac{1}{4}\int_0^{2\pi} d\varphi - \frac{1}{4}\int_0^{\pi} \cos \vartheta\, d\vartheta$$

$$= \frac{a^4 \pi^2}{4\pi}$$

$$T = 2\pi \int_0^a \int_0^\pi r^4 dr\, d\varphi \sin^3\varphi$$

$$= \frac{2\pi e^5}{5} \int_0^\pi \sin^3\varphi\, d\varphi$$

$$\int_0^\pi [\sin\varphi - \sin\varphi \cos^2\varphi]\, d\varphi = -\cos\varphi + \frac{\cos^3\varphi}{3}\Big|_0^\pi = \frac{4}{3}$$

$$T = \frac{8\pi e^5}{15} = M \cdot \frac{2e^2}{5}$$

θ

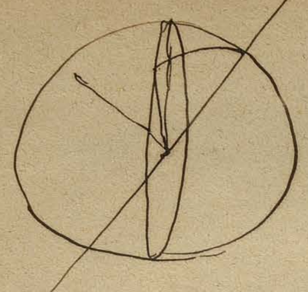
$$\frac{g}{10000} \cdot M = K$$

$$M g \frac{v}{V}$$

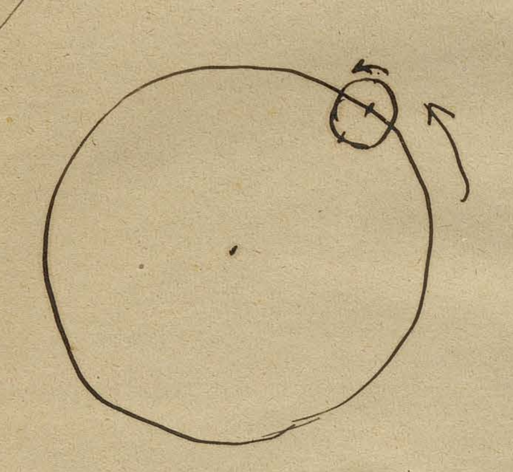
$$MR \frac{d\varphi^2}{dt} = M g \frac{R}{V} \frac{d\varphi}{dt}$$

$$\log \frac{d\varphi}{dt} = g\frac{t}{V} + \text{Const}$$

$$\frac{d\varphi}{dt} = C e^{-g\frac{t}{V}} \qquad \varphi = C' e^{g\frac{t}{V}} + C$$



$\frac{2}{3}\ 0{\cdot}917 + 0{\cdot}399$



$\operatorname{tg}\ \varepsilon = \frac{1}{10000}$ ‖ $1^{0} = \frac{1}{60}$ $\qquad$ $\frac{6 \cdot 60 \cdot 60}{10000} \cdot 4$ $\qquad$ $\frac{14{\cdot}4}{14\ \text{sec}}$

$\varepsilon = \frac{6}{1000} = \left(\frac{1}{167}\right)^{0} = \left(\frac{1}{2{\cdot}5}\right)' = 20''$

$20''$

$1^{0} = 4^{m}$

$1' = 4^{s}$

$20'' = \frac{4}{3}^{s}$

$\frac{1}{390000000}$

$= 500000 : 25 = 20000^{t}$

$=$

$t = \frac{10{,}000.000}{2}$

innerer — äußeren Druck bei festen Körpern

Capillaritätskonstante derselben bestimmen [illegible]

[illegible] Stefan = [illegible] = [illegible] etc.

164

$1 + \vartheta \cos\varphi = x$ $\qquad \cos\varphi = \frac{x-1}{\vartheta}$ $\qquad \sin\varphi = \frac{\sqrt{\vartheta^2 - (x-1)^2}}{\vartheta}$

$-\vartheta \sin\varphi \, d\varphi = dx$

$d\varphi = \frac{-dx}{\sqrt{\vartheta^2 - (x-1)^2}}$

$\frac{1}{1+\vartheta\cos\varphi} = x$ $\qquad 1 = x + \vartheta x \cos\varphi$

$\cos\varphi = \frac{1-x}{\vartheta x}$ $\qquad \sin\varphi = \frac{\sqrt{\vartheta^2 x^2 - (1-x)^2}}{\vartheta x}$

$\frac{-\vartheta \sin\varphi \, d\varphi}{(1+\vartheta\cos\varphi)^2} = dx$

$J = -\int \frac{x\,dx}{\sqrt{(\vartheta^2-1)x^2 + 2x - 1}} = \frac{-1}{\vartheta^2-1}\sqrt{(\vartheta^2-1)x^2+2x-1} + \frac{1}{\vartheta^2-1}\int\frac{dx}{\sqrt{}}$

$= \frac{1}{2\sqrt{\vartheta^2-1}} \lg \left\{ \frac{2 + 2(\vartheta^2-1)x + 2\sqrt{}}{2 + 2(\vartheta^2-1)x - 2\sqrt{(\vartheta^2-1)[(\vartheta^2-1)x^2+2x-1]}} \right.$

$\sqrt{} = \sqrt{\frac{\vartheta^2-1}{(1+\vartheta\cos\varphi)^2} + \frac{2}{1+\vartheta\cos\varphi} - 1}$

$= \frac{\vartheta^2 - 1 + 2 + 2\vartheta\cos\varphi - 1 - 2\vartheta\cos\varphi - \vartheta^2\cos^2\varphi}{(1+\vartheta\cos\varphi)^2}$

$= \frac{\vartheta \sin\varphi}{1+\vartheta\cos\varphi}$

$J = \frac{1}{1-\vartheta^2}\frac{\vartheta\sin\varphi}{1+\vartheta\cos\varphi} + \frac{1}{2(\vartheta^2-1)^{\frac{3}{2}}} \lg \frac{1 + \frac{\vartheta^2-1}{1+\vartheta\cos\varphi} + \frac{\sqrt{\vartheta^2-1}\,\cdot\,\vartheta\sin\varphi}{1+\vartheta\cos\varphi}}{1 + \qquad - \qquad}$

$\frac{\cos\varphi + \vartheta + \sqrt{\vartheta^2-1}\sin\varphi}{\cos\varphi + \vartheta - \sqrt{\vartheta^2-1}\sin\varphi}$

$$\frac{v_0^2}{2} - \frac{\mu}{r_0} = 0$$

$$v_A^2 = \frac{2\mu}{r_0}$$

$$r_0 = \frac{2\mu}{v_0^2} = \frac{2\rho v_0^2}{\sqrt{2}\, v^2} = \sqrt{2}\,\rho \frac{v_0^2}{v^2}$$

$$\frac{\rho v_0^2}{\sqrt{2}} = 2k\rho^2 \tfrac{4}{3} x\, s$$

$$k = \frac{v_0^2}{10 \ldots \rho^2 s}$$

$$k\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \right.$$
$$+ \frac{1}{2^2} + \frac{1}{3^2} \cdots$$
$$+ \frac{1}{3^2} + \cdots$$
$$+ \cdots$$

$$\frac{B^2 - 1 + B - [B^2 + 2B]\cos\varphi}{(1 + B\cos\varphi)(1-\cos\varphi)} = \frac{(B^2+2B)\sin\varphi}{(1+B\cos\varphi)(1-\cos\varphi)} \ldots \frac{[B^2-1+B-(B^2+2B)\cos\varphi]}{(1+B\cos\varphi)^2(1-\cos\varphi)^2}$$

$$[(1+B\cos\varphi)\sin\varphi - B\sin\varphi(1-\cos\varphi)]$$

$$2B\cos\varphi\sin\varphi$$

$$= \frac{(B^2+2B)(1+B\cos\varphi)(1-\cos\varphi) - [B^2-1+B-(B^2+2B)\cos\varphi]\, 2B\cos\varphi}{(1+B\cos\varphi)^2(1-\cos\varphi)^2} \quad B\sin\varphi$$

$$2 + B\cos\varphi - 2\cos\varphi - 2B\cos^2\varphi$$
$$+ B + B^2\cos\varphi - [illegible] - B^2\cos^2\varphi - [illegible]$$

$$= 2 + B \; [illegible] - \cos\varphi + \cos^2\varphi\,[illegible]$$

$$\frac{(B + 2 - \cos\varphi)\; B\sin\varphi}{(1+B\cos\varphi)^2(1-\cos\varphi)^2}$$

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[illegible] Infanterie Regiment Kaiser No. 1

[illegible]

Programm

der dritten zwanglosen Zusammenkunft

1. „Soldaten Geist" [illegible] Marsch

2. Ouverture „Dichter und Bauer" von Franz von Suppé

[illegible] Dichter [illegible]

3. „Eine pikante Geschichte" Corp. Reber

4. „Airs variés" für Violine Spanhauser

5. „Komischer Vortrag" Schmutzek

6. „Komische Vorträge [illegible]" [illegible]

7. „Couplets" Müller

8. „Gimpellied" aus der Operette [illegible] [illegible]

[illegible]

9. „Ein Ständchen in der Leihbibliothek"

[illegible] Sachmann

10. „Deutsche Tänze" für Violine von Schubert

[illegible] Corp. Spanhauser

11. [illegible] aus der Operette „Der Vogelhändler" Zeller

12. „Komische Vorlesung" [illegible] Müller

13. „Couplets"

14. „Obersteiger Marsch" von Zeller

Wien, am 16. April 189[illegible] Anfang [illegible]

